

Doctoral Thesis

Massive Neutrinos and Cosmology

By

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RESUMO

Os neutrinos desempenham um papel fundamental tanto na física de partículas quanto na cosmologia, fornecendo uma conexão importante entre a física das escalas muito pequenas e das maiores escalas do Universo. Embora os experimentos de oscilação de neutrinos tenham estabelecido firmemente que os neutrinos são massivos, sua escala de massa absoluta e o mecanismo responsável por sua geração de massa permanecem desconhecidos. Na cosmologia, os neutrinos deixam marcas distintas na evolução do Universo, afetando observáveis importantes, como a Radiação Cósmica de Fundo e a estrutura em larga escala. Nesta tese, investigamos o papel dos neutrinos na evolução do Universo e usamos observações cosmológicas para restringir suas propriedades em uma variedade de estruturas teóricas. Analisamos o impacto das massas dos neutrinos na formação de estruturas e exploramos sua relação com extensões do modelo cosmológico padrão, incluindo cenários inflacionários e modelos de energia escura dinâmica. Nossos resultados mostram que os dados cosmológicos fornecem limites rigorosos para a soma das massas dos neutrinos, oferecendo informações importantes e complementares aos experimentos de laboratório. Em particular, demonstramos como diferentes hipóteses cosmológicas, como a natureza da energia escura ou a inclusão da curvatura, podem afetar essas restrições. Apresentamos também previsões para futuros levantamentos de galaxias, destacando seu potencial para aprimorar ainda mais a sensibilidade à massa total dos neutrinos.

ABSTRACT

Neutrinos play a fundamental role in both particle physics and cosmology, providing an important bridge between the physics of the very small and the largest scales of the Universe. While neutrino oscillation experiments have firmly established that neutrinos are massive, their absolute mass scale and the mechanism responsible for their mass generation remain unknown. In cosmology, neutrinos leave distinctive imprints on the evolution of the Universe, affecting key observables such as the Cosmic Microwave Background and the large-scale structure. In this thesis, we investigate the role of neutrinos in the evolution of the Universe and use cosmological observations to constrain their properties within a variety of theoretical frameworks. We analyze the impact of neutrino masses on structure formation and explore their correlation with extensions of the standard cosmological model, including inflationary scenarios and dynamical dark energy models. Our results show that cosmological data provide stringent bounds on the sum of neutrino masses, offering important and complementary information to laboratory experiments. In particular, we demonstrate how different cosmological assumptions, such as the nature of dark energy or the inclusion of curvature, can affect these constraints. We also present forecasts for future galaxy surveys, highlighting their potential to further improve the sensitivity to the absolute neutrino mass.

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INTRODUCTION

Back in 1930, a tiny, neutral, and nearly undetectable particle, emitted alongside the electron in beta decay, was hypothesized by Wolfgang Pauli to explain the continuous energy spectrum of electrons observed in this process. In 1934, Enrico Fermi developed the first complete theoretical description of beta decay, incorporating this particle into his formalism and naming it the neutrino. Even though Fermi beta-decay theory would be already very well accepted, the neutrino was definitively detected only in 1956 by the so-called “Poltergeist” experiment, led by Frederick Reines and Clyde Cowan.

In 1958, Maurice Goldhaber, Lee Grodzins, and Andrew Sunyar measured the neutrino helicity, demonstrating that neutrinos are left-handed, with their spin oriented opposite to their momentum. By the mid-1970s, the Standard Model of particle physics (SM) was established, incorporating neutrinos as massless particles. From this initial picture of a massless, neutral, and weakly interacting particle, neutrino physics has undergone a profound transformation. The establishment that neutrinos are massive particles through oscillation experiments represents one of the major achievements in modern particle physics and provides compelling evidence for the incompleteness of the SM. We now know that there are three neutrino flavors, and oscillation experiments have demonstrated that at least two of them are massive. Although the precise values of the individual neutrino masses remain undetermined, oscillations experiments have established lower bounds on the sum of their masses. Determining the absolute neutrino mass scale and understanding the origin of neutrino masses remain among the most important open questions in physics.

Massive neutrinos have been shown to play a fundamental role not only in particle physics but also in the evolution of the Universe. Neutrinos are the second most abundant particles in the Universe, behind only the photons of the Cosmic Microwave Background (CMB). Since the late 1990s, observations of Type Ia supernovae have established that the expansion of the

Universe is accelerating, implying the existence of a component with negative pressure, commonly referred to as dark energy. The standard cosmological model, Λ CDM, describes a spatially flat Universe composed of dark energy, dark matter, baryonic matter, radiation, and neutrinos. In this framework, dark energy is modeled as a cosmological constant, corresponding to a constant vacuum energy density, and the model provides an excellent fit to a wide range of observations. Throughout cosmic history, neutrinos behave as both relativistic and a non-relativistic particle. Due to their large number density, they contribute a non-negligible fraction to the total energy density of the Universe. As a result, neutrinos leave distinct imprints through the Universe evolution. They affect both the expansion rate and the formation of structures.

In the early Universe, neutrinos behave as relativistic particles and therefore contribute to the radiation energy density. At this stage, the number of neutrino species, usually parametrized by the effective number of relativistic degrees of freedom, N_{eff} , affects the expansion rate and key cosmological observables, such as the angular power spectrum of the CMB, and the production of light elements during Big Bang nucleosynthesis. At late times, when neutrinos transition from relativistic to non-relativistic particles, they behave as a form of warm dark matter. This occurs when their masses become comparable to, and eventually larger than, their thermal velocity ($m_\nu \gtrsim T_\nu$), allowing them to contribute to the matter energy density. However, their thermal velocities remain significant even after this transition. As a result, neutrinos are unable to efficiently cluster within gravitational potential wells, they instead suppress the growth of cold dark matter perturbations at large wave numbers k , or small scales. Consequently, the absolute neutrino mass leaves an important imprint on the shape of the matter power spectrum observable. Therefore, the Universe itself provides a powerful laboratory for probing the fundamental properties of these particles, offering insights that are complementary to those obtained from terrestrial experiments. In this chapter, we discuss the impact of neutrinos on the evolution of the Universe.

1.1 Neutrino Physics

Understanding the fundamental properties of neutrinos is essential for uncovering their role in both particle physics and cosmology. In this section, we analyze neutrinos from a theoretical and phenomenological perspective, focusing on their interactions within the framework of the SM. We examine their involvement in weak interactions, highlight the experimental discovery that they are massive, contrary to the original SM assumption, and discuss theoretical extensions beyond the SM that aim to explain the origin and nature of these masses, particularly the so-called seesaw mechanism.

1.1.1 The Standard Model of Particle Physics

To begin with, we introduce the highly successful theoretical framework that describes the Universe at its most fundamental level, the SM. The SM unifies three of the four known fundamental interactions, the electromagnetic, weak, and strong forces, leaving gravity as the only fundamental interaction not incorporated. The SM is fundamentally governed by a non-Abelian gauge symmetry group:

$$(1.1) \quad G_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y,$$

where C, L and Y stands for colour, left-handed chirality and weak hypercharge. Each factor corresponds to one of the gauge interactions: strong force under $SU(3)_C$, the weak force under $SU(2)_L$, and hypercharge under $U(1)_Y$, which combines with $SU(2)_L$ to yield electromagnetism via spontaneous symmetry breaking.

To understand the Standard Model, one must begin with the symmetries that define its underlying structure. The already mentioned local gauge symmetry, which dictates the form of interactions and determines the structure of the Lagrangian. In addition, the SM is built upon Poincare symmetry, the symmetry group of Minkowski spacetime, which constrains the field content to include scalar fields, spinor fields (fermions), and vector fields (gauge bosons). Global symmetries also play an important role. These include approximate symmetries such as baryon number (B) and lepton number (L) conservation, which arise dynamically from the SM's gauge structure and renormalizable interactions. It also includes various approximate flavor symmetries. Finally, discrete symmetries, charge conjugation (C), parity (P), and time reversal (T), and their combinations (notably CP and CPT). While C and P are individually and jointly maximally violated in the weak sector, the CPT theorem guarantees that any local, Lorentz-invariant quantum field theory is invariant under the combined CPT transformation. This result remains one of the cornerstones of modern quantum field theory (QFT). Our following analysis will focus exclusively on the electroweak sector, since neutrinos, like all leptons, are color-neutral and do not interact strongly.

1.1.1.1 Spontaneous Symmetry Breaking and the Higgs Mechanism for fermions

Symmetries dictate the structure of physical laws, but their "breaking" profoundly alters observable phenomena. In QFT, symmetry breaking manifests in two distinct ways, each with unique implications. The first one is an explicit symmetry breaking, a symmetry is explicitly broken if the action (or Lagrangian) contains terms that are not invariant under the symmetry transformation. The second is the spontaneous symmetry breaking (SSB), this happens if the action is invariant under the symmetry, but the vacuum state (ground state) is not. The symmetry persists in the dynamics but is "hidden" in the vacuum structure. This mechanism plays a central role in the SM, particularly in the electroweak sector, where it is responsible for generating the masses of the weak gauge bosons and all charged fermions. In the absence of SSB, the gauge invariance of

the electroweak theory under $SU(2)_L \times U(1)_Y$ would forbid any explicit mass terms for the $W^\pm$ and Z^0 bosons, contradicting experimental observations. The solution is to allow the vacuum itself to spontaneously break the symmetry. This is achieved by introducing a complex scalar field known as the Higgs field which transforms as a doublet under $SU(2)_L$:

$$(1.2) \quad \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \Phi \sim (1, 2)_{1/2}.$$

The dynamics of this field are governed by the Higgs potential:

$$(1.3) \quad V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2,$$

where $\lambda > 0$ ensures the potential is bounded from below, and $\mu^2 < 0$ ensures the minimum occurs away from the origin. Minimizing the potential yields a non-zero vacuum expectation value (VEV) for the neutral component:

$$(1.4) \quad \langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \text{with} \quad v = \sqrt{-\mu^2/\lambda} \simeq 246 \text{ GeV}.$$

This VEV breaks the electroweak symmetry:

$$(1.5) \quad SU(2)_L \times U(1)_Y \longrightarrow U(1)_{\text{EM}},$$

where the residual $U(1)_{\text{EM}}$ corresponds to electromagnetism, generated by the conserved electric charge $Q = T_3 + Y$.

The spontaneous breaking of gauge symmetry results in the Higgs mechanism: three of the four gauge bosons, $W^\pm$ and Z^0 , acquire mass by 'eating' the corresponding Goldstone bosons, which become the longitudinal degrees of freedom of the massive vector fields. The photon remains massless as the gauge boson of the unbroken $U(1)_{\text{EM}}$. The masses of the gauge bosons are:

$$(1.6) \quad m_W = \frac{1}{2} g v,$$

$$(1.7) \quad m_Z = \frac{1}{2} \sqrt{g^2 + g'^2} v,$$

$$(1.8) \quad m_\gamma = 0,$$

where g and g' are the gauge couplings of $SU(2)_L$ and $U(1)_Y$, respectively. Beyond the gauge sector, the Higgs mechanism provides a framework for fermion mass generation through Yukawa couplings. For a given Dirac fermion ψ , the generation of a mass term requires a formal coupling between its left-handed (ψ_L) and right-handed (ψ_R) chiral components. In this context, it is critical to distinguish between chirality and helicity. Chirality is an intrinsic, Lorentz-invariant property that dictates how a field enters the Lagrangian and interacts with gauge bosons. Helicity, the projection of spin onto momentum, is a physical observable that is frame-dependent for massive

particles. While these two concepts coincide for massless particles, a massive particle must possess both chiralities to maintain a consistent physical identity across all reference frames; because a Lorentz boost can "overtake" a massive particle and flip its helicity, the underlying Lagrangian must provide both chiral "building blocks" to describe the state. In the SM, fermion masses arise from Yukawa interactions with the Higgs field,

$$(1.9) \quad \mathcal{L}_{\text{Yukawa}} = -y_f \bar{\psi}_L \Phi \psi_R + \text{h.c.}$$

where ψ_L and ψ_R denote the left- and right-chiral components of the fermion field. After spontaneous symmetry breaking, the Higgs field acquires a vacuum expectation value, $\langle \Phi \rangle = v/\sqrt{2}$, generating a Dirac mass term,

$$(1.10) \quad m_f = \frac{y_f v}{\sqrt{2}}.$$

While this mechanism successfully accounts for the masses of charged fermions, it cannot be directly applied to neutrinos within the minimal Standard Model. Experimental observations show that weak interactions involve only left-handed neutrinos and right-handed antineutrinos [Goldhaber et al. \[1958\]](#). Accordingly, the SM includes left-handed lepton doublets and right-handed charged lepton singlets,

$$(1.11) \quad L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (1, 2)_{-1/2}, \quad e_R \sim (1, 1)_{-1},$$

but does not introduce a right-handed neutrino field ν_R . As a consequence, it is not possible to construct a renormalizable, gauge-invariant Yukawa interaction of the form $\bar{L} \tilde{\Phi} \nu_R$, where $\tilde{\Phi} = i\sigma_2 \Phi^*$. Therefore, no Dirac mass term for neutrinos can be generated through the Higgs mechanism in the minimal Standard Model.

One might attempt to construct a Majorana mass term using only the left-handed neutrino field already present in the Standard Model, through an operator of the form $\overline{(\nu_L)^C} \nu_L$. However, since ν_L is a component of the lepton doublet $L \sim (1, 2)_{-1/2}$, such a bilinear cannot be arranged into an $SU(2)_L \times U(1)_Y$ gauge singlet. As a result, no renormalizable, gauge-invariant operator of this form can be constructed within the Standard Model.

The description of neutrinos as massless particles is therefore not an ad hoc assumption, but a structural prediction of the SM. It follows directly from the experimentally established chiral nature of the weak interaction, the absence of right-handed neutrino fields, and the requirement of gauge invariance. This prediction, however, stands in direct conflict with the experimental observation of neutrino oscillations, as will be discussed in the next section.

1.1.1.2 Neutrino Oscillations

Neutrino oscillations arise from the fact that neutrino flavor eigenstates are quantum superpositions of mass eigenstates. As a consequence, a neutrino produced in a given flavor state (e.g.,

ν_e) can change its flavor as it propagates. This phenomenon is evidence for nonzero neutrino masses and mixing, and it cannot be explained within the minimal Standard Model. The relation between the flavor eigenstates ν_α ($\alpha = e, \mu, \tau$) and the mass eigenstates ν_i ($i = 1, 2, 3$) is encoded in the unitary Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix $U_{\alpha i}$,

$$(1.12) \quad \nu_\alpha = \sum_{i=1}^3 U_{\alpha i} \nu_i.$$

The standard parameterization of the PMNS matrix is given by

$$(1.13) \quad U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\text{CP}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\text{CP}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\text{CP}}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\text{CP}}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\text{CP}}} & c_{23}c_{13} \end{pmatrix},$$

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$ and δ_{CP} is the Dirac CP-violating phase. In the simplified two-flavor scenario, the probability for a neutrino of flavor α to oscillate into a neutrino of flavor β after traveling a distance L with energy E is given by:

$$(1.14) \quad P_{\nu_\alpha \rightarrow \nu_\beta} = \sin^2(2\theta) \sin^2\left(\frac{\Delta m_{ij}^2 L}{E}\right),$$

where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ denotes the mass-squared difference between the neutrino mass eigenstates, and θ is the effective mixing angle. For a full three-flavor description, the oscillation probabilities depend on all three mixing angles θ_{12} , θ_{23} , θ_{13} , the CP-violating phase δ_{CP} , and the two independent mass-squared differences Δm_{21}^2 and Δm_{31}^2 (or Δm_{32}^2). A wide range of experiments have precisely established neutrino oscillations, providing measurements of the mixing angles and mass-squared differences, as well as constraints on the CP-violating phase [Navas et al. \[2024\]](#). The current best-fit values are

$$(1.15) \quad \begin{aligned} \sin^2\theta_{12} &= 0.307 \pm 0.012, & \sin^2\theta_{23} &= 0.534_{-0.019}^{+0.015} (0.537 \pm 0.020), \\ \sin^2\theta_{13} &= (2.16 \pm 0.06) \times 10^{-2}, \\ \Delta m_{21}^2 &= (7.50 \pm 0.19) \times 10^{-5} \text{ eV}^2, \\ \Delta m_{32}^2 &= (2.451 \pm 0.026) \times 10^{-3} \text{ eV}^2 \quad (-2.527 \pm 0.034) \times 10^{-3} \text{ eV}^2, \end{aligned}$$

where, for $\sin^2\theta_{23}$ and Δm_{32}^2 , the values correspond to normal (inverted) mass ordering. The CP-violating phase is currently constrained to $\delta_{\text{CP}} = (1.21_{-0.22}^{+0.19})\pi$, although with relatively large uncertainties.

Oscillation experiments are insensitive to the absolute neutrino mass scale. However, the measured mass-squared differences imply a robust lower bound on the sum of neutrino masses. For the normal hierarchy (NH), $m_1 < m_2 \ll m_3$, one finds $\sum m_\nu \gtrsim 0.06 \text{ eV}$, while for the inverted hierarchy (IH), $m_3 \ll m_1 < m_2$, the bound becomes $\sum m_\nu \gtrsim 0.10 \text{ eV}$. These observations establish that neutrinos are massive particles and that their flavor eigenstates mix, providing clear

evidence for physics beyond the minimal Standard Model. Nevertheless, several important questions remain open, including the neutrino mass ordering, the absolute mass scale, and the mechanism responsible for neutrino mass generation.

1.1.2 Dirac and Majorana Mass Terms and the Seesaw Mechanism

As discussed before, neutrinos are massless in the minimal SM due to the absence of right-handed neutrino fields, which prevents the construction of Dirac mass terms via Yukawa couplings to the Higgs field. A minimal and well-motivated extension therefore consists in introducing right-handed neutrino states, ν_R , which are singlets under the SM gauge group. This extension not only allows for the generation of Dirac mass terms via Yukawa interactions, but also permits the existence of Majorana mass terms for the gauge-singlet neutrino fields. The most general neutrino mass Lagrangian can be written as

$$(1.16) \quad \mathcal{L} = m_D(\bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L) + \frac{1}{2} M_L \overline{(\Psi_L)^C} \Psi_L + \frac{1}{2} M_R \bar{\Psi}_R (\Psi_R)^C,$$

where m_D is the Dirac mass term, while M_L and M_R are Majorana mass terms for left- and right-handed neutrinos, respectively. In the basis of Majorana fields, this can be written as

$$(1.17) \quad \mathcal{L} = (\bar{\phi}_1, \bar{\phi}_2) \begin{pmatrix} M_L & m_D & m_D & M_R \end{pmatrix} \begin{pmatrix} \phi_1 & \phi_2 \end{pmatrix}.$$

Diagonalizing this matrix yields the mass eigenvalues

$$(1.18) \quad m_{1,2} = \frac{M_R + M_L}{2} \mp \sqrt{\frac{(M_R - M_L)^2}{4} + m_D^2}.$$

A particularly important regime is obtained in the limit $M_R \gg m_D$ and $M_L \simeq 0$, leading to the so-called Type-I seesaw mechanism. In this case, the eigenvalues simplify to

$$(1.19) \quad m_\nu \simeq \frac{m_D^2}{M_R}$$

corresponding to a light neutrino state and a heavy sterile state. This mechanism naturally explains the smallness of neutrino masses. For instance, if the Dirac mass is of the order of the electroweak scale, $m_D \sim 10^2$ GeV, and $m_\nu \sim 0.1$ eV, one obtains a heavy mass scale $M_R \sim 10^{14}$ GeV. Alternatively, if m_D is much smaller, e.g. $m_D \sim 1$ MeV, the corresponding scale is lowered to $M_R \sim 10$ TeV, opening the possibility of phenomenological signatures at colliders.

While the discussion above focuses on the simplest realization of neutrino mass generation, many alternative mechanisms have been proposed. These include radiative models, inverse seesaw scenarios, and frameworks involving extra dimensions. A comprehensive overview of these possibilities can be found in the literature [Cai et al. \[2017\]](#), [Navas et al. \[2024\]](#).

1.2 Neutrinos In Cosmology

In this section, we present neutrinos role on the cosmic evolution, beginning with their decoupling in the early Universe and going through their influence on key observables such as the cosmic microwave background and the formation of large-scale structure. We discuss how the properties of neutrinos, particularly their mass and number of species, affect the expansion history and growth of perturbations, and how cosmological observations provide powerful constraints on these properties, complementing those from laboratory experiments.

1.2.1 Standard Cosmological Model

One can begin by introducing the current paradigm in cosmology, the Standard Cosmological Model (SCM). The SCM has been remarkably successful in describing a wide range of observational data, from the cosmic microwave background to the large-scale structure of the Universe. However, as we will discuss throughout the following section, there are growing tensions and anomalies brought by the current observations that suggest the need for extensions or modifications to this model. Neutrinos, with their subtle yet significant cosmological imprints, play a central role in this context. The SCM is built upon two fundamental pillars, the Theory of General Relativity, which provides the theoretical framework for gravity, and the Cosmological Principle, which assumes that the Universe is statistically homogeneous and isotropic on sufficiently large scales (typically above ~ 100 Mpc). Understanding these foundations is essential before we explore how neutrinos fit into the broader cosmological picture. In this section, we adopt natural units in which $\hbar = c = 1$, unless otherwise stated. Physical units are restored where needed for observational quantities.

1.2.1.1 Friedmann equations

Under the assumptions of homogeneity and isotropy, the spacetime interval is described by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric,

$$(1.20) \quad ds^2 = c^2 dt^2 - a^2(t) \left[\frac{dr^2}{1 - k r^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right],$$

where, r, θ, ϕ are comoving coordinates, t is the cosmic time, $a(t)$ is the scale factor and $k \in \{+1, 0, -1\}$ sets the spatial curvature (closed, flat, open). The dynamics of the Universe are obtained by inserting the FLRW metric into the field equations of General Relativity. This allows us to determine the evolution of the scale factor in terms of the Universe's energy content, namely,

$$(1.21) \quad G_{\mu\nu} - \Lambda g_{\mu\nu} = \chi_G T_{\mu\nu},$$

where $G_{\mu\nu}$ is the Einstein tensor, defined by

$$(1.22) \quad G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R,$$

with $R_{\mu\nu} \equiv R_{\mu\alpha\nu}^{\alpha}$ the Ricci tensor and $R \equiv R_{\mu\nu} g^{\mu\nu}$ the Ricci scalar. Here, $\chi_G = 8\pi G$ is Einstein's gravitational constant, Λ is the cosmological constant, $g_{\mu\nu}$ is the background metric, and $T_{\mu\nu}$ is the energy-momentum tensor. We thus arrive at the fundamental equation of General Relativity, which relates the distribution of matter and energy to the geometry of spacetime:

$$(1.23) \quad R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}.$$

From the 00 component of (1.23), we have the two fundamental Friedmann equations,

$$(1.24) \quad H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3} + \frac{8\pi G \rho}{3} - \frac{k}{a^2}.$$

$$(1.25) \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3},$$

where H is the Hubble parameter, which describes the expansion rate of the Universe, and $\frac{\ddot{a}}{a}$ represents the cosmic acceleration. The total energy density ρ and the pressure of the cosmic fluid p , satisfy the continuity equation,

$$(1.26) \quad \dot{\rho} + 3H(\rho + p) = 0.$$

1.2.1.2 Energy Content of the Universe

Having introduced the equations governing the dynamics of a homogeneous and isotropic Universe, we now describe its energy content. We know from an observational fact that the Λ CDM Universe is composed roughly of $\sim 70\%$ dark energy, $\sim 25\%$ dark matter, $\sim 5\%$ baryonic matter, with radiation contributing with a small fraction of today's energy content $< 1\%$. The total energy density of the Universe is given by $\rho = \sum_w \rho_w$, where ρ_w is the energy density of a component w . Each component can be characterized by a barotropic equation of state of the form $p_w = w \rho_w$, with w being the equation-of-state parameter. The total pressure is then $p = \sum_w p_w = \sum_w w \rho_w$. From the continuity equation we find that the energy density evolves with the scale factor as, $\rho_w(a) = \rho_{w,0} a^{-3(1+w)}$, where $\rho_{w,0}$ is the present-day energy density of the w component. Different components of the Universe evolve differently throughout cosmic expansion, depending on their equation-of-state parameter w ,

$$(1.27) \quad \rho_m(a) = \rho_{m,0} a^{-3}, \quad (w = 0, \text{ non-relativistic matter})$$

$$(1.28) \quad \rho_r(a) = \rho_{r,0} a^{-4}, \quad (w = \frac{1}{3}, \text{ radiation and relativistic neutrinos})$$

$$(1.29) \quad \rho_\Lambda(a) = \rho_{\Lambda,0}, \quad (w = -1, \text{ dark energy})$$

One can note that the evolution of relativistic components decays much more abruptly $\sim a^{-4}$, followed by matter $\sim a^{-3}$ and dark energy. This tells us that each of these components must have dominated the Universe for a period of time. In an early Universe radiation must have dominated, followed by matter, and the cosmological constant. An important feature of cosmic neutrinos is their dual behavior throughout the evolution of the Universe. At early times, when their

thermal velocity are high compared to their masses, neutrinos behave as relativistic particles and contribute to the total radiation energy density, ρ_r . As the Universe expands and cools, they eventually become non-relativistic, transitioning to a behavior akin to matter, thereby contributing to the matter energy density, ρ_m . This transition affects not only the expansion history of the Universe but also leaves distinctive imprints on the formation and growth of cosmic structures. In particular, massive neutrinos suppress the clustering of matter on small scales, an effect that is clearly manifested in the shape of the matter power spectrum, as we shall discuss in detail later.

We can also express the Hubble parameter directly in terms of the energy densities of the Universe. Using the critical density $\rho_c = \frac{3H^2}{8\pi G}$ (energy density required for the Universe to be spatially flat) and defining the density parameters $\Omega_w = \rho_w/\rho_c$, we have,

$$(1.30) \quad H(z) = H_0 \sqrt{\Omega_{r,0}(1+z)^4 + \Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{\Lambda,0}}.$$

where $\Omega_{k,0} = -\frac{k}{H_0^2}$. The so-called standard cosmological model as flat and composed of cold dark matter, baryons, radiation, and the cosmological constant agrees remarkably well with the majority of current cosmic observations. CMB data provides stringent constraints on the fundamental parameters at the 1σ confidence level [Planck Collaboration \[2020\]](#):

$$(1.31) \quad H_0 = 67.4 \pm 0.5$$

$$(1.32) \quad \Omega_{\Lambda,0} = 0.6847 \pm 0.0073$$

$$(1.33) \quad \Omega_{m,0} = 0.315 \pm 0.007$$

$$(1.34) \quad \Omega_{k,0} = 0.0007 \pm 0.0019 \sim 10^{-3}$$

$$(1.35) \quad \Omega_{r,0} \sim 10^{-4}.$$

Furthermore, the combination of CMB data and BAO has historically supported a constant dark energy equation of state, yielding, i.e, $w = -1.04 \pm 0.10$ at the 2σ confidence level [Planck Collaboration \[2020\]](#). However, this paradigm has recently been challenged by recent analyses from the Dark Energy Spectroscopic Instrument Collaboration (DESI). DESI measurements have recently provided evidence for a dynamical dark energy component. In particular, the combination of BAO data with Type Ia supernova observations shows a statistical preference for dynamical dark energy over the standard Λ CDM model, with a significance ranging from 1.7 to 3.3σ , depending on the supernova sample considered. This preference becomes even stronger, reaching up to 4.2σ , when Planck CMB data are included, especially for the combination DESI BAO + CMB + 5-year Dark Energy Survey supernova sample (DESY5) [Abdul Karim et al. \[2025b\]](#), [Lodha et al. \[2025\]](#).

These results are obtained within the framework of the phenomenological Chevallier–Polarski–Linder (CPL) parameterization, $w(a) = w_0 + w_a(1 - a)$ [Chevallier and Polarski \[2001\]](#), [Linder \[2003\]](#). Assuming CPL, DESI analysis shows a significant and intriguing preference for a

phantom-like dark energy equation of state ($w < -1$)¹. In addition, the standard Λ CDM model faces other significant observational challenges, most notably the Hubble tension, which currently exceeds the 5σ level [Casertano et al. \[2025\]](#). This discrepancy between early- and late-Universe determinations of the Hubble constant may point to the presence of new physics beyond the standard cosmological model.

1.2.2 Thermal Evolution of the Universe

The thermal history of the Universe depends directly on its evolving energy content. As discussed in the previous section, as the Universe expands, the energy densities of its various components dilute at different rates. This has led to a sequence of distinct cosmological epochs: an initial radiation-dominated era, followed by a period of matter domination, and a dark-energy-dominated Universe today. In this section, we detail this thermal evolution, beginning with the inflationary epoch and proceeding through baryogenesis, the Electroweak (EW) phase transition, and the Quantum Chromodynamics (QCD) phase transition. We then give particular emphasis to neutrino decoupling, electron-positron annihilation, Big Bang nucleosynthesis, matter-radiation equality, recombination, and finally photon decoupling. Table 1.1 presents the approximate time, redshift, and temperature at which event occurred.

Event	Time	Redshift	Temperature
Inflation	?	—	—
Baryogenesis	?	?	?
EW phase transition	20 ps	10^{15}	100 GeV
QCD phase transition	20 μ s	10^{12}	150 MeV
Neutrino decoupling	1 s	6×10^9	1 MeV
Electron-positron annihilation	6 s	2×10^9	500 keV
Big Bang nucleosynthesis	3 min	4×10^8	100 keV
Matter-radiation equality	60 kyr	3400	0.75 eV
Recombination	260–380 kyr	1100–1400	0.25–0.30 eV
Photon decoupling	380 kyr	1100	0.25 eV

Table 1.1: Key events in the thermal history of the Universe, showing their approximate times, redshifts, and temperatures.

¹Phantom dark energy corresponds to an effective equation of state $w < -1$, violating the null energy condition and often associated with exotic scalar field models or modifications of general relativity.

1.2.2.1 Inflation and the Initial Conditions

The standard cosmological model successfully describes a wide range of observations; however, it faces several conceptual problems related to the initial conditions of the Universe. Among these are the flatness (or curvature) problem, the monopole problem, and the horizon problem.

The flatness problem arises from the extreme fine-tuning required for the density parameter Ω to be close to unity today. From the Friedmann equation, deviations from spatial flatness grow with time during both radiation- and matter-dominated eras, implying that the Universe must have been extraordinarily close to flat in the early Universe. **The monopole problem** emerges in the context of Grand Unified Theories, which predict the formation of topological defects, as for instance magnetic monopoles, during early phase transitions. These relics are expected to be produced in large abundances, in contradiction with current observations. **The horizon problem** is the question of why regions of the sky that were causally disconnected at the time of recombination nevertheless exhibit nearly identical temperatures, as evidenced by the large-scale homogeneity of the CMB.

Primordial inflation as a solution to these problems was first proposed by [Starobinsky \[1980\]](#), [Guth \[1981\]](#). Inflation corresponds to an early epoch of the Universe during which the scale factor undergoes accelerated expansion, $\ddot{a} > 0$. From the second Friedmann equation Eq. (1.25) it follows that accelerated expansion requires $\rho + 3p < 0$. This condition can be expressed in terms of the equation-of-state parameter $w = p/\rho$, leading to $w < -\frac{1}{3}$, which corresponds to a component with sufficiently negative pressure. A defining feature of this accelerated expansion is the decrease of the comoving Hubble radius,

$$(1.36) \quad \ddot{a} > 0 \quad \Rightarrow \quad \frac{d}{dt}(aH)^{-1} < 0$$

Both the curvature and horizon problems are closely related to the behavior of the Hubble radius. A phase of accelerated expansion in the early Universe, during which the comoving Hubble radius shrinks, provides a natural mechanism to resolve these issues by driving the Universe toward spatial flatness and by stretching a single causally connected patch to a size exceeding the observable Universe today, while simultaneously diluting any unwanted relics produced prior to the end of inflation.

In order to have a period of accelerated expansion we need an equation of state $w < -1/3$. In fact, a Universe dominated by dark energy $w = -1$, for example, gives us an accelerated (exponentially) expansion. However, in the primordial inflation we need the accelerated expansion to stop so that the transition from the inflationary period to the Big Bang cosmology period is possible. This happens with a variable equation of state $w(t)$. The simplest way to achieve this is to consider a scalar field ϕ , called Inflaton, with a potential $V(\phi)$, as the responsible agent.

The inflaton scalar field should have an approximately constant energy density, and its energy density should dominate over the others at the very early Universe to cause the accelerated expansion. Therefore, at this time, magnetic monopoles would not be a problem. After this,

inflation predicts a reheating period. This reheating period is a transition between this accelerated expansion phase and the Universe described by the standard cosmology we discussed earlier. This reheating occurs in a way that prevents the resurgence of these monopoles. For this, the characteristic temperature of this epoch must be T_{rec} less than the mass of these monopoles (10^{16} GeV). The energy density and pressure for the inflaton field is given by

$$(1.37) \quad \rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$(1.38) \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi).$$

and the equations that describe the dynamics of the Universe dominated for this scalar field is given by,

$$(1.39) \quad \ddot{\phi} + 3H\dot{\phi} + V' = 0$$

$$(1.40) \quad H^2 = \frac{8\pi G}{3} \left[\frac{1}{2}\dot{\phi}^2 + V(\phi) \right]$$

$$(1.41) \quad \frac{\ddot{a}}{a} = -\frac{8\pi G}{3} [\dot{\phi}^2 - V(\phi)].$$

One can see that for inflation to occur ($w_\phi = -1$) we must have potential energy dominating over the kinetic energy $V(\phi) \gg \frac{1}{2}\dot{\phi}^2$. Fig. 1.1 shows a scalar field positioned in regions that correspond to a high value of potential energy, where $\dot{\phi}$ represents the speed at which this field changes its position. In our approximation $V(\phi) \gg \frac{1}{2}\dot{\phi}^2$ we have a very small $\dot{\phi}$. This is called the Slow-Roll approximation. As this field changes position relative to the potential, it will actually lose potential energy, gain kinetic energy, and exit the inflationary phase. The friction term in the Klein-Gordon equation ($3H\dot{\phi}$) will allow the field to stop oscillating, instead of going back and forth in the potential. The first Friedmann equation, in this Slow-Roll scenario, takes the form, $H^2 \approx \frac{8\pi G}{3} V(\phi)$.

We can define some parameters to describe these slow roll conditions,

$$(1.42) \quad \epsilon_V = \frac{1}{16\pi G} \left(\frac{V'}{V} \right)^2 \ll 1$$

$$(1.43) \quad \eta_V = \frac{1}{8\pi G} \left(\frac{V''}{V} \right) \ll 1.$$

Another parameter that will give us information about the amount of inflation, and is also related to the duration of the inflationary period, is the number of e-folds N .

$$(1.44) \quad N = -8\pi G \int_{\phi_i}^{\phi_f} \frac{V}{V'} d\phi.$$

One last important thing to note is that the scalar field ϕ driving inflation is typically treated as a homogeneous background field, $\bar{\phi}(t)$, which provides a good description of the large-scale dynamics of the Universe. However, this is only an approximation, as small inhomogeneous

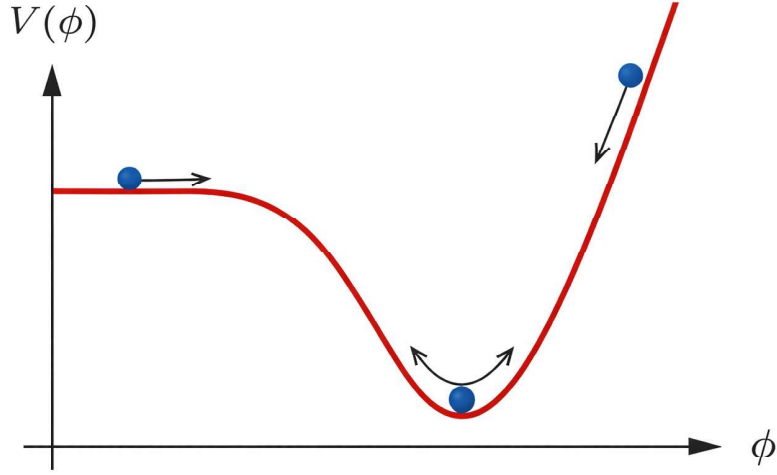


Figure 1.1: Evolution of the inflaton scalar field with the potential energy $V(\phi)$.

fluctuations are always present. These fluctuations are crucial, as they are the origin of the structures observed in the Universe today. The inflaton field can be decomposed as

$$(1.45) \quad \phi(\mathbf{x}, t) = \bar{\phi}(t) + \delta\phi(\mathbf{x}, t),$$

where $\delta\phi$ represents small quantum fluctuations around the homogeneous background. During inflation, these fluctuations are generated by quantum effects and stretched to super-Hubble scales due to the accelerated expansion. These primordial fluctuations provide the seeds for the density perturbations that later grow under gravitational instability, leading to the formation of cosmic structures such as galaxies and clusters. A useful heuristic picture is that fluctuations in the inflaton field lead to spatial variations in the time at which inflation ends. This can be expressed as $\delta t = \frac{\delta\phi}{\dot{\phi}}$, implying that different regions of the Universe undergo slightly different amounts of expansion. This results in curvature perturbations, which can be related to perturbations in the energy density and gravitational potential. After horizon re-entry, these curvature perturbations evolve into the density fluctuations that ultimately give rise to the large-scale structure of the Universe.

As mentioned, after the primordial inflationary epoch, the Universe must undergo a transition to the standard hot Big Bang evolution, dominated by radiation, baryonic matter, and dark matter. This transition is known as reheating. During inflation, the energy density of the Universe is dominated by the potential energy of the inflaton field, $V(\phi)$. As the field evolves toward the minimum of its potential, this energy is converted into kinetic energy, and inflation comes to an end. The inflaton field then begins to oscillate around the minimum of the potential, with the

total energy density stored in the form of coherent oscillations of the field.

In order to recover the standard thermal history of the Universe, this energy must be transferred to SM particles. This process occurs through the decay of the inflaton into lighter degrees of freedom and is referred to as reheating. The detailed mechanism responsible for this energy transfer remains model-dependent, with possibilities including perturbative decay and nonperturbative effects such as preheating. Assuming a decay rate Γ for the inflaton, the evolution of its energy density is governed by

$$(1.46) \quad \frac{\partial \rho_\phi}{\partial t} + (3H + \Gamma)\rho_\phi = 0.$$

The decay products subsequently interact and thermalize, leading to the formation of a hot plasma characterized by a reheating temperature T_{rea} . This temperature can be estimated as

$$(1.47) \quad T_{\text{rea}} = \left(\frac{30 \rho_{\text{rea}}}{\pi^2 g_{\text{rea}}} \right)^{1/4},$$

where g_{rea} is the effective number of relativistic degrees of freedom at reheating and ρ_{rea} is the energy density at the end of this process. Once thermal equilibrium is established, the Universe enters the standard radiation-dominated phase, and its subsequent evolution follows the well-known thermal history of the Big Bang model. Despite its importance, reheating remains poorly constrained observationally, as no direct probes of this epoch are currently available.

A key process that must occur within this thermal history is baryogenesis, which refers to the generation of a net baryon number in the early Universe and explains the observed asymmetry between matter and antimatter. Had matter and antimatter been produced in equal amounts, they would have annihilated almost completely, leaving behind a radiation-dominated Universe with no baryonic content. Instead, observations indicate a small but nonvanishing asymmetry, quantified by the baryon-to-photon ratio,

$$(1.48) \quad \eta_B \equiv \frac{(n_B - n_{\bar{B}})}{n_\gamma} \simeq 6 \times 10^{-10},$$

consistent with both BBN and CMB measurements. Any viable baryogenesis mechanism must satisfy the three Sakharov conditions: baryon number violation, C and CP violation, and a departure from thermal equilibrium. Among the proposed mechanisms, leptogenesis provides a particularly compelling scenario, in which a lepton asymmetry is first generated through the out-of-equilibrium decay of heavy Majorana neutrinos and subsequently converted into a baryon asymmetry via nonperturbative electroweak sphaleron processes, which violate $B + L$ while preserving $B - L$.

Following reheating and baryogenesis, the Universe continues to cool and undergoes a sequence of important transitions. The heaviest particles decouple first as the temperature drops below their mass scale. For instance, the top quark, with mass $m_t \simeq 173 \text{ GeV}$, is among the first Standard Model particles to become non-relativistic and disappear from the thermal

plasma. At temperatures $T \sim 100 \text{ GeV}$, the electroweak phase transition occurs, during which the Higgs field acquires a vacuum expectation value, giving mass to the $W^\pm$ and Z^0 bosons and breaking the electroweak symmetry. If dark matter consists of weakly interacting massive particles (WIMP), thermal freeze-out takes place shortly after, when the WIMP interaction rate drops below the Hubble expansion rate, typically at temperatures of order $T \sim \mathcal{O}(1\text{--}10) \text{ GeV}$ for weak-scale candidates. At lower temperatures, $T \sim 150 \text{ MeV}$, the QCD phase transition marks the confinement of quarks and gluons into hadrons such as protons and neutrons. At lower temperatures, $T \sim 150 \text{ MeV}$, the QCD phase transition takes place, marking the confinement of quarks and gluons into hadrons such as protons and neutrons. These processes collectively shape the particle content of the Universe and set the initial conditions for later epochs such as Big Bang nucleosynthesis and recombination.

1.2.2.2 The Cosmic Neutrino Background (CνB)

While the nature of dark matter remains unknown, neutrinos provide a well established example of a relic species whose thermal history can be accurately described. In the early Universe, neutrinos remain in thermal equilibrium with the primordial plasma through weak interactions such as $p + e^- \leftrightarrow n + \nu$, $p + \bar{\nu} \leftrightarrow n + e^+$, and $n \leftrightarrow p + e^- + \bar{\nu}$.

Before proceeding, it is useful to recall that the thermal properties of particle species in the early Universe are described in terms of their phase-space distribution function. For particles in thermal equilibrium, this distribution is given by the Fermi–Dirac or Bose–Einstein form,

$$(1.49) \quad f_{\text{EQ}}(p) = \frac{1}{\exp[(E - \mu)/T] \pm 1},$$

where the plus (minus) sign corresponds to fermions (bosons), $E = \sqrt{p^2 + m^2}$ is the particle energy, μ is the chemical potential, and T is the temperature of the thermal bath. Macroscopic quantities such as the number density, energy density, and pressure can be obtained by integrating the distribution function over momentum space,

$$(1.50) \quad n = \frac{g}{(2\pi)^3} \int f(p) d^3 p,$$

$$(1.51) \quad \rho = \frac{g}{(2\pi)^3} \int E(p) f(p) d^3 p,$$

$$(1.52) \quad p = \frac{g}{(2\pi)^3} \int \frac{p^2}{3E(p)} f(p) d^3 p,$$

where g denotes the number of internal degrees of freedom, accounting for spin, polarization states, and particle–antiparticle contributions. The entropy density of the system is given by

$$(1.53) \quad s = \frac{\rho + p}{T},$$

which plays a central role in the thermal evolution of the Universe, particularly in processes such as particle decoupling and entropy transfer between species.

The probability per unit of time for a particle to interact with another particle is given by the interaction rate $\Gamma = n\sigma v$, where n is the particle number density, σ is the cross section and v is the relative velocity between the projectile and the targets. The expansion rate of the Universe during the radiation dominated era is given by the Hubble parameter

$$(1.54) \quad H \sim \sqrt{\frac{8\pi G}{3}\rho} \sim \frac{T^2}{M_{\text{Pl}}},$$

where M_{Pl} is the Planck mass and we have used $\rho \sim gT^4$. Throughout the universe's history, particles remain in thermal equilibrium until their interaction rate is equal or larger than the expansion rate of the universe. Neutrino decoupling occurs when $\Gamma_\nu \sim H$. Neutrinos only interact via weak interactions, with a rate $\Gamma_\nu \simeq T^3 G_F^3 T^2 \sim G_F^3 T^5$. One can obtain the temperature in which neutrinos decouple as,

$$(1.55) \quad G_F^2 T^5 \sim \frac{T^2}{M_{\text{Pl}}} \Rightarrow T_{\text{dec}} \sim (G_F^2 M_{\text{Pl}})^{-1/3} \sim 1 \text{ MeV}.$$

At temperatures below 1 MeV, neutrinos decouple from the plasma and begin to free-stream, forming a relic background known as the Cosmic Neutrino Background (CvB).

After neutrino decoupling, electrons and positrons annihilate when $T \sim m_e$, transferring their entropy to photons but not to neutrinos. As a result, the photon temperature becomes higher than the neutrino temperature. In an expanding universe the entropy density per comoving volume is conserved, where we have the Boson's entropy density contribution $s_b = \frac{2\pi^2}{45} T^3 g_s$, and the Fermion's entropy density contribution $s_f = \frac{7}{8} \frac{2\pi^2}{45} T^3 g_s$. Entropy conservation provides a simple way to compute the ratio between photon and neutrino temperatures. Before e^+e^- annihilation, electrons ($g = 2$), positrons ($g = 2$), neutrinos (g_ν), antineutrinos ($g_{\bar{\nu}}$) and photons ($g = 2$) contribute to the effective number of entropy degrees of freedom g_s . The entropy density is given by,

$$(1.56) \quad s(a_1) = \frac{2\pi^2}{45} T^3 \left[2 + \frac{7}{8}(2+2) + \frac{7}{8}N_\nu(g_\nu + g_{\bar{\nu}}) \right]$$

where, N_ν is the number of neutrino family. Before the annihilation process we assume that neutrinos and photons have the same temperature, since both came from the original thermal bath. After annihilation, we have only neutrinos, antineutrinos and photons but at different temperature,

$$(1.57) \quad s(a_2) = \frac{2\pi^2}{45} \left[2T_\gamma^3 + \frac{7}{4}N_\nu g_\nu T_\nu^3 \right]$$

From the entropy conservation we have,

$$(1.58) \quad s(a_1)a_1^3 = s(a_2)a_2^3$$

$$(1.59) \quad (a_1 T_1)^3 \left[2 + \frac{7}{4}(2 + N_\nu g_\nu) \right] = (a_2 T_\nu)^3 \left[2 \left(\frac{T_\gamma}{T_\nu} \right)^3 + \frac{7}{4}N_\nu g_\nu \right]$$

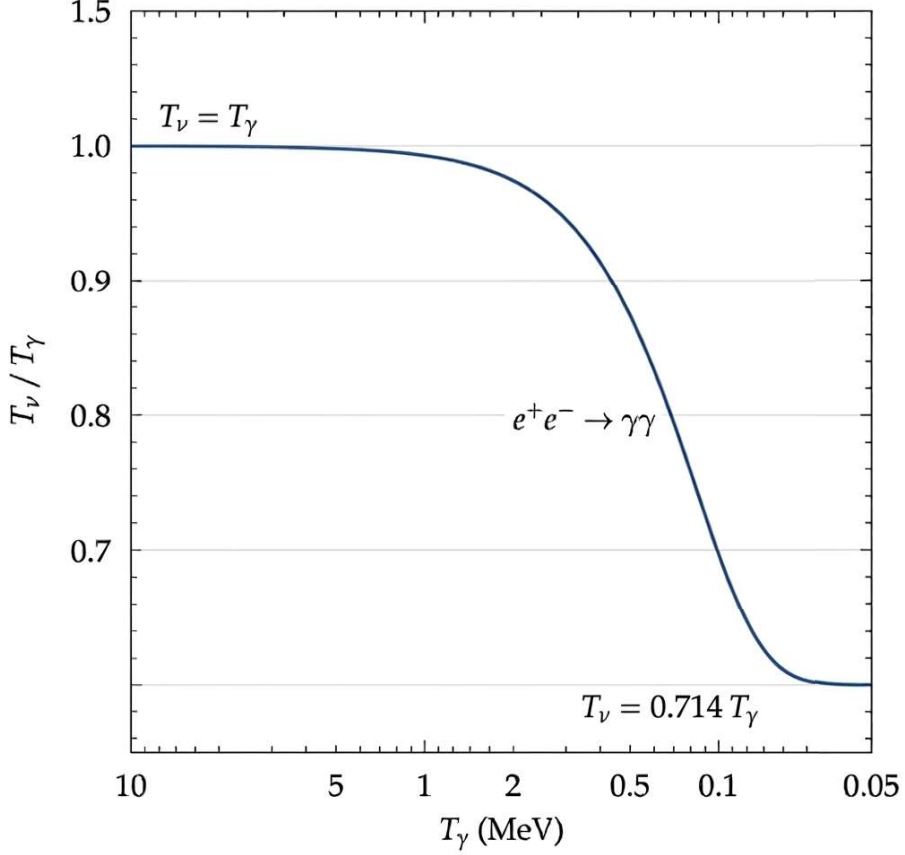


Figure 1.2: Evolution of the neutrino-to-photon temperature ratio T_ν/T_γ as a function of the photon temperature T_γ .

Since neutrinos are already decoupled, their temperature evolves simply as $T_\nu \propto a^{-1}$, so that $(a_1 T_1)^3 = (a_2 T_\nu)^3$, and one finds that after electron–positron annihilation the neutrino temperature is lower than the photon temperature,

$$(1.60) \quad \frac{T_\nu}{T_\gamma} = \left(\frac{4}{11} \right)^{1/3} \approx 0.714.$$

This is illustrated in Fig. 1.2. Before annihilation, neutrinos and photons share a common thermal bath, so $T_\nu/T_\gamma = 1$. Once electrons and positrons annihilate, they transfer their entropy exclusively to the photon bath, since neutrinos have already decoupled at $T \sim 2\text{--}3$ MeV. The photon temperature receives an additional increase from the annihilation process, while the neutrino temperature continues to redshift freely, and the ratio drops to the asymptotic value $T_\nu/T_\gamma = (4/11)^{1/3} \approx 0.714$. The CMB temperature is measured with remarkable precision, $T_{\gamma,0} = 2.725$ K, one can then infer a relic neutrino background temperature of $T_{\nu,0} \approx 1.945$ K.

As mentioned before, early Universe neutrinos behave as relativistic particles, from Eq. (1.60) we can relate neutrino energy density with the photons energy density. Relativistic neutrinos

energy density are given by,

$$(1.61) \quad \rho_\nu(m_\nu < T_\nu) = \frac{7}{8} \frac{\pi^2}{15} N_\nu g_\nu T_\nu^4.$$

on the other hand photons energy density are given by,

$$(1.62) \quad \rho_\gamma = \frac{\pi^2}{15} T_\gamma^4.$$

One can easily find the relation,

$$(1.63) \quad \rho_\nu = \frac{7}{8} N_\nu g_\nu \left(\frac{4}{11} \right)^{4/3} \rho_\gamma,$$

The total radiation energy density can be written as,

$$(1.64) \quad \rho_r = \rho_\gamma + \rho_\nu = \rho_\gamma \left[1 + \frac{7}{8} N_\nu g_\nu \left(\frac{4}{11} \right)^{4/3} \right].$$

In the instantaneous decoupling approximation, the product $N_\nu g_\nu$ corresponds to the number of relativistic neutrino degrees of freedom N_{eff} , leading to $N_{eff} = 3$ for three neutrino species. In the standard model, however, neutrino decoupling is not instantaneous. As a result, a small fraction of the entropy released during electron–positron annihilation is transferred to neutrinos. In addition, finite-temperature QED corrections and neutrino flavor oscillations further modify the neutrino energy density, leading to a slightly larger value.

Neutrinos are the second most abundant particle in the Universe today, after photons. Their number density follows directly from the phase-space integral. The number density of a species in thermal equilibrium is

$$(1.65) \quad n = \frac{g}{2\pi^2} \int_0^\infty \frac{p^2}{e^{p/T} \pm 1} dp.$$

Using the standard results

$$(1.66) \quad \int_0^\infty \frac{x^2}{e^x - 1} dx = 2\zeta(3), \quad \int_0^\infty \frac{x^2}{e^x + 1} dx = \frac{3}{2}\zeta(3),$$

where $\zeta(3) \approx 1.202$ is the Apéry constant, one finds

$$(1.67) \quad n_{\text{boson}} = \frac{\zeta(3)}{\pi^2} g T^3,$$

$$(1.68) \quad n_{\text{fermion}} = \frac{3}{4} \frac{\zeta(3)}{\pi^2} g T^3,$$

so that fermions contribute a factor 3/4 relative to bosons with the same g . For photons ($g = 2$, bosons):

$$(1.69) \quad n_\gamma = \frac{2\zeta(3)}{\pi^2} T_\gamma^3.$$

For neutrinos, each flavor contributes $g_{\nu_i} = 2$ degrees of freedom (one helicity state each for the neutrino and antineutrino). The number density per flavor is then

$$(1.70) \quad n_{\nu_i} = \frac{3}{4} \frac{2\zeta(3)}{\pi^2} T_\nu^3.$$

Substituting $T_\nu = (4/11)^{1/3} T_\gamma$ from Eq. (1.60):

$$(1.71) \quad n_{\nu_i} = \frac{3}{4} \cdot \frac{4}{11} \cdot \frac{2\zeta(3)}{\pi^2} T_\gamma^3 = \frac{3}{11} n_\gamma.$$

Summing over $N_\nu = 3$ active flavors:

$$(1.72) \quad n_\nu = \frac{9}{11} n_\gamma.$$

Restoring SI units via $T^3 \rightarrow (k_B T / \hbar c)^3$, and using the measured CMB temperature $T_{\gamma,0} = 2.725$ K, one finds $n_{\gamma,0} \approx 410.7 \text{ cm}^{-3}$, giving

$$(1.73) \quad n_{\nu,0} \approx \frac{9}{11} \times 410.7 \text{ cm}^{-3} \approx 336 \text{ cm}^{-3},$$

or $\approx 112 \text{ cm}^{-3}$ per flavor. As the Universe expands and cools, neutrinos transition from relativistic to non-relativistic particles once their temperature drops below their mass, $T_\nu \lesssim m_\nu$. In this regime, their energy density is given by

$$(1.74) \quad \rho_\nu(m_\nu \gtrsim T_\nu) = n_\nu \sum m_\nu,$$

which together with the critical density $\rho_{c,0} = 1.879 \times 10^{-29} h^2 \text{ g cm}^{-3}$ yields

$$(1.75) \quad \Omega_{\nu,0} = \frac{\rho_{\nu,0}}{\rho_{c,0}} = \frac{n_{\nu,0} \sum m_\nu}{\rho_{c,0}} = \frac{\sum m_\nu}{94.07 \text{ eV } h^2}.$$

1.2.2.3 Big Bang Nucleosynthesis

After electron positron annihilation Big Bang Nucleosynthesis (BBN) took place. BBN corresponds to the epoch during which the light elements were formed in the early Universe. The primordial abundances of light nuclei, particularly ^4He , D, and ^3He , are highly sensitive to the physical conditions of the early Universe and therefore provide a powerful probe of fundamental physics. A key quantity governing BBN is the neutron-to-proton ratio. While weak interactions are in thermal equilibrium, this ratio is given by

$$(1.76) \quad \frac{n_n}{n_p} = \exp\left(-\frac{\Delta m}{T}\right),$$

where $\Delta m = m_n - m_p = 1.293 \text{ MeV}$. This relation follows from chemical equilibrium between neutrons and protons through weak processes such as $n + \nu_e \leftrightarrow p + e^-$ and $n + e^+ \leftrightarrow p + \bar{\nu}_e$. As the Universe expands and cools, the interaction rate of these weak processes decreases and eventually falls below the Hubble expansion rate,

$$(1.77) \quad \Gamma_{n \leftrightarrow p} \sim H.$$

At this point, weak interactions freeze out, and the neutron-to-proton ratio departs from its equilibrium value. The freeze-out temperature is therefore determined by the expansion rate of the Universe. Since the expansion rate depends on the total radiation energy density, it is sensitive to the effective number of relativistic species, N_{eff} . An increase in N_{eff} leads to a higher expansion rate, causing weak interactions to decouple earlier, at higher temperatures. As a consequence, the neutron-to-proton ratio at freeze-out is larger,

$$(1.78) \quad \frac{n}{p} \simeq \exp\left(-\frac{\Delta m}{T_{\text{freeze}}}\right).$$

After freeze-out, neutrons decay until they are incorporated into nuclei. Since most neutrons end up bound in ${}^4\text{He}$ due to its large binding energy, the primordial helium abundance can be approximated as

$$(1.79) \quad Y_p = \frac{2(n/p)}{1 + (n/p)}.$$

Therefore, an increase in N_{eff} results in a higher neutron fraction and consequently a larger primordial helium abundance. This sensitivity makes BBN a powerful probe of additional relativistic degrees of freedom in the early Universe. Current analysis combining primordial helium data with Planck CMB and BAO measurements reports $N_{\text{eff}} = 2.990 \pm 0.070$ at 1σ C.L. [Goldstein and Hill \[2026\]](#). This result is consistent with the existence of three neutrino families and no significant additional relativistic degrees of freedom.

1.2.2.4 Matter–Radiation Equality

Following Big Bang Nucleosynthesis, the Universe remained radiation-dominated until the epoch of matter–radiation equality, $\rho_m = \rho_r$. This epoch is particularly important for the evolution of cosmological perturbations, as it marks the transition to matter domination, allowing structures to grow efficiently. As discussed previously, the present-day radiation density includes a contribution from relativistic neutrinos, parametrised by the effective number of relativistic species, N_{eff} . The scale factor at equality is obtained from

$$(1.80) \quad \frac{\Omega_{r,0}}{a_{\text{eq}}^4} = \frac{\Omega_{m,0}}{a_{\text{eq}}^3},$$

$$(1.81) \quad a_{\text{eq}} = \frac{\Omega_{r,0}}{\Omega_{m,0}} = \frac{\Omega_{\gamma,0}}{\Omega_{m,0}} \left[1 + \frac{7}{8} N_{\text{eff}} \left(\frac{4}{11} \right)^{4/3} \right].$$

Adopting the standard value $N_{\text{eff}} = 3.044$, one finds

$$(1.82) \quad a_{\text{eq}} \simeq \frac{4.15 \times 10^{-5}}{\Omega_{m,0} h^2}, \quad 1 + z_{\text{eq}} \simeq 2.4 \times 10^4 \Omega_{m,0} h^2.$$

These expressions are valid provided all neutrino species remain relativistic at equality. If, however, one or more species become non-relativistic before this epoch, they cease to contribute

to the radiation density and instead augment the matter density. The effective matter density is then increased, advancing the epoch of equality to

$$(1.83) \quad 1 + z_{\text{eq}} \simeq 2.79 \times 10^4 (\Omega_{m,0} + \Omega_{\nu,0}) h^2,$$

To determine which scenario is realised, one compares the neutrino mass scale with the neutrino temperature at equality. Using $T_{\nu,0} \simeq 1.95$ K, $T \propto (1+z)$, and $\Omega_{m,0} = 0.3$, the neutrino temperature at equality is

$$(1.84) \quad T_{\nu,\text{eq}} = T_{\nu,0}(1 + z_{\text{eq}}) \simeq 1.7 \times 10^{-4} \text{ eV} (1 + z_{\text{eq}}) \sim 0.5\text{--}0.6 \text{ eV}.$$

Neutrinos remain relativistic at equality provided $m_\nu < T_{\nu,\text{eq}}$, while they become non-relativistic when $m_\nu \gtrsim T_{\nu,\text{eq}}$. For standard cosmological parameters values, this implies that neutrinos would need $m_\nu \gtrsim 0.6$ eV to become non-relativistic before equality. Current cosmological data, however, place the upper bound $\sum m_\nu \lesssim 0.07\text{--}0.1$ eV at 2σ C.L. [Elbers et al. \[2025b\]](#), well below this threshold. Consequently, all three active neutrino species are still relativistic at matter–radiation equality and contribute to the radiation density at that epoch, becoming non-relativistic only later during the matter-dominated era.

1.2.2.5 Recombination and the Cosmic Microwave Background

As the Universe expanded and cooled, it eventually reached temperatures at which electrons and protons could combine to form neutral hydrogen. Prior to recombination, the Universe consisted of a tightly coupled photon–baryon plasma, in which photons were frequently scattered by free electrons via Thomson scattering. The high interaction rate maintained thermal equilibrium and prevented photons from propagating freely over large distances. The interaction rate of photons with electrons is given by

$$(1.85) \quad \Gamma_{\gamma e} = n_e \sigma_T,$$

where n_e is the free electron number density and σ_T is the Thomson cross section. As the temperature decreased, the formation of neutral hydrogen led to a rapid drop in the free electron density, reducing the interaction rate. When $\Gamma_{\gamma e} \lesssim H$, photons decoupled from the plasma and began to free-stream throughout the Universe. This epoch, known as photon decoupling, marks the origin of the CMB, which provides a picture of the Universe at a redshift $z \sim 1100$. The CMB is observed today as an almost perfect blackbody radiation with temperature $T_0 = 2.725$ K. Small temperature anisotropies at the level of $\Delta T/T \sim 10^{-5}$ encode a wealth of cosmological information. These anisotropies arise from several physical effects, including acoustic oscillations in the photon–baryon fluid, gravitational redshift (Sachs–Wolfe effect), Doppler contributions from velocity perturbations, and diffusion damping (Silk damping) at small scales. The angular power spectrum of the CMB therefore provides a powerful probe of the composition and evolution of the Universe.

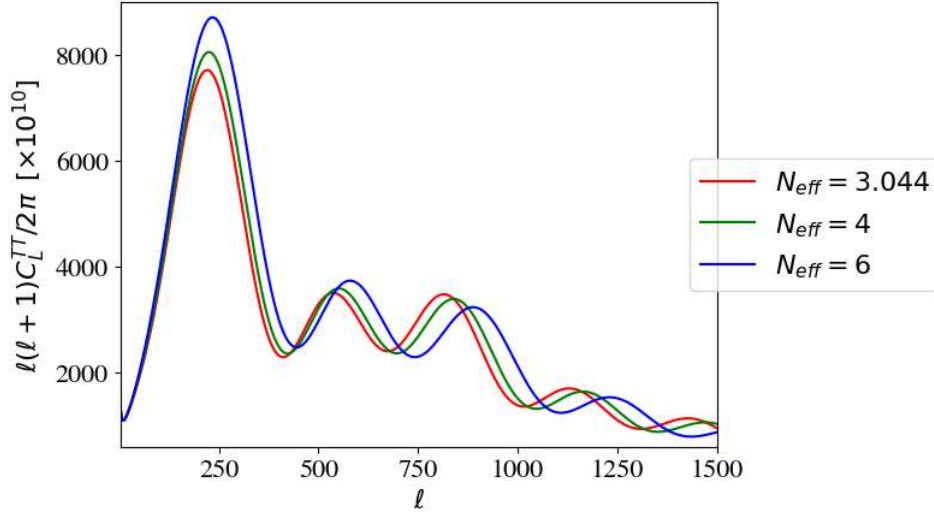


Figure 1.3: CMB angular spectrum for different values of the effective number of neutrino species (N_{eff}) within the standard Λ CDM cosmological model.

Relativistic neutrinos leave an important imprint on the angular power spectrum of the CMB. Fig. 1.3 shows the CMB angular spectrum for different values of N_{eff} within the Λ CDM model. At low multipoles, $\ell \lesssim 1000$, the effect of N_{eff} is primarily encoded through neutrino anisotropic stress. After decoupling, neutrinos free-stream, propagating at nearly the speed of light, which is larger than the sound speed of the photon–baryon fluid. This free-streaming behavior generates anisotropic stress that modifies the evolution of gravitational potentials, leading to a suppression of the amplitude of acoustic oscillations for modes that enter the horizon during the radiation-dominated era. However, at these scales, degeneracies with other cosmological parameters limit the constraining power on N_{eff} . Stronger constraints arise from smaller angular scales, $\ell \gtrsim 1000$, where N_{eff} affects the damping tail of the CMB spectrum by modifying the expansion rate prior to recombination. An increase in N_{eff} leads to a higher expansion rate, which reduces the diffusion length and enhances Silk damping, leaving a characteristic signature in the high- ℓ region of the spectrum. Current cosmological observations provide tight constraints on this parameter. In particular, measurements of the Cosmic Microwave Background (CMB) anisotropies by the Planck collaboration yield $N_{\text{eff}} = 2.99^{+0.34}_{-0.33}$ at 2σ [Planck Collaboration \[2020\]](#).

In addition to their relativistic effects, neutrinos that become non-relativistic at late times also affects the CMB anisotropy spectrum. As the Universe expands and cools, neutrinos transition from relativistic to non-relativistic particles once their temperature drops below their mass, $T_\nu \lesssim m_\nu$. Massive neutrinos affect the CMB through several distinct physical mechanisms. First, they modify the expansion history of the Universe, which alters both the angular diameter distance to the last scattering surface and the sound horizon at recombination. This leads to a

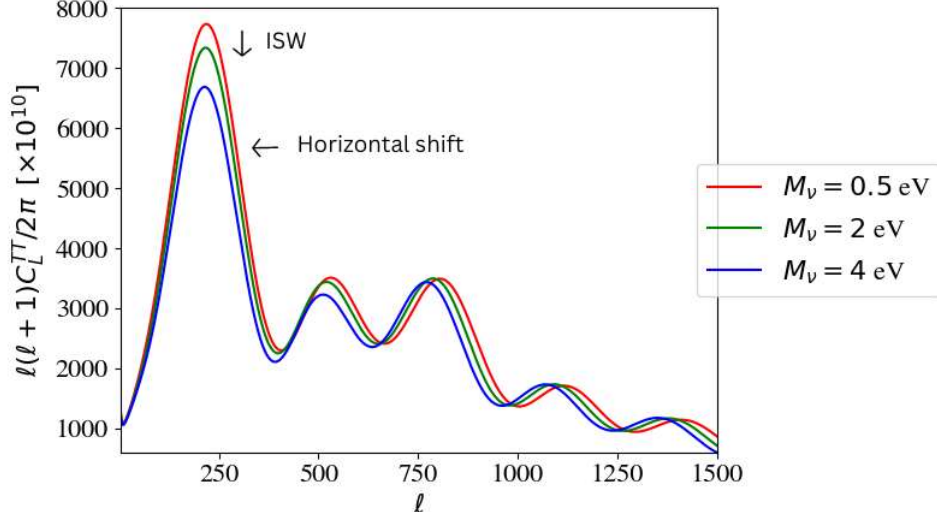


Figure 1.4: CMB angular spectrum for different values of total neutrino mass $\sum m_\nu$ in the Λ CDM model.

horizontal shift in the angular positions of the acoustic peaks, governed by the angular scale

$$(1.86) \quad \theta_s = \frac{r_s}{D_A},$$

where r_s is the comoving sound horizon and D_A is the angular diameter distance,

$$(1.87) \quad D_A = \int_0^{z_{\text{rec}}} \frac{dz}{H(z)}.$$

Massive neutrinos also affect the evolution of gravitational potentials. The observed temperature anisotropies can be written as

$$(1.88) \quad \Theta(\hat{n}) = \frac{\delta T}{T}(\hat{n}) = \Theta_0 + \Psi + \hat{n} \cdot (\vec{v}_e - \vec{v}) + \int (\dot{\Psi} + \dot{\Phi}) d\eta,$$

where the last term corresponds to the Integrated Sachs–Wolfe (ISW) effect. The transition of neutrinos from relativistic to non-relativistic behavior modifies the time evolution of the gravitational potentials, leading to changes in both the early and late ISW contributions and affecting the amplitude of the CMB anisotropies. In Fig. 1.4 we show the change in the shape of the angular power spectrum for different values of the total neutrino mass, $\sum m_\nu \equiv M_\nu$, within the Λ CDM framework. These effects can be partially mimicked by variations in other cosmological parameters, leading to degeneracies, in particular between $\sum m_\nu$ and the Hubble constant H_0 .

Another important effect arises from gravitational lensing of the CMB. The observed temperature field is remapped according to

$$(1.89) \quad \Theta(\hat{n}) \rightarrow \Theta(\hat{n} + \nabla\phi(\hat{n})),$$

where ϕ is the lensing potential. This potential probes the integrated matter distribution along the line of sight,

$$(1.90) \quad \phi(\hat{n}) = -2 \int_0^{z_{\text{rec}}} \frac{dz}{H(z)} \Psi(z, D(z) \hat{n}) \left[\frac{D(z_{\text{rec}}) - D(z)}{D(z_{\text{rec}})D(z)} \right].$$

Since massive neutrinos behave as a form of hot dark matter, their large thermal velocities suppress the growth of structure on small scales. This leads to a reduction in the matter clustering amplitude and, consequently, to a suppression of the CMB lensing signal. Importantly, this effect helps break degeneracies present at the level of primary CMB anisotropies and allows for robust constraints on the sum of neutrino masses. Current observations combining temperature, polarization, and lensing data constrain the sum of neutrino masses to $\Sigma m_\nu \lesssim 0.24 \text{ eV}$ at 2σ confidence level [Planck Collaboration \[2020\]](#).

1.2.3 Neutrinos and the Large-Scale Structure

Neutrinos have an important impact on the growth of cosmic structure. The large-scale structure (LSS) can be observed with great sensitivity and offers a powerful window to infer neutrino properties. In the framework of General Relativity, assuming dark energy perturbations are negligible, the Large Scale Structure (LSS) of the Universe is statistically characterized by the matter power spectrum. This spectrum is derived from the two-point correlation function of the matter density contrast, $\delta(\mathbf{x})$, which represents the fractional fluctuation of the matter density $\rho(\mathbf{x})$ relative to the mean background density $\bar{\rho}$:

$$\delta(\mathbf{x}) = \frac{\rho(\mathbf{x}) - \bar{\rho}}{\bar{\rho}}.$$

In Fourier space, the two-point correlation function of the density contrast $\delta(\mathbf{k})$ is defined as:

$$(1.91) \quad \langle \delta(\mathbf{k}) \delta(\mathbf{k}') \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}') P(k, z),$$

where $P(k, z)$ is the matter power spectrum at redshift z .

1.2.3.1 Neutrino Free-Streaming and Impact on Density Perturbations

A key property distinguishing massive neutrinos from CDM is their large thermal velocity. Because neutrino masses are very small, their typical momentum at temperature T_ν is $\langle p_\nu \rangle \sim 3T_\nu$, giving a thermal velocity

$$(1.92) \quad v_t = \frac{\langle p_\nu \rangle}{m_\nu} \simeq 150(1+z) \left(\frac{\text{eV}}{m_\nu} \right) \text{ km s}^{-1}.$$

On the other hand, CDM particles have negligible thermal velocities by definition and can cluster at any scale. Neutrinos, however, free-stream out of gravitational potential wells on scales below their free-streaming length, suppressing the growth of structure. This is a very important effect

to infer neutrino absolute mass from cosmology. To understand this quantitatively, consider the perturbation equation for a single pressurized fluid in an expanding Universe. In the linear regime, the matter overdensity $\delta = \delta\rho/\bar{\rho}$ satisfies

$$(1.93) \quad \ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - \frac{c_s^2 k^2}{a^2}\delta = 4\pi G\rho\delta,$$

where c_s is the sound speed of the fluid and k is the comoving wavenumber. The right-hand side represents gravitational attraction, which drives clustering, while the pressure term $c_s^2 k^2 \delta/a^2$ opposes it. The balance between these two terms defines the Jeans wavenumber,

$$(1.94) \quad k_J = \sqrt{\frac{4\pi G\rho}{c_s^2(1+z)^2}},$$

which separates two distinct regimes: for $k < k_J$, gravity dominates and perturbations grow; for $k > k_J$, pressure dominates and perturbations oscillate and decay rather than collapse.

For neutrinos, the role of the sound speed is played by their thermal velocity v_t . The analogous scale is the free-streaming wavenumber

$$(1.95) \quad k_{\text{fs}}(z) = \sqrt{\frac{3}{2}} \frac{H(z)}{(1+z)v_t(z)},$$

which upon substituting Eq. (1.92) and the Hubble parameter Eq. (1.30) evaluates to

$$(1.96) \quad k_{\text{fs}}(z) \simeq 0.677 \sqrt{\frac{\Omega_{m,0}}{1+z}} \left(\frac{m_\nu}{1 \text{ eV}} \right) h \text{ Mpc}^{-1}.$$

On scales $k < k_{\text{fs}}$, neutrinos behave as a pressureless fluid and cluster together with CDM and baryons, contributing to the growth of structure. On scales $k > k_{\text{fs}}$, neutrino perturbations are washed out by free-streaming, and neutrinos contribute only to the homogeneous background density.

This has an important consequence for the total matter power spectrum. The impact of neutrino free-streaming on structure formation can be understood by considering the total matter perturbation,

$$(1.97) \quad \delta_m = \frac{\rho_{\text{cdm}}\delta_{\text{cdm}} + \rho_b\delta_b + \rho_\nu\delta_\nu}{\rho_m}.$$

On large scales ($k < k_{\text{fs}}$), neutrino perturbations follow those of CDM, $\delta_\nu \simeq \delta_{\text{cdm}}$, and the standard growth of structure is recovered. On small scales ($k > k_{\text{fs}}$), neutrino perturbations are strongly suppressed ($\delta_\nu \ll \delta_{\text{cdm}}$), and neutrinos contribute only to the homogeneous background. As a result, the gravitational source term driving the growth of CDM and baryon perturbations is reduced. Defining the neutrino fraction $f_\nu = \frac{\Omega_\nu}{\Omega_m}$, the growth rate of matter perturbations in the matter-dominated era is approximately modified to

$$(1.98) \quad \delta_m \propto a^{1-\frac{3}{5}f_\nu},$$

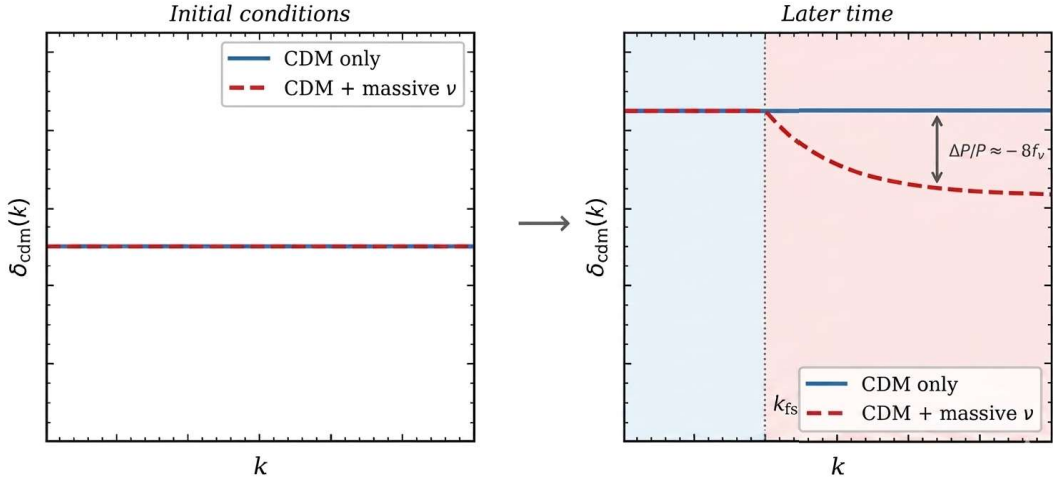


Figure 1.5: Schematic evolution of the CDM density perturbation $\delta_{\text{cdm}}(k)$ in a CDM-only universe (solid blue) and in a mixed CDM + neutrinos universe (dashed red).

instead of $\delta_m \propto a$ in a pure CDM universe. This scale-dependent suppression accumulates over time, resulting in a characteristic step-like suppression of the matter power spectrum on scales $k > k_{\text{fs}}$, approximately given by

$$(1.99) \quad \frac{\Delta P}{P} \simeq -8f_\nu.$$

in Fig. 1.5 we show a schematic evolution of the CDM density perturbation, one can see that at early times both spectra are identical. At later times, massive neutrinos free-stream out of potential wells on scales $k > k_{\text{fs}}$, slowing the growth of CDM perturbations and imprinting a characteristic step-like suppression $\Delta P/P \approx -8f_\nu$ on the matter power spectrum.

The matter power spectrum suppression is therefore completely sensitive to the total neutrino mass. For $\sum m_\nu \sim 0.1$ eV, $f_\nu \sim 0.005$ and the suppression reaches the percent level, which is well within the sensitivity of current and upcoming galaxy surveys. The mass dependence is illustrated in Fig. 1.6, which shows the ratio of the matter power spectrum with massive neutrinos to that with massless neutrinos; the characteristic suppression on small scales is clearly visible and grows with increasing neutrino mass.

In addition to the impact of massive neutrinos, the effective number of relativistic species, N_{eff} , also modifies the matter power spectrum. As discussed in the context of matter–radiation equality, increasing N_{eff} enhances the radiation energy density and would delay equality. However, since the redshift of equality is tightly constrained by CMB observations, variations in N_{eff} are typically compensated by an increase in the cold dark matter density $\omega_c \equiv \Omega_c h^2$, while the baryon density ω_b remains well constrained. As a consequence, the baryon-to-CDM ratio Ω_b/Ω_c decreases with increasing N_{eff} .

This change in the relative abundances of baryons and dark matter affects the evolution of density perturbations prior to recombination. While baryons are tightly coupled to photons and

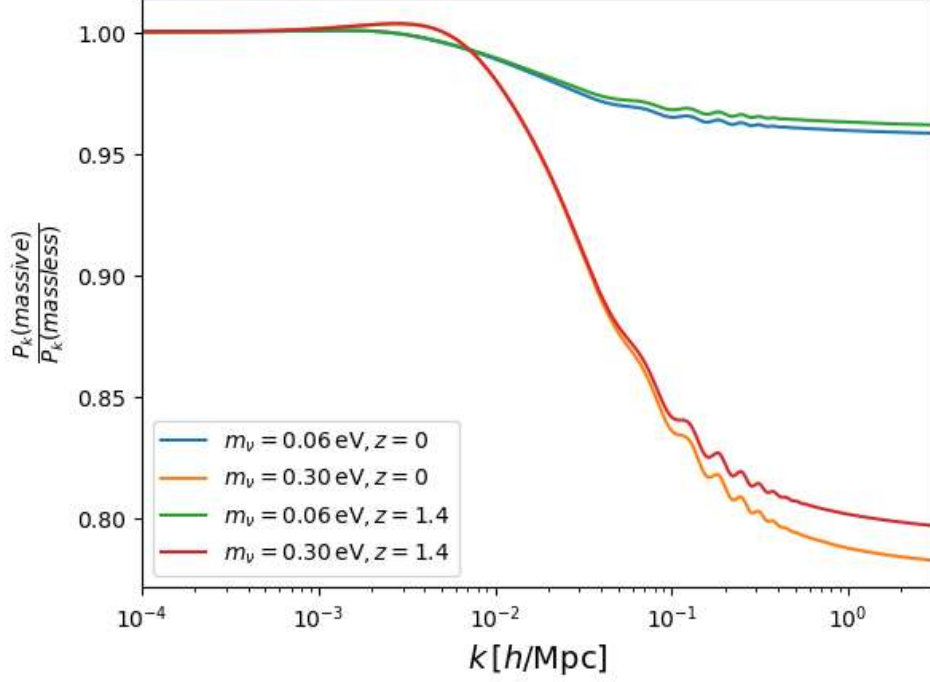


Figure 1.6: Ratio of the matter power spectrum in cosmologies with massive neutrinos to that with massless neutrinos, for different values of the total neutrino mass $\sum m_\nu$.

undergo acoustic oscillations, cold dark matter can cluster gravitationally. A lower baryon fraction therefore enhances the contribution of the more strongly clustered CDM component, leading to an increase in the small-scale matter power spectrum. At the same time, the amplitude of baryon acoustic oscillations (BAO) is reduced, since these features originate from the baryon–photon fluid. Variations in N_{eff} induce also a small phase shift in the BAO features, analogous to the phase shift observed in the CMB acoustic peaks due to neutrino free-streaming.

1.2.3.2 Linear and Non-Linear Matter Power Spectrum

On large scales, where density fluctuations are small ($\delta \ll 1$), the evolution of perturbations is well described by linear perturbation theory, as shown in Eq. (1.93). In this regime, different Fourier modes evolve independently, and the matter power spectrum can be written as

$$(1.100) \quad P(k, z) = P_{\text{prim}}(k) T^2(k) D^2(k, z),$$

where $P_{\text{prim}}(k) = A_s k^{n_s}$ is the primordial power spectrum generated during inflation, $T(k)$ is the transfer function encoding the evolution of perturbations through horizon entry and the radiation-dominated era, and $D(k, z)$ is the growth factor describing the late-time evolution of structures. It is important to distinguish between the primordial and the late-time matter power spectra. The primordial power spectrum encodes the initial conditions of cosmological perturbations and

is independent of the particle content of the Universe at later times. The matter power spectrum $P(k, z)$ describes the distribution of density fluctuations at a given redshift and incorporates the effects of all relevant physical processes through the transfer function and the growth factor. The transfer function $T(k)$ describes how primordial perturbations are processed by the microphysics of the early Universe. More precisely, it is obtained by solving the coupled Einstein–Boltzmann equations with initial conditions set by inflation. In this framework, primordial perturbations are described by a stochastic field, typically the curvature perturbation, whose two-point correlation function defines the primordial power spectrum. The transfer function then propagates these initial perturbations forward in time, accounting for all relevant physical processes such as horizon entry, radiation driving, baryon–photon interactions, and neutrino free-streaming.

In the standard Λ CDM model without massive neutrinos, the growth factor is scale-independent, $D(k, z) \equiv D(z)$, and the time and scale dependence of $P(k, z)$ factorize. However, in the presence of massive neutrinos, free-streaming suppresses the growth of structure below the free-streaming scale, leading to a scale-dependent growth factor $D(k, z)$. As a consequence, the matter power spectrum exhibits a characteristic suppression on small scales ($k > k_{\text{fs}}$), as discussed in the previous section. Importantly, massive neutrinos do not modify the primordial power spectrum, but affect both $T(k)$ and $D(k, z)$, making the matter power spectrum the primary observable to constrain their mass.

On sufficiently small scales, once density fluctuations become non-linear ($\delta \sim 1$), different Fourier modes couple and linear theory breaks down. In this regime, analytic treatments are no longer sufficient, and one must rely on numerical N -body simulations or semi-analytic prescriptions. A widely used approach is the *halofit* model [Smith et al. \[2003\]](#), [Takahashi et al. \[2012\]](#), which provides a fitting formula for the non-linear matter power spectrum calibrated against simulations. Extensions of *halofit* including massive neutrinos have been developed [Bird et al. \[2012\]](#), allowing accurate predictions across a wide range of scales. The combination of large-scale linear information, where neutrino effects are theoretically clean, with small-scale non-linear measurements, where the statistical power is greater, makes large-scale structure surveys a particularly sensitive probe of the total neutrino mass Σm_ν .

1.3 Current and Future Constraints on Neutrino Properties

In this section, we present an overview of current constraints on neutrino properties derived from both cosmological observations and laboratory experiments, as well as prospects for future improvements. In particular, we focus on bounds on the sum of neutrino masses and the effective number of relativistic species. We also discuss the expected sensitivity of upcoming experiments and surveys, and their potential to significantly improve our understanding of neutrino physics and to address outstanding questions beyond the Standard Model. Neutrino properties are constrained through two complementary observational approaches; terrestrial particle physics

experiments and cosmological observations. Laboratory based experiments provide direct measurements of neutrino properties, such as mass splittings and mixing angles, with remarkable precision. Cosmology offers a unique large-scale laboratory in which neutrinos leave distinct imprints on the evolution of the Universe, affecting observables such as the CMB and the LSS. As discussed throughout this chapter, neutrinos influence several key cosmological processes, including Big Bang Nucleosynthesis, recombination, and structure formation. These effects allow cosmological data to place competitive and complementary constraints on neutrino properties.

We begin by considering the effective number of relativistic species, N_{eff} , which parametrizes the energy density in relativistic particles in the early Universe. From particle physics experiments, the number of light neutrino species has been precisely measured at LEP through the Z boson production in e^+e^- collision, yielding $N_\nu = 2.984 \pm 0.008$ [Schael et al. \[2006\]](#), consistent with three active neutrinos. More recently, improved analyses have revisited this measurement by accounting for correlated luminosity systematics and using updated calculations of the Bhabha scattering cross section. These refinements lead to the value $N_\nu = 2.9963 \pm 0.0074$ [Janot and Jadach \[2020\]](#), [Voutsinas et al. \[2020\]](#). To compare with cosmological measurements, this value can be rescaled to the effective number of relativistic species by accounting for Standard Model corrections, $N_{\text{eff}} = N_\nu \times \frac{3.044}{3} = 3.040 \pm 0.0075$. From a cosmological perspective, the effective number of relativistic species, N_{eff} , is tightly constrained by both BBN and CMB observations. Recent analyses combining a new determination of the primordial helium abundance from the Large Binocular Telescope (LBT) Y_P project with Planck CMB data give us $N_{\text{eff}} = 2.968 \pm 0.083$ at 1σ C.L [Yeh et al. \[2026\]](#). High-resolution CMB temperature and polarization measurements by the Atacama Cosmology Telescope (ACT) and the South Pole Telescope (SPT) combined with BAO measurements from DESI, constraints $N_{\text{eff}} = 2.86 \pm 0.13$ at 1σ C.L [Calabrese et al. \[2025\]](#). More recently, a combination of primordial helium abundance measurements from the LBT Y_P Project, primordial deuterium data, Planck, ACT and SPT CMB, together with BAO measurements from DESI, has yielded one of the most precise constraints so far, $N_{\text{eff}} = 2.990 \pm 0.070$ at 1σ C.L [Goldstein and Hill \[2026\]](#).

We now consider constraints on the absolute neutrino mass scale. As discussed previously, neutrino oscillation experiments provide precise measurements of the mass-squared differences, implying a robust lower bound on the sum of neutrino masses, $\sum m_\nu \gtrsim 0.06$ eV for the normal hierarchy and $\sum m_\nu \gtrsim 0.10$ eV for the inverted hierarchy. However, oscillation experiments are insensitive to the absolute mass scale. Laboratory experiments provide direct constraints on neutrino masses through kinematic measurements. In particular, the KATRIN experiment, based on precision studies of the tritium β -decay spectrum, currently sets an upper bound on the effective electron antineutrino mass of $m_\beta < 0.45$ eV at 2σ C.L [Aker et al. \[2024\]](#). Cosmological observations currently provide the most stringent bounds on the sum of neutrino masses. Massive neutrinos affect both the expansion history of the Universe and the growth of cosmic structures. Their impact on the CMB anisotropies and, in particular, on the matter power spectrum allows

for precise constraints on $\sum m_\nu$. Planck analysis combining CMB temperature and polarization data with BAO and Supernovae measurements have established the bound $\sum m_\nu \lesssim 0.12$ eV at 2σ C.L for the Λ CDM model [Aghanim et al. \[2020c\]](#). More recently, the inclusion of baryon acoustic oscillation (BAO) data from the DESI collaboration has significantly tightened cosmological constraints on the sum of neutrino masses, yielding limits approaching $\sum m_\nu \lesssim 0.07$ eV at 2σ C.L [Elbers et al. \[2025b\]](#). Even stronger bounds have been reported when combining CMB data with DESI BAO, galaxy cluster measurements, and a prior on the Hubble constant from SH0ES, leading to $\sum m_\nu < 0.042$ eV at 2σ C.L [Jiang et al. \[2025\]](#). These constraints are obtained within the context of the Λ CDM model. Such stringent upper limits are particularly interesting when compared to the lower bounds inferred from neutrino oscillation experiments. This shows that current cosmological data, assuming a standard Λ CDM like scenario, presents a statistical preference for the normal ordering of neutrino masses [Abdul Karim et al. \[2025b\]](#), [Jimenez et al. \[2022a\]](#), [Gariazzo et al. \[2022a\]](#).

It is important to note, however, that cosmological constraints on neutrino masses are model-dependent. In particular, the total neutrino mass is known to be degenerate with the dark energy equation of state parameter w [Hannestad \[2005\]](#), [Roy Choudhury and Hannestad \[2020\]](#). As a result, in phenomenological dynamical dark energy models, the bounds on $\sum m_\nu$ can relax to $\sum m_\nu \lesssim 0.11$ eV at 2σ C.L. On the other hand, in physically motivated dynamical dark energy models based on scalar fields, the constraints remain close to those obtained in the minimal cosmological model, with $\sum m_\nu \lesssim 0.071$ eV at 2σ C.L, which excludes the inverted ordering at approximately 3.3σ significance (**see chapter 5 for this results**). The absolute neutrino mass also exhibits a degeneracy with the curvature of the Universe, Ω_k . When the curvature is allowed to deviate from flatness in a Λ CDM+ Ω_k model, the constraints on the neutrino mass are significantly relaxed, yielding $\sum m_\nu < 0.10$ eV at 2σ C.L [Chen and Zaldarriaga \[2025\]](#) (**see chapter 3 for a discussion on this degeneracy**). Interestingly, regardless of the model, current cosmological data show an intriguing preference for zero or even negative values of $\sum m_\nu$, possibly indicating limitations of these models in describing the role of neutrinos in the evolution of the Universe [Elbers et al. \[2025b\]](#), [Yang et al. \[2026\]](#). Looking ahead, future experiments such as CMB-S4, Euclid, and next-generation galaxy surveys are expected to significantly improve the sensitivity to neutrino masses, potentially reaching $\sigma(\sum m_\nu) \sim 0.02$ eV. Such a precision would enable a high-significance detection of the minimal neutrino mass scale and provide critical insights into neutrino physics [Archidiacono and et al. \[2025\]](#) (**see chapter 4 for neutrino mass forecasts on Future J-PAS stage IV galaxy survey**).

1.4 This Thesis

The main goal of this thesis is to infer neutrino properties from cosmological observations and to investigate how the neutrino sector impacts the evolution of the Universe within the context

of different cosmological models. We achieve this by analyzing current cosmological data across a range of theoretical frameworks in order to constrain neutrino properties, in particular their absolute mass scale, and to assess their influence on the dynamics of the Universe. The structure of this thesis is as follows.

Chapter 2 presents a detailed analysis of the small-field double-well inflationary model and its observational viability in light of current Cosmic Microwave Background data. Considering both minimal and nonminimal couplings between the inflaton field and the Ricci scalar, we perform a Markov Chain Monte Carlo analysis to explore the model parameter space. We further investigate the implications of these results within the context of the canonical type-I seesaw mechanism, deriving constraints on the vacuum expectation value of the inflaton field and on the strength of its coupling to gravity in the nonminimal scenario.

Chapter 3 explores the framework of quintessential inflation, which provides a unified description of the early and late accelerated phases of the Universe through a single scalar field. Within the context of α -attractor models, we investigate the impact of this unification on cosmological constraints of the sum of neutrino masses.

Chapter 4 explores the forecasting the sensitivity of the J-PAS large-scale structure survey to the sum of neutrino masses. Using the Fisher matrix formalism we forecast neutrino mass under various observational configurations of galaxy clustering. We explore different tracers, the impact of nonlinear corrections, and varying sky coverage.

Chapter 5 investigates the interplay between massive neutrinos and dynamical dark energy models. In particular, we assess how different parameterizations of the dark energy equation of state, such as the CPL, Barboza–Alcaniz, and Jassal–Bagla–Padmanabhan models, as well as a physically motivated thawing scenario, affect cosmological constraints on $\sum m_\nu$.

Chapter 6 presents a summary of the main results obtained in this thesis, along with a discussion of future prospects for neutrino cosmology.

DOUBLE-WELL INFLATION: OBSERVATIONAL CONSTRAINTS AND THEORETICAL IMPLICATIONS

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We revisit the small field double-well inflationary model and investigate its observational viability in light of the current Cosmic Microwave Background data. In particular, considering scenarios with minimal and nonminimal coupling between the inflaton field and the Ricci scalar, we perform a Monte Carlo Markov chain analysis to probe the model's parameter space. We also investigate the consequences of the cosmological results in the canonical type-I seesaw mechanism context and obtain constraints on the vacuum expectation value of the inflaton field, together with the amplitude of the coupling to gravity in the nonminimal case. We employ a Bayesian procedure to compare the model's predictions with the Starobinsky inflationary scenario and find a strong statistical preference for the latter against the minimal and nonminimal coupled double-well scenario.

2.1 Introduction

The inflationary scenario provides a mechanism to explain the initial conditions of the observable universe [Starobinsky \[1980\]](#), [Guth \[1981\]](#), [Linde \[1982\]](#). In the standard picture, the cosmic acceleration is driven by the dynamics of a scalar field, the inflaton, through the slow-roll mechanism. Quantum fluctuations are generated in the chaotic regime, inducing curvature perturbations and, ultimately, the fluctuations present in the Cosmic Microwave Background (CMB) [Ade et al. \[2018a\]](#), [Aghanim et al. \[2020a\]](#), as well as the Large-Scale Structures (LSS) formation [Ross et al. \[2015\]](#), [Alam et al. \[2017\]](#), [Scolnic et al. \[2018\]](#).

In the previous decades, there has been an extensive effort to determine which inflationary scenario best describes the cosmological data [Martin et al. \[2014\]](#). One particularly simple realization arises from the Double-Well potential [Linde and Linde \[1994\]](#), [Vilenkin \[1994\]](#), [Garcia-Bellido and Liddle \[1997\]](#), [Green and Liddle \[1997\]](#), [Martin et al. \[2014\]](#), first introduced as a toy symmetry-breaking model [Goldstone \[1961\]](#), with the vacuum expectation value (vev) v_ϕ setting the symmetry-breaking scale. For a sufficiently large v_ϕ value, one expects a flat region for amplitudes of the field close enough to the origin of the potential, allowing for slow-roll inflation. Despite the slow-roll analysis pointing to tension with current data [Akrami et al. \[2020\]](#), [Aghanim et al. \[2020c\]](#), extensions to the canonical scenario may displace the model predictions to the sweet spot of the constrained parameters. This becomes particularly true when a non-minimal coupling between the inflaton field and gravity is evoked, as demonstrated in [Linde et al. \[2011\]](#), [Bostan et al. \[2018\]](#). Nevertheless, a complete, updated statistical analysis of these models is still lacking in the literature.

From another perspective, it is instructive to investigate which other possible roles the inflaton may assume during the subsequent evolution of the Universe. One interesting conjecture relates the inflaton field to the seesaw mechanism. The seesaw mechanism, proposed in the late '70s, hatches as an elegant description of neutrino masses origin [Minkowski \[1977\]](#), [Yanagida \[1979\]](#), [Gell-Mann et al. \[1979\]](#), [Mohapatra and Senjanovic \[1980\]](#), [Schechter and Valle \[1980\]](#).

The high-energy scale M defines the lepton number breaking ($\Delta L = 2$), interweaving its diverse realizations. The canonical type-I seesaw includes three sterile fermions, N_i , and one singlet scalar, ϕ , to the standard model of fundamental particles (SM) content to generate M dynamically. The high energies associated with the lepton number breaking scale M build a challenging scenario for probing ϕ , taking into account its production at collider experiments or even its off-shell effects on the flavor physics. Still, such phenomena can leave a singular signature in the primordial Universe when related to the inflationary dynamics.

In this work, we investigate the observational viability of the Double-Well inflation in the face of the current CMB data. In particular, we performed a Monte Carlo Markov Chain analysis to apply the Bayesian parameter selection procedure to constrain the usual cosmological parameters and the inflaton vev v_ϕ . We also explore the impact of a non-zero, non-minimal coupling of the inflaton to gravity, with the strength given by a parameter ξ , to establish possible correlations between values of the coupling and the remaining cosmological parameters. Furthermore, we discuss the physical consequences of associating the inflaton field to the scale of the lepton number breaking in the type-I seesaw mechanisms, $M = v_\phi$. In particular, we consider the effects of the sterile sector on possible rare lepton decays.

This paper is organized as follows: Sections II and III review the Double-Well model and present our methodology and results, respectively. Section IV details the type-I Seesaw mechanism and how it relates to our results, while Section V is dedicated to the discussion and our concluding remarks.

2.2 The Model

In the Double-Well scenario, the slow-roll predictions are obtained from the prototype symmetry breaking potential,

$$(2.1) \quad V(\phi) = \frac{\lambda}{4}(\phi^2 - v_\phi^2)^2,$$

where λ is a dimensionless coupling constant, and the vev v_ϕ that determines the minimum of the potential. In what follows, we explore the possibility of a flat region for small values of the inflaton field, $\phi < v_\phi$, characteristic of small field inflation scenarios [Kolb \[1999\]](#). Such configuration can also be understood as a particular case of the Hilltop Inflation ($p = 2$) (see e.g. [Boubekeur and Lyth \[2005\]](#), [Tzirakis and Kinney \[2007\]](#)).

The study of the inflationary dynamics follows from the slow-roll parameters,

$$(2.2) \quad \epsilon = \frac{M_P^2}{2} \left(\frac{V'}{V} \right)^2 \quad \text{and} \quad \eta = M_P^2 \left(\frac{V''}{V} \right),$$

where $'$ indicates the derivative with respect to the inflaton field ϕ and $M_P = \frac{1}{\sqrt{8\pi G}}$ stands for the reduced Planck mass. The accelerated expansion occurs mainly in the slow-roll regime where $\epsilon, \eta \ll 1$, and continues until $\epsilon, \eta \sim 1$. The standard approach quantifies the cosmological

perturbations according to the power-law expansion of the primordial power spectrum. For scalar perturbations,

$$(2.3) \quad \ln \frac{P_{\mathcal{R}}(k)}{P_{\mathcal{R}}(k_*)} = (n_s - 1) \ln \left(\frac{k}{k_*} \right) + \frac{\alpha_s}{2} \ln \left(\frac{k}{k_*} \right)^2 + \dots,$$

and similarly for the tensor perturbations, where the ellipsis represents high order in the logarithm and k_* is the pivot scale. Deep inside the slow-roll regime, the inflationary observables and the slow-roll parameters are associated in a simple way. In particular, the predictions for the scalar tilt n_s and the tensor-to-scalar ratio r are computed via:

$$(2.4) \quad n_s = 1 - 6\epsilon_* + 2\eta_* \quad \text{and} \quad r = 16\epsilon_*,$$

where the subscript $*$ denotes that predictions are evaluated for the amplitude $\phi = \phi_*$, associated with the time at which the pivot scale crosses the Hubble horizon. The Planck Collaboration [Akrami et al. \[2020\]](#) estimates $n_s = 0.9651 \pm 0.0041$ at 68% confidence level (C.L.). An upper limit is also obtained for the tensor-to-scalar ratio, $r < 0.06$, at 95% (C.L.).

To complete the description of the scalar perturbations one needs to specify the amplitude of scalar perturbations,

$$(2.5) \quad A_s = P_{\mathcal{R}}(k_*) = \frac{V}{24M_P^4 \pi^2 \epsilon} \bigg|_{\phi=\phi_*}.$$

The Planck Collaboration has set $\ln(10^{10} A_s) = 3.044 \pm 0.014$, also at 68% (C.L.). Its explicit relation to V allows to obtain a correlation between the free parameters of the potential. Particularly for the Double-Well, one may have $\lambda(v_\phi)$ or $\lambda(v_\phi, \xi)$, where the latter stands for the non-minimal scenario.

For the minimally coupled case, the inflationary predictions of the potential (2.1) are known, and Fig. 5.1 depicts its behaviour in the the solid blue lines. Note that while a transplanckian vev ($v_\phi/M_P > 15$) is necessary for n_s to lie into the viable region allowed by Planck, relatively high values for the tensor-to-scalar ratio r are obtained as consequence, assigning a tension between the predictions of the model and the recent data from the B mode polarization of the CMB as obtained by BICEP/Keck Array telescopes [Ade et al. \[2018b, 2021\]](#). This can motivate a variety of extensions to the standard inflationary picture, including interacting neutrinos [Roy Choudhury et al. \[2022\]](#), [Cuesta et al. \[2023, 2022\]](#), [Berryman et al. \[2023\]](#), [Escudero and Witte \[2020\]](#), [Bostan and Choudhury \[2024\]](#), inflation with thermal effects [Berera \[1995\]](#), [Berera et al. \[2009\]](#), [Bartrum et al. \[2014\]](#), and non-minimal couplings of the scalar field with the gravitational sector [Bostan et al. \[2018\]](#), [Accetta et al. \[1985\]](#). This last avenue has been explored for decades, and it has been shown that for a variety of models that are excluded when considered in the minimal setting, a non-minimal coupling can be able to restore concordance with data [Starobinsky \[1980\]](#), [Spokoyny \[1984\]](#), [Salopek et al. \[1989\]](#), [Bezrukov and Shaposhnikov \[2008\]](#).

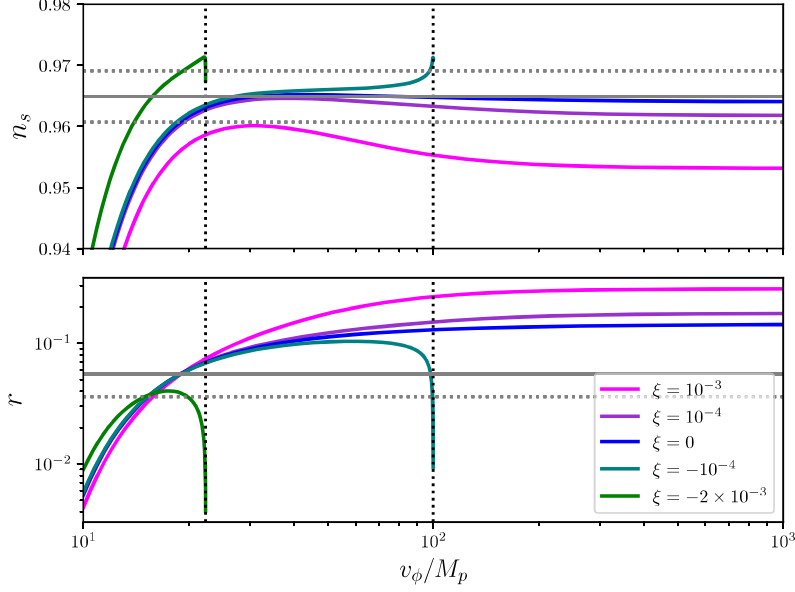


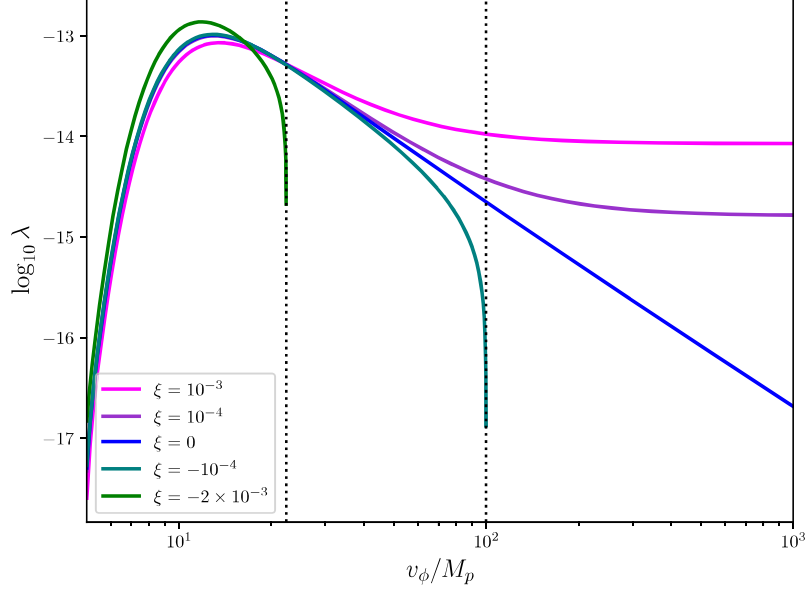
Figure 2.1: n_s and r as a function of the vev v_ϕ/M_P for different values of the non-minimal coupling parameter ξ , including the minimally coupled model (blue line). In the upper panel, the horizontal lines refer to the Planck constraint of $n_s = 0.9649 \pm 0.0042$ (at 68% C.L.), while in the lower panel, the horizontal lines refer to the upper limit on the tensor-to-scalar ratio from BK15 and BK18 of $r < 0.056$ and $r < 0.036$ (at 95% C.L.), respectively. The dotted vertical lines indicate the maximum value of v_ϕ/M_P for which the slow-roll regime is valid, when the coupling is negative. We fixed the number of e-folds before the end of inflation to $N_\star = 55$.

Here, we test this approach by considering a gravitational sector enhanced by a non-minimal coupling with the inflaton of the form,

$$(2.6) \quad S_J \supset \int d^4x \sqrt{-g} \frac{M_P^2}{2} \Omega^2(\phi) R,$$

where $\Omega^2(\phi) = 1 + \xi\phi^2/M_P^2$ and R is the Ricci scalar. When the non-minimal coupling constant ξ equals zero, we recover the minimally coupled model. By using the formalism applied in previous works [Campista et al. \[2017\]](#), [Rodrigues et al. \[2020, 2021a, 2023\]](#), [Santos et al. \[2024\]](#), we can readily compute the predictions for the spectral index n_s and the tensor-to-scalar ratio r as shown in fig. 5.1. We plot both n_s and r as a function of the vev v_ϕ/M_P , and we immediately see that when $\xi < 0$, there is an upper limit on the vev. This result emerges from the condition that the effective gravitational coupling, settled by $\Omega^2(\phi)$, remains positive during the whole field region where inflation takes place. In the Einstein frame, this limit translates into an infinitely large potential $V_E(\phi) \equiv V(\phi)/\Omega^4$ when the inflaton ϕ approaches $M_P/\sqrt{|\xi|}$, which is not suitable for the slow-roll inflation [Linde et al. \[2011\]](#), [Reyimuaji and Zhang \[2021\]](#).

We notice how the sign of ξ makes all the difference in the ability of the extended model to provide predictions that agree with the current constraints. For $\xi > 0$, the predictions worsen as


 Figure 2.2: v_ϕ vs $\log_{10} \lambda$ for $N_\star = 55$.

n_s becomes too low while r increases (as shown by the purple curve). Conversely, a negative ξ can guarantee the agreement, as the curves for n_s can pass the viable region while r can be lower than the current upper limits on r . This, in turn, also results in a significant decrease in the maximum viable value for v_ϕ/M_P , as seen when we compare the teal curve ($\xi = -10^{-4}$) and the green one ($\xi = -2 \times 10^{-3}$). As already pointed out in [Linde et al. \[2011\]](#), in the limit of $|\xi|v^2 \rightarrow M_P^2$, the predictions of the small field Double-Well scenario coincide with the canonical Higgs Inflation with $\xi \gg 1$.

From the amplitude of scalar perturbations (2.5), one can constrain one of the free parameters of the inflationary potential once the Planck value for A_s is considered. In Fig. (5.2) we present the results in the v_ϕ vs $\log_{10}(\lambda)$ plane. Again, the dotted vertical lines denote the upper limit on v_ϕ allowed by a negative ξ .

2.3 Methodology and Results

We perform a comprehensive numerical analysis to constrain the cosmological parameters of the models above discussed. As we also seek to determine the statistical preference for a given model through a robust criterion, we use the cobaya sampler [Torrado and Lewis \[2021a\]](#), [Torrado and Lewis \[2019\]](#), whose Monte Carlo Markov Chains (MCMC) produced will be analysed by the MCEvidence Python code [Heavens et al. \[2017\]](#) to compute the Bayesian evidence for each model. The theoretical input will be given through an implementation of the double-well potential on the Code for Anisotropies of the Microwave Background (CAMB) [Lewis et al. \[2000\]](#), [Howlett](#)

et al. [2012]. For this analysis, we employ a model with the following free parameters: the baryon density parameter ω_b , the cold dark matter density parameter ω_{cdm} , the reionization optical depth τ , and the angular scale of the sound horizon at the drag epoch θ_s . Within the Double-Well model, n_s and r are both derived parameters, as well as λ , determined from eq. (2.5); as a consequence, both v_ϕ/M_P and ξ are free to vary. In this manner, we choose to vary the parameter $\log_{10} v_\phi/M_P$ and leave a free ξ or set $\xi = 0$ in the non-minimally and minimally coupled models, respectively.

As for the priors of the new parameters, we note that the model supports a weaker coupling with gravity, of order $|\xi| \sim 10^{-3} - 10^{-2}$. We then set $\xi : [-0.05, 0.05]$, as it is consistent with our initial tests. For $\log_{10} v_\phi/M_P$, we leave the lower limit above the one allowed for the model to achieve slow-roll inflation, being $\log_{10} v_\phi/M_P = 0.5$, while we set $\log_{10} v_\phi/M_P = 2$ as a general upper limit, but being controlled by the condition on ξ described above. It's worth mentioning that we fix the sum of neutrino masses at 0.06 eV and utilize the pivot scale set by Planck for the primordial spectrum, $k_\star = 0.05 \text{ Mpc}^{-1}$. We also fixed the number of e-folds remaining until the end of inflation as $N_\star = 55$.

We perform our analysis using the latest Planck data Aghanim et al. [2020c]. Specifically, we use the high-multipole `HiLLiPoP TTTEEE` likelihood Tristram et al. [2024], add the low-multipole data from the Commander low- ℓ TT and the `LoLLiPoP EE` likelihoods. Combined with the updated CMB lensing potential likelihood Carron et al. [2022], we denote these data sets as PR4. We also consider the 2018 BICEP/Keck Array B-mode polarization data (BK18) from the BICEP3 and Keck Array CMB Experiments Ade et al. [2021].

Fig. 5.3 and Table 2.1 summarize the main results of our analyses. When considering PR4+BK18 data, we obtained vacuum expectation values for the minimally and non-minimally coupled cases, respectively, of $v_\phi/M_P = 1.258^{+0.035}_{-0.048}$ and $v_\phi/M_P = 0.92^{+0.084}_{-0.26}$ at a 68% (C.L.). We also obtain a constraint on the coupling strength for the non-minimal case, $\xi = -0.0169^{+0.015}_{-0.0059}$ also at 68% (C.L.). This result seems to exclude completely the minimally-coupled case, in which a negative ξ is preferred, in agreement with the n_s and r predictions shown in Fig. 5.1. We also observe a clear correlation between the ξ and $\log_{10} v_\phi/M_P$ parameters, in that a lower vev is achieved if the coupling becomes more negative.

There is a slight difference in the constraints on the inflationary parameters between the minimal and non-minimal cases, with the most significant difference concerning the vacuum expectation values. We also find a small shift towards lower values of H_0 in the minimally coupled scenario.

For completeness, we also analyse whether the minimally or non-minimally coupled cases are preferred by the data when compared to the concordance model. We perform this analysis by using a Bayesian model-selection Trotta [2008]. From the Bayes' theorem we can write the posterior probability of one model given a dataset d as,

$$(2.7) \quad p(m|d) \propto p(d|m)\pi(m),$$

Parameter	Minimal DW		Non-minimal DW	
	Mean value	Best-fit	Mean value	Best-fit
PR4+BK18				
ω_b	0.02216 ± 0.00012	0.02213	0.02224 ± 0.00012	0.02224
ω_c	0.12044 ± 0.00089	0.12031	0.11905 ± 0.00096	0.11984
$100\theta_{MC}$	1.04069 ± 0.00024	1.04063	1.04083 ± 0.00024	1.0407
τ_{reio}	0.0554 ± 0.0054	0.0519	0.0591 ± 0.0059	0.0619
$\ln 10^{10} A_s$	3.038 ± 0.011	3.031	3.046 ± 0.012	3.051
H_0	66.96 ± 0.40	66.96	67.53 ± 0.42	67.24
$\log_{10} v_\phi/M_P$	$1.258^{+0.035}_{-0.048}$	1.247	$0.920^{+0.084}_{-0.26}$	0.9287
ξ	–	–	$-0.0169^{+0.015}_{-0.0059}$	-0.01119
$\ln B$	-7.815		-5.689	

Table 2.1: The 68% (C.L.) estimates and best-fit values for the cosmological parameters obtained from the PR4+BK18 data.

where $p(d|m)$ is the marginal likelihood for the model, or often called evidence $\mathcal{Z}$, and $\pi(m)$ is the prior probability of the model before the data. If we consider two models, m_0 and m_1 , the *Bayes factor* is given by the ratio of the evidence for the two models,

$$(2.8) \quad B_{0,1} = \frac{p(d|m_0)}{p(d|m_1)} = \frac{\mathcal{Z}(m_0)}{\mathcal{Z}(m_1)}.$$

Here, we are assuming that $\pi(m_0) = \pi(m_1) = 1/2$, that is, the priors are noncommittal about the alternatives. The strength of evidence will be defined by $|\ln B_{0,1}|$, and is interpreted using the Jeffreys' scale [Grandis et al. \[2016\]](#). For a $|\ln B_{0,1}| < 1$, we have no preference for either model, is inconclusive. For a value of $|\ln B_{0,1}| = 1$, it shows a positive evidence for the model m_0 over the model m_1 . For $|\ln B_{0,1}| = 2.5$ and $|\ln B_{0,1}| = 5$, it gives a moderate and strong evidence in favors of m_0 , respectively.

We analyze our generated MCMC chains with the MCEvidence code [Heavens et al. \[2017\]](#). Following the latest Planck report, we compare our results with the Starobinsky model. Table 2.1 shows the Bayes factor ($\ln B$) for the considered models. We find $\ln B = -7.815$ and $\ln B = -5.689$ for the minimally and non-minimally coupled models, respectively, showing a clear preference for the Starobinsky model; however, the addition of a nonminimal coupling alleviates this result, as it helps bring the inflationary predictions to the acceptable limits, at the cost of increased complexity. There is strong evidence against these models, which is understandable since previous results on the minimally coupled model already indicated moderate evidence against it, confirmed and exacerbated by the updated data sets. Although the non-minimally coupled model alleviates this issue, the inclusion of ξ , which increases the parameter space, will also contribute to the computation of the evidence. Therefore, we find a result compatible with the model's added

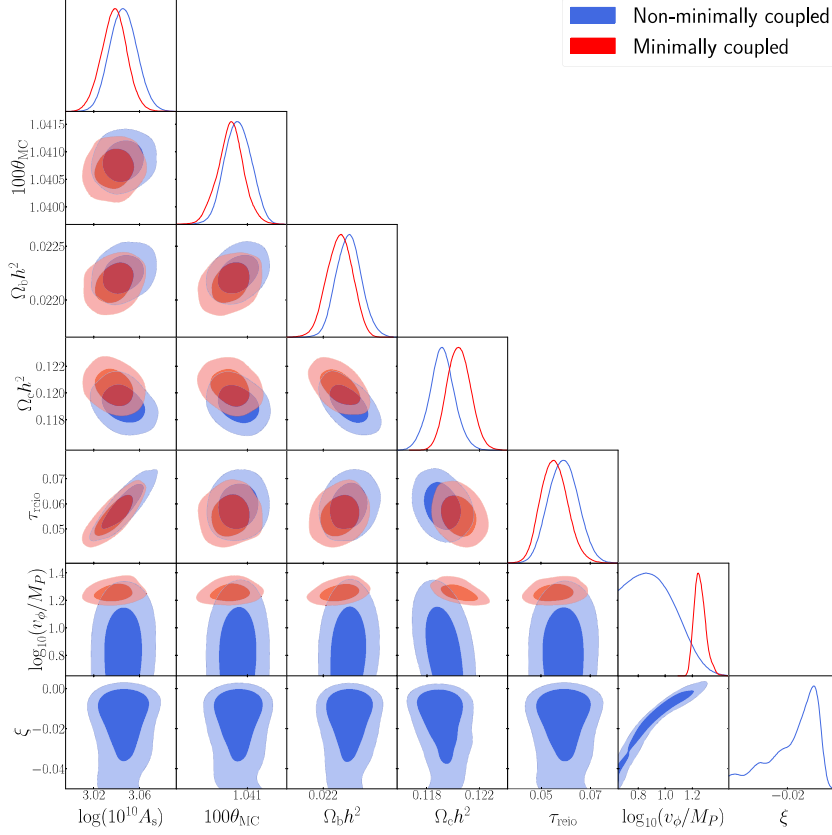


Figure 2.3: Confidence contours at 68% and 95% C.L. for the minimally and non-minimally coupled double-well models, constrained using Planck+BK18 data.

complexity.

2.4 Rare Lepton Decays

We proceed with our analysis by exploring possible consequences of the Double-Well inflation in the context of the seesaw mechanism. The canonical structure of type-I seesaw includes three sterile fermions and a scalar $SU(2)_L$ singlet to the SM content in order to introduce the Lagrangian terms,

$$(2.9) \quad \mathcal{L} \supset h_{ij} \bar{f}_{iL} i \sigma_2 H^\dagger N_j + h'_{ij} \phi \bar{N}_i^C N_j,$$

where $f = (\nu, e)_L^T$ and $H = (H^+, H^0)^T$ are the standard leptons and the Higgs doublets, respectively. By allowing a non-zero vev for the singlet field one obtains a Majorana mass term for the sterile fermions, $M = h' v_\phi / \sqrt{2}$. Here, v_ϕ is constrained by cosmological perturbation data (Table 2.1). A Dirac mass term is also obtained once the electroweak symmetry breaking is accomplished, $m_D = h v_h / \sqrt{2}$. Once the block diagonalization procedure [Schechter and Valle \[1982\]](#), [Hettmansperger et al. \[2011\]](#), [Dias et al. \[2012\]](#) has been applied, it is possible to use

the information on the neutrino oscillation pattern (mixing angles and quadratic mass differences [Workman and Others \[2022\]](#)) to estimate their Yukawa couplings, h_{ij} and h'_{ij} . The latter is particularly useful when estimating extra contributions to flavor violating rare lepton decays.

Such processes can be triggered through active-sterile neutrinos superposition in the charged current Lagrangian,

$$\begin{aligned}
 \mathcal{L}_{CC} &= -\frac{g}{\sqrt{2}} \bar{e}_{iL} \gamma^\mu \nu_{iL} W_\mu^- + h.c. \\
 (2.10) \quad &\simeq -\frac{g}{\sqrt{2}} \bar{e}_{iL} \gamma^\mu \left\{ U_{ij}^0 n_{jL}^0 + F_{ij} n_{jL}^1 \right\} W_\mu^- + h.c.,
 \end{aligned}$$

where $F = m_D M^{-1}$ and U^0 is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix. The contribution of the sterile (heavy) neutrinos to the charged current is suppressed by the ratio v_h/v_ϕ . We consider the predictions of the model only to the decay channel $\mu \rightarrow e \gamma$, as the current observational bound to this process is the most restrictive one [Workman and Others \[2022\]](#).

The branching ratio for the decay mediated by the three sterile neutrinos is evaluated by [Ilakovac and Pilaftsis \[1995\]](#)

$$(2.11) \quad B(\mu \rightarrow e \gamma) = \frac{\alpha_W^3 \sin^2(\theta_W)}{256\pi^2} \frac{m_\mu^5}{m_W^4 \Gamma_\mu} \left| \sum_{i=1}^3 F_{2i} F_{1i} I\left(\frac{M^2}{m_W^2}\right) \right|^2,$$

where

$$(2.12) \quad I(x) = -\frac{2x^3 + 5x^2 - x}{4(1-x)^3} - \frac{3x^3 \ln x}{2(1-x)^4}.$$

In the expression above, α_W stands for $\frac{g^2}{4\pi}$ and g is the weak gauge coupling. θ_W is the Weinberg angle, while Γ_μ is the total decay width of the muon. m_μ and m_W are the pole mass for the muon and the W boson, respectively. The values for these quantities can be found in [Workman and Others \[2022\]](#). We also consider the normal hierarchy case and a degenerated sterile sector with $h' \sim 10^{-41}$, the best-fit value from the MCMC analysis for the inflaton vacuum $v_\phi \simeq 17.66 M_P$ and the electroweak breaking scale $v_H \simeq 246$ GeV to obtain

$$(2.13) \quad B(\mu \rightarrow e \gamma) \simeq 3.93 \times 10^{-53},$$

which is negligible when compared to the highly restrictive superior bound on $B(\mu \rightarrow e \gamma) < 4.2 \times 10^{-13}$ [Workman and Others \[2022\]](#).

The same process can be employed to obtain the corresponding branching ratio in the non-minimal scenario of the Double-Well inflation. In this case, one needs to re-scale the Dirac and Majorana mass terms according to the Weyl factor $\Omega^2 = 1 + \xi \phi^2/M_P^2$, $\tilde{m}^D = m^D/\Omega$ and $\tilde{M} = M/\Omega$, following the conformal transformation of the metric to the Einstein frame [Garcia-Bellido et al.](#)

¹ $h' \lesssim 10^{-4}$ in order to preserve the unitary aspect of neutrino-neutrino scattering.

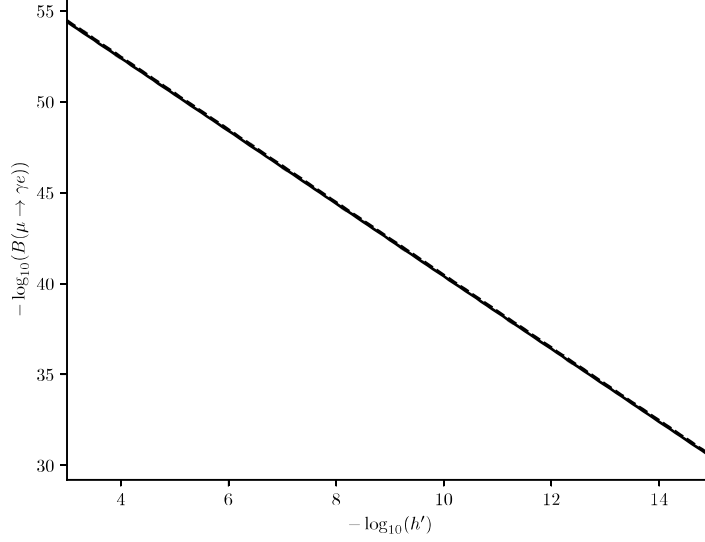


Figure 2.4: Branching ratio of the rare lepton decay $\mu \rightarrow e \gamma$ against the Yukawa coupling h' for the minimal (solid black) and non-minimal (dashed black) scenarios of the Double-Well inflation. We consider $\sum m_i = 0.057$ eV.

[2009]. For $h' = 10^{-4}$ and considering the best-fit value for $\xi \simeq -0.011$ and $v_\phi = 8.49 M_P^2$, one obtains $B(\mu \rightarrow e \gamma) \simeq 3.31 \times 10^{-53}$.

Fig. (2.4) shows the branching ratio in the minimal and non-minimal scenarios for different values of h' , where it is evident that a sizable contribution to the Lepton violating decay is obtained only for tiny values of the Yukawa coupling of the sterile neutrinos, roughly $h' \lesssim 10^{-20}$. Considering that one of the goals of the seesaw mechanism is to obtain the correct neutrino mass pattern without resorting to huge fine-tuning in the Yukawa sector, one may infer that such a signature of the seesaw mechanism is highly improbable to be detected, relegating the sterile neutrinos of the Double-Well scenario to the dark sector of the Universe. The decay in the nonminimal scenario is slightly smaller when compared to the minimal one.

2.5 Discussion

The double-well potential arises in the context of spontaneous symmetry breaking, where the vacuum expectation value v_ϕ determines the energy scale of the transition from a symmetric to an asymmetric phase of the system. Once settled in the vacuum state, the scalar field acquires a

²One should be aware that some combinations of the parameters ξ and v_ϕ can lead to a negative conformal factor, $\Omega^2 < 0$, even within the 1σ confidence limit for their integrated posterior probabilities. The distorted 2D confidence region obtained for these parameters is a symptom of these nonphysical combinations.

positive non-zero squared mass term $m_\phi^2 = \lambda v_\phi^2$. This transition is not instantaneous in general, allowing for a slow-roll phase at the cost of a rather unnatural large vev.

In this paper, we performed a complete statistical analysis of the observational viability of the small-field Double-Well inflationary model. In particular, we applied a Bayesian procedure of parameter selection and comparison of models to probe the likelihood of the model against the Starobinsky scenario, as well as to investigate the phenomenological consequences of the underlying field theory. From the comparison of models, we obtained a preference for the reference (Starobinsky) model against the minimally coupled double-well model, in agreement with results discussed in the Planck 2015 report [Ade et al. \[2016\]](#), but now with a more significant difference due to the updated data sets. Including a non-minimal coupling with gravity improves the results, but the higher complexity still makes the model less favorable.

For the scenario minimally coupled to gravity, we obtain $v_\phi \simeq 17.66 M_P$ according to the selection of parameters procedure (best-fit value). Once the Planck normalization to the amplitude of scalar perturbation is considered, one can compute the inflaton's quartic coupling $\lambda \simeq 2.86 \times 10^{-15}$, which enables $m_\phi \simeq 9.45 \times 10^{-7} M_P$ for the mass of the inflaton field.

In the non-minimal scenario these parameters are slightly different, following the effect of the direct coupling between the inflaton field and the Ricci scalar. Assuming $v_\phi \simeq 8.49 M_P$ for the vev, $\lambda \simeq 3.28 \times 10^{-13}$ for the inflaton's coupling and $\xi \simeq -0.011$ for the non-minimal coupling strength, one should obtain for the mass of the inflaton $m_\phi \simeq 1.10 \times 10^{-5} M_P$.

Such high values for the inflaton mass prevent this particle from being produced thermally in the early universe, as well as produced on-shell by collider experiments. One may be tempted to investigate the secondary effects triggered by the inflaton field immersed in a context beyond the cosmological set up. We investigated the consequences for the inflaton field immersed in the framework of canonical type-I seesaw and computed the contributions of the right-handed neutrinos for the rare lepton decay $\mu^- \rightarrow e^- \gamma$. We found such a contribution to be negligible when compared to the current experimental upper bound on this processes.

The large values inferred for the inflaton's vev in the Double-Well scenarios seclude the associated sector to complete darkness. Rescuing the Double-Well models for the visible sector must necessarily cover a realization of the inflation period for lower values of v_ϕ . A particularly attractive scenario may arise in the framework of warm inflation [Berera \[1995\]](#), [Berera et al. \[2009\]](#), where the dissipation coefficient over-dampens the inflaton's trajectory in field space, optimizing the slow-roll. The predicted results of this configuration will appear in a forthcoming communication.

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COSMOLOGICAL CONSTRAINTS ON NEUTRINO MASSES IN QUINTESSENTIAL INFLATION

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Quintessential inflation provides a unified description of the early and late accelerated phases of the Universe, linking the inflationary epoch to the present-day dark energy–dominated era through a single scalar degree of freedom. In this work, we explore the implications of this unification for cosmological constraints on the sum of neutrino masses. Focusing on the α -attractor scenario, we implement the model in a modified version of the Boltzmann solver CLASS to compute the relevant cosmological observables and perform a Bayesian parameter estimation analysis using data from the cosmic microwave background (CMB), baryon acoustic oscillations (BAOs), and Type Ia supernovae. The model naturally breaks the degeneracy between the dark energy equation of state and the total neutrino mass, yielding tight upper bounds of $\sum m_\nu < 0.067$ eV for flat spatial geometry and $\sum m_\nu < 0.116$ eV when curvature is included. We also provide forecasts for future probes, showing that the Simons Observatory, LiteBIRD, and Euclid configurations may reduce the uncertainty on $\sum m_\nu$ by $\approx 9\%$, while the precision on the quintessential parameter α_{QI} is improved by $\approx 72\%$. These results highlight the importance of consistently accounting for neutrino mass when assessing the viability of extensions to the standard cosmological model.

3.1 Introduction

In standard cosmology, the observable Universe originated in an early phase of accelerated expansion, dubbed Inflation, whose dynamics is commonly described by a scalar field known as the inflaton. The transition to a radiation-dominated Universe is achieved through the reheating mechanism, where the energy stored in the inflaton condensate is transferred to the standard particles. The subsequent expansion of the cosmos is dominated by an ordinary fluid composed initially of radiation and, as this fluid cools down, by a cold dark matter component. More recently, observations of distance–redshift relation of Type Ia supernovae revealed a late accelerated expansion [Riess et al. \[1998\]](#), [Perlmutter et al. \[1999\]](#) compatible with the description of an exotic component, generically dubbed dark energy (DE), whose energy density remains constant, or nearly so, as the Universe expands.

The scenario drawn from the cosmological model enables the use of probes from distinct moments of the cosmic evolution to constrain the very same parameters. A remarkable example is the present rate of the cosmic expansion, H_0 . One could choose to constrain the Hubble parameter indirectly from the angular scale of the acoustic peaks in the cosmic microwave background (CMB) [Aghanim et al. \[2020c\]](#), for example, or by the observations of the type Ia supernovae light curves and the cosmic ladder techniques [Riess et al. \[2021\]](#). While such complementarity can be useful to break possible degeneracies between the cosmological model parameters, the increasing amount and precision of cosmological data open up room for tensions between the values inferred from different probes. In the former example, the tension between early and late time probes of H_0 ranges between $4\sigma - 6\sigma$ [Di Valentino et al. \[2021b\]](#).

The cohesion between the different cosmological eras can also be explored from the model building perspective. One particularly interesting example can be found in the quintessential inflation (QI) models [Peebles and Vilenkin \[1999\]](#), [Peloso and Rosati \[1999\]](#), [Dimopoulos and Valle \[2002\]](#), [Tashiro et al. \[2004\]](#). In this scenario, inflation and quintessence are unified in the same framework, with the Lagrangian of the inflaton/quintessence field prescribing the characteristics of the primordial and late universe. By connecting early- and late-time cosmic expansion, this class of models enables the use of different probes to infer conceptually uncorrelated parameters, further increasing precision. Such a feature is already explored in the literature [Geng et al. \[2015\]](#), [Akrami et al. \[2018, 2021\]](#), [Dimopoulos and Owen \[2017\]](#), [Geng et al. \[2017\]](#), [Alestas et al. \[2024\]](#), [Giarè et al. \[2024\]](#), [Rodrigues et al. \[2025c\]](#), where particularly in [Akrami et al. \[2021\]](#) an increase in the forecasted precision of the spectral index, n_s , was obtained.

In this context, QI models can help disentangle well-known parameter degeneracies. At small scales, the gravitational backreaction of massive neutrinos affects the clustering of cold dark matter in a way similar to dark energy, slowing down the late-time growth of matter perturbations (see [Lesgourgues and Pastor \[2006\]](#) for more details). A degeneracy is then present on the effects of $\sum m_{\nu_i}$ and the equation of state w . Phenomenological parametrizations of w , which have recently shown a preference for a phantom crossing with $w < -1$, lead to a loosening of the bounds on $\sum m_{\nu_i}$. On the other hand, when considering physical models for dark energy, where the equation of state is, by construction, constrained to never cross the phantom divide, $w \geq -1$, the bounds on $\sum m_{\nu_i}$ are tightened [Hannestad \[2005\]](#), [Roy Choudhury and Hannestad \[2020\]](#), [Lorenz et al. \[2017\]](#), [Rodrigues et al. \[2025b\]](#).

From a particle physics perspective, the oscillatory behavior of the three neutrino flavors is observed in solar and atmospheric neutrino measurements. This points to the existence of neutrino mass terms in the neutrino Lagrangian, despite predictions from the Standard Model of particles. Oscillation experiments are insensitive to the absolute mass scale of the neutrinos, but are useful for probing their squared mass differences. For atmospheric neutrinos, one can settle $\Delta m_{32}^2 \approx 2.45 \times 10^{-3} \text{ eV}^2$, assuming the Normal Ordering (NO), or $\Delta m_{32}^2 \approx -2.53 \times 10^{-3} \text{ eV}^2$ for the Inverted Ordering (IO), while from solar neutrinos one obtains $\Delta m_{21}^2 \approx 7.5 \times 10^{-5} \text{ eV}^2$ [Navas et al. \[2024\]](#). This configuration enables at least two non-relativistic species of neutrinos, while lower limits on the sum of their masses can be estimated. For the NO one may have $\sum m_\nu > 0.06 \text{ eV}$, while IO requires $\sum m_\nu > 0.10 \text{ eV}$.

In a scenario where cosmological upper limits for neutrino masses are increasingly approaching the lower limits obtained when considering oscillation experiments [Palanque-Delabrouille et al. \[2020\]](#), [Di Valentino et al. \[2021a, 2022\]](#), [Adame et al. \[2024c\]](#), [Elbers et al. \[2025b\]](#), the analysis of neutrino masses in the context of quintessential inflation may emerge as a crucial test for the viability of these models.

In this work, we perform a Monte-Carlo Markov Chain (MCMC) analysis of the sum of neutrino masses in the framework of the α -attractor quintessence inflation model in light of

current CMB, baryon acoustic oscillations, and supernovae data. The forecasted sensitivity of the model to the configurations of future large-scale structure and cosmic microwave background (CMB) surveys has also been considered. Specifically, we use data from Planck, DESI DR2 BAO, and Pantheon+ for the real-data analysis, while the forecasts are made for the next-generation CMB experiments, LiteBIRD and Simons Observatory, combined with the Euclid survey configurations.

As main results, we obtain the constraints on the parameter space for two configurations of the cosmological model, with the most restrictive case arising from the QI model in a flat spatial geometry. For the analysis with real data, we established an upper bound for the total mass of neutrinos, $\sum m_\nu < 0.067$ eV, while the predicted sensitivity of the next-generation experiments was set at 0.0192 eV at 68% confidence level (C.L.). We also managed to constrain the typical parameter of the α -attractor scenarios to $\alpha_{QI} = 1.70^{+1.0}_{-0.41}$, while the forecast analysis returned 0.286 as the sensitivity of the next generation experiments, both at 68% C.L.

3.2 The α -attractor Quintessential Inflation

The α -attractor models were initially conceived as an inflationary scenario based on supergravity theories [Kallosh and Linde \[2013\]](#), [Kallosh et al. \[2013, 2014\]](#). The underlying Lagrangian density governing the inflaton dynamics has the general form,

$$(3.1) \quad \mathcal{L} = \sqrt{-g} \left(\frac{M_p^2}{2} R - \frac{1}{2} \frac{\partial_\mu \phi \partial^\mu \phi}{\left(1 - \frac{\phi^2}{6\alpha_{QI}}\right)^2} - V(\phi) \right),$$

where M_p is the reduced Planck mass. The typical non-canonical kinetic term has a pole structure controlled by α_{QI} which, in turn, is defined by the curvature of the scalar manifold in the Kähler potential [Kallosh et al. \[2013\]](#).

The features of the α -attractor models are better understood in terms of the canonical field, φ , obtained according to the relation,

$$(3.2) \quad \phi \equiv \sqrt{6\alpha_{QI}} \tanh \frac{\varphi}{\sqrt{6\alpha_{QI}}}.$$

Such transformation drives the poles to large amplitudes of the canonical field, $|\varphi| \rightarrow \infty$, yielding asymptotic flat directions for the potential energy when expressed in terms of the canonical variable [Kallosh et al. \[2013\]](#), [Linde \[2017\]](#). Those are the key ingredients that make the α -attractor structure suitable for inflation and quintessence. For the appropriate choice of the underlying supergravity theory, the scalar potential may develop an inflationary plateau in the limit of large and positive field, $\varphi \rightarrow +\infty$, while acquiring a runaway behavior in the negative asymptotic values, $\varphi \rightarrow -\infty$, where the scalar field drives the late accelerated expansion.

The inflationary stage is dictated, as usual, by the slow-roll parameters, ϵ , η . In particular, the contour conditions for the metric perturbations are described in terms of the power-law expansion

of the primordial power spectrum. For scalar perturbations,

$$(3.3) \quad \ln \frac{P_{\mathcal{R}}(k)}{P_{\mathcal{R}}(k_*)} = (n_s - 1) \ln \left(\frac{k}{k_*} \right) + \frac{\alpha_s}{2} \ln \left(\frac{k}{k_*} \right)^2 + \dots,$$

and similarly for the tensor perturbations, where k_* is the pivot scale and the ellipse portrays the higher order terms. When the slow-roll regime is considered, $\epsilon, \eta \ll 1$, the spectral index and the tensor-to-scalar ratio are computed in terms of the slow-roll parameters,

$$(3.4) \quad n_s = 1 - 6\epsilon + 2\eta, \quad r = 16\epsilon,$$

respectively. One especially appealing feature of this inflationary scenario follows from its attractor behavior, which allows for general predictions for the inflationary observables [Linde \[2017\]](#),

$$(3.5) \quad n_s \simeq 1 - \frac{2}{N_*}, \quad r \simeq \frac{12\alpha_{QI}}{N_*^2},$$

where N_* is the number of e-folds, $N \equiv \ln(a)$. In particular, the relation of r to the Lagrangian parameter α_{QI} enables one to tune the spectral index and the tensor-to-scalar ratio simultaneously to the sweet-spot of Planck's observations ¹ [Aghanim et al. \[2020c\]](#). To complete the description of the primordial power spectrum, one may consider the amplitude of the scalar perturbations,

$$(3.6) \quad P_{\mathcal{R}}(k_*) \equiv A_s(k_*) = \left. \frac{V}{24M_P^4 \pi^2 \epsilon} \right|_{\varphi=\varphi_*},$$

where the field amplitude φ_* is computed at the time of horizon crossing for the pivot scale.

Unlike the other inflationary parameters, A_s is particularly sensitive to the potential energy amplitude. Here, we consider one of the simplest forms of the potential energy that can address inflation and quintessence² [Akrami et al. \[2018\]](#), [Aresté Saló et al. \[2021\]](#), [Dimopoulos et al. \[2018\]](#),

$$(3.7) \quad V(\varphi) = M^2 e^{\gamma \left(\tanh \varphi / \sqrt{6\alpha_{QI}} - 1 \right)}.$$

This potential assumes a step-like form, with the inflationary period taking place in the positive large field regime, where $V \rightarrow M^2$. According to the Klein-Gordon equation, the field will eventually roll down the potential hill into the negative region of field space. In the limit of large negative amplitudes, it will mimic a cosmological constant, $V \rightarrow M^2 e^{-2\gamma}$, establishing the late accelerated expansion of the Universe. The steepness of the potential function is tuned by γ

¹See [Rodrigues et al. \[2021b\]](#) for a MCMC analysis of the inflationary α -attractor scenario with a Higgs-like potential.

²Near the kinetic pole, canonical normalization implies $\phi = \sqrt{6\alpha_{QI}} (1 - 2e^{-2\varphi/\sqrt{6\alpha_{QI}}} + \dots)$. As a result, any regular (e.g. polynomial) $V(\phi)$ near $\phi \simeq \sqrt{6\alpha_{QI}}$ is mapped into a plateau with exponential corrections characteristic of α -attractor models [Kallosh et al. \[2013\]](#), [Linde \[2017\]](#).

in order to reproduce the correct vacuum energy density observed today, $\rho_\Lambda \sim 2.5 \times 10^{-11} \text{ eV}^4$ [Aghanim et al. \[2020c\]](#). Meanwhile, the measured value of A_s fixes the amplitude of the scalar potential through (3.6), giving

$$(3.8) \quad \frac{M^2}{M_P^4} = \frac{144\pi^2 \alpha_{QI} N_\star A_s}{(2N_\star - 3\alpha_{QI})^3}.$$

This set of conjectures brings the early and late-time conditions for accelerated expansion into agreement. In the following, we will detail the numerical methods performed to constrain the model and cosmological parameters.

3.3 Methodology

In this work, we perform a MCMC analysis to obtain the posterior probability distribution of the cosmological parameters. In particular, the Cobaya [Torrado and Lewis \[2021a\]](#) sampler code is used to build the chains, following the Metropolis-Hastings algorithm. The predictions for cosmological observables are obtained from the Cosmic Linear Anisotropy Resolution System (CLASS) [Blas et al. \[2011b\]](#) code, which has been modified to solve the Boltzmann equations for the quintessential α -attractor model.

We also perform an MCMC analysis with simulated data in order to forecast the sensitivity of the total neutrino mass, in the context of the quintessential inflation scenario, to future CMB and LSS experiments. In this case, the MontePython [Brinckmann and Lesgourgues \[2018\]](#) code is used to sample the likelihood of the mock data. For both real and simulated data analysis, the covariance matrix is updated until the Gelman-Rubin convergence criterion reaches the values of $R - 1 < \mathcal{O}(10^{-2})$. Finally, the chains obtained are analyzed and plotted using the GetDist code [Lewis \[2025\]](#). Below, we discuss the details of the cosmological model and the datasets used in this analysis.

3.3.1 The Model

The base parameters utilized in the Bayesian parameter estimation are specified as follows,

$$(3.9) \quad (\Omega_b h^2, \Omega_c h^2, 100\theta_s, \tau_{\text{reio}}, \ln(10^{10} A_s), n_s, \alpha_{QI}, \sum m_\nu),$$

where $\Omega_b h^2$ and $\Omega_c h^2$ represent the weighted abundances of baryons and cold dark matter, respectively; $100\theta_s$ denotes the angular acoustic scale; τ_{reio} is the optical depth; $\ln(10^{10} A_s)$ indicates the amplitude of scalar perturbations; n_s is the spectral index; α_{QI} refers to the α parameter from α -attractors; and $\sum m_\nu$ corresponds to the total neutrino mass. Hereafter dubbed CosmoI, this is the minimal set of quantities capable of describing the observables of a flat Universe, including the damping effect of the neutrinos' free-streaming and the angular distances in the latter Universe.

In order to consider the effects of a non-zero curvature, we also include the energy component associated with the curvature of space-time, Ω_k , as a varying parameter in the MCMC analysis,

$$(3.10) \quad (\Omega_b h^2, \Omega_c h^2, 100\theta_s, \tau_{\text{reio}}, \ln(10^{10} A_s), n_s, \Omega_k, \alpha_{\text{QI}}, \sum m_\nu).$$

Hereafter called CosmoII model. The curvature is particularly important in the late-time Universe as it influences the cosmic expansion and, consequently, displaces the angular distance between the observer and the source of the signal.

In time, it is useful to remind that the quintessential α -attractor scenario reconciles early and late expansion with the same physical mechanism. Numerically, this is accomplished by the shooting process, which adjusts one of the parameters of the quintessential potential to recover the correct budget equation at later times. In this work, the CLASS code was modified to perform shooting with the parameter γ of (3.7), making it a derived parameter in the MCMC analysis. Another important quantity in the analysis of the late-time behavior of the quintessential field is its initial position, or freezing value, φ_F Akrami et al. [2018, 2021]. The possible values of φ_F vary essentially due to the reheating mechanism that takes place after inflation. For our purposes, we follow Akrami et al. [2021], Rodrigues et al. [2025c] and fix $\varphi_F = -10$ in our analysis, which corresponds to the case of instant reheating.

3.3.2 The Data

3.3.2.1 Real data

As previously discussed, we can integrate early and late-time observations to explore how QI models can help address parameter degeneracies. In this context, we have combined observations from both early and late periods. For the early times, we utilized the latest Planck data release (PR4), processed using the NPIPE pipeline, which includes measurements of temperature (TT), polarization (EE), and the cross-correlation (TE) Rosenberg et al. [2022], Aghanim et al. [2020b]. The temperature power spectrum at low multipoles is modeled using the publicly available Commander likelihood. For the CMB lensing, we use the Planck PR4 likelihood, obtained through the reconstruction of the lensing potential maps Carron et al. [2022].

The late-time measurements include SNe Ia data from the Pantheon+ catalog Brout et al. [2022], Scolnic et al. [2022] and the latest geometric BAO observations from the DESI DR2 Abdul Karim et al. [2025b]. The BAO measurements can determine, at different redshifts, the angular scale imprinted in the large-scale structures at the baryon drag epoch, thereby providing constraints on the matter content of the Universe. It is important to note that BAO data alone cannot distinguish between the effects of massive neutrinos and those of the standard cold dark matter component. For this reason, it is necessary to combine them with CMB measurements. These can break the degeneracy between Ω_ν and Ω_c . They also allow one to constrain the total neutrino mass, $\sum m_\nu$. This is because massive neutrinos alter the time of matter–radiation

equality and suppress the growth of small-scale perturbations through free-streaming effects [Lesgourgues and Pastor \[2006\]](#).

In our analyses, we considered the CMB dataset and the combination of CMB, BAO, and SNe Ia.

3.3.2.2 Simulated data

For the forecast analysis, we consider a combined analysis of future CMB and galaxy clustering data. For the CMB part, we include the temperature (T) and polarization (E and B) modes, which are jointly described by the covariance matrix of angular power spectra,

$$(3.11) \quad \mathbf{C}_\ell = \begin{pmatrix} C_\ell^{TT} + N_\ell^{TT} & C_\ell^{TE} & 0 \\ C_\ell^{TE} & C_\ell^{EE} + N_\ell^{EE} & 0 \\ 0 & 0 & C_\ell^{BB} + N_\ell^{BB} \end{pmatrix},$$

where C_ℓ denotes the theoretical temperature, polarization, and cross-correlation power spectra, while N_ℓ represents the corresponding instrumental noise. The noise term for each channel is modeled as

$$(3.12) \quad N_\ell^{XX} = \sigma_X^2 \exp \left[\frac{\ell(\ell+1)\theta_{\text{FWHM}}^2}{8 \ln 2} \right],$$

with σ_X being the noise level for temperature or polarization, and θ_{FWHM} the full width at half maximum of the beam, expressed in radians.

For galaxy surveys, one is interested in computing the galaxy power spectrum, modeled as

$$(3.13) \quad P_{\text{obs}}(k_{\text{fid}}, \mu_{\text{fid}} z) = \frac{1}{q_\perp^2(z) q_\parallel(z)} \left\{ \frac{[b\sigma_8(z) + f(k, z)\sigma_8(z)\mu^2]^2}{1 + f(k, z)^2 k^2 \mu^2 \sigma_p^2(z)} \right. \\ \left. \times \frac{P_{\text{dw}}(k, \mu; z)}{\sigma_8^2(z)} F_z(k, \mu; z) + P_s(z) \right\},$$

where $f(k, z)$ is the linear growth rate, $P_{\text{dw}}(k, \mu; z)$ the de-wiggled (BAO-damped) power spectrum, and $P_s(z)$ the shot noise. The model incorporates three key physical effects: (i) the Alcock–Paczynski effect, represented by the prefactor $\frac{1}{q_\perp^2 q_\parallel}$, with $q_\perp = D_A(z)/D_{A, \text{fid}}(z)$ and $q_\parallel = H_{\text{fid}}(z)/H(z)$; (ii) redshift-space distortions (Kaiser effect), through the factor $[b(z)\sigma_8(z) + f(k, z)\sigma_8(z)\mu^2]$; and (iii) the Fingers-of-God suppression, encoded in the denominator term $1 + f(k, z)^2 k^2 \mu^2 \sigma_p^2(z)$, which accounts for small-scale random motions. The redshift uncertainty is modeled by the damping term $F_z(k, \mu; z) = \exp[-k^2 \mu^2 \sigma_r^2(z)]$, with $\sigma_r(z) = c\sigma_z/H(z)$.

In what follows, we describe the specific surveys used in our forecast analysis:

- **Simons Observatory:** The Simons Observatory [Ade et al. \[2019\]](#) is a ground-based survey located in Chile, whose focus is on smaller scales in the CMB spectra, expected to cover

Parameter	CMB	CMB + DESI + Pantheon+
$\Omega_b h^2$	0.02216 ± 0.00014	0.02232 ± 0.00013
$\Omega_c h^2$	0.1198 ± 0.0012	0.11786 ± 0.00070
$100\theta_s$	1.04172 ± 0.00024	1.04189 ± 0.00023
τ_{reio}	0.0545 ± 0.0071	0.0566 ± 0.0071
n_s	0.9629 ± 0.0040	0.9678 ± 0.0034
$\ln(10^{10} A_s)$	3.042 ± 0.014	3.042 ± 0.014
Σm_ν	$< 0.252 \text{ eV}$	$< 0.0670 \text{ eV}$
α_{QI}	< 3.45	$1.70^{+1.0}_{-0.41}$

Table 3.1: 1σ projections and 2σ superior limits of the posterior distribution of the CosmoI model parameters.

Parameter	CMB	CMB + DESI + Pantheon+
$\Omega_b h^2$	0.02219 ± 0.00015	0.02221 ± 0.00014
$\Omega_c h^2$	0.1190 ± 0.0013	0.1193 ± 0.0012
$100\theta_s$	1.04179 ± 0.00025	1.04175 ± 0.00024
τ_{reio}	$0.0473^{+0.0082}_{-0.0070}$	0.0544 ± 0.0072
n_s	0.9646 ± 0.0045	0.9642 ± 0.0042
$\ln(10^{10} A_s)$	$3.024^{+0.018}_{-0.015}$	3.041 ± 0.014
Σm_ν	$< 0.629 \text{ eV}$	$< 0.116 \text{ eV}$
α_{QI}	—	$1.69^{+1.0}_{-0.44}$
Ω_k	$-0.022^{+0.014}_{-0.010}$	0.0021 ± 0.0013

Table 3.2: 1σ projections and 2σ superior limits of the posterior distribution of the CosmoII model parameters.

$40 \leq \ell \leq 5000$ in the multipole range, thus being complementary to the current Planck data in the determination of cosmological parameters. We consider the above mentioned multipole range in our analysis, while taking $f_{\text{sky}} = 0.4$ as the sky fraction coverage. As for the noise modeling, we use the curves made available by the collaboration in Ade et al. [2019] for a baseline configuration (see their fig. 5) ³.

- **LiteBIRD:** The LiteBIRD survey Matsumura et al. [2014], Hazumi et al. [2019], Allys et al. [2023] primarily constrains the large angular scales, providing sensitivity to the low multipoles of the CMB temperature and polarization spectra. In our case, we use Eq. 3.12 to model the instrument noise, with sky fraction coverage of $f_{\text{sky}} = 0.7$, while $\sigma_{E,B}/\sqrt{2} = 4.17 \text{ } \mu\text{K-arcmin}$. The beam full width at half maximum is $\theta_{\text{FWHM}} = 23 \text{ arcmin}$,

³https://github.com/simonsobs/so_noise_models

Parameter	Simons Observatory + LiteBIRD	Simons Observatory + LiteBIRD + Euclid
$\Omega_b h^2$	0.000059	0.0000503
$\Omega_c h^2$	0.000819	0.000407
$100\theta_s$	0.000154	0.000113
τ_{reio}	0.00200	0.00194
n_s	0.00257	0.00196
$\ln(10^{10} A_s)$	0.00420	0.00382
$\sum m_\nu$	0.0530	0.0192
α_{QI}	0.479	0.286

 Table 3.3: 1σ forecasted constraints on the cosmological parameters.

in agreement with the 140 GHz frequency channel as displayed in [Paoletti and Finelli \[2019\]](#). In order to properly combine with Simons Observatory, we limit the multipole range as $2 \leq \ell < 40$, enough to cover the primordial part of the B-mode spectrum, as well as the reionization contribution to the E-mode CMB spectrum.

- **Euclid:** The Euclid mission [Casas et al. \[2024b\]](#), [Collaboration \[2024b\]](#), launched in mid-2023, began collecting scientific data in early 2024, to provide three-dimensional clustering measurements described by the observed galaxy power spectrum. The survey aims to map a sky fraction of approximately $f_{\text{sky}} = 0.3636$, with its first public data release expected in October 2026 [Collaboration \[2025\]](#). In this analysis, we focus exclusively on the galaxy clustering component, considering the spectroscopic part of the survey.

For the spectroscopic analysis, we consider scales up to $k_{\text{max}} = 0.25 \, h/\text{Mpc}$, corresponding to the conservative configuration defined in [Archidiacono and et al. \[2025\]](#), while taking $\sigma_z = 0.002$ for the spectroscopic redshift uncertainty. The likelihood implementation follows the official Euclid likelihood module available in MontePython [Casas et al. \[2024b\]](#).

3.4 Results and Discussion

In Figure (5.1), we present the contour regions obtained for the CosmoI model, and the corresponding 68% constraints in Table (5.1), in light of early- and late-time data. In particular, we explore two combinations of the datasets in our analysis. The first one considers the CMB temperature, polarization, and lensing data from Planck, while the second adds the BAO measurements from DESI's year two data release and the SNe Ia luminosity distances from Pantheon+ catalog.

From the figure, one can observe a substantial improvement in parameter constraints when early- and late-time observables are analyzed jointly—particularly for $\Omega_c h^2$, α_{QI} , and $\sum m_\nu$. The CMB-only contours for $\sum m_\nu$ and $\Omega_c h^2$ reveal a positive correlation, reflecting the geometric

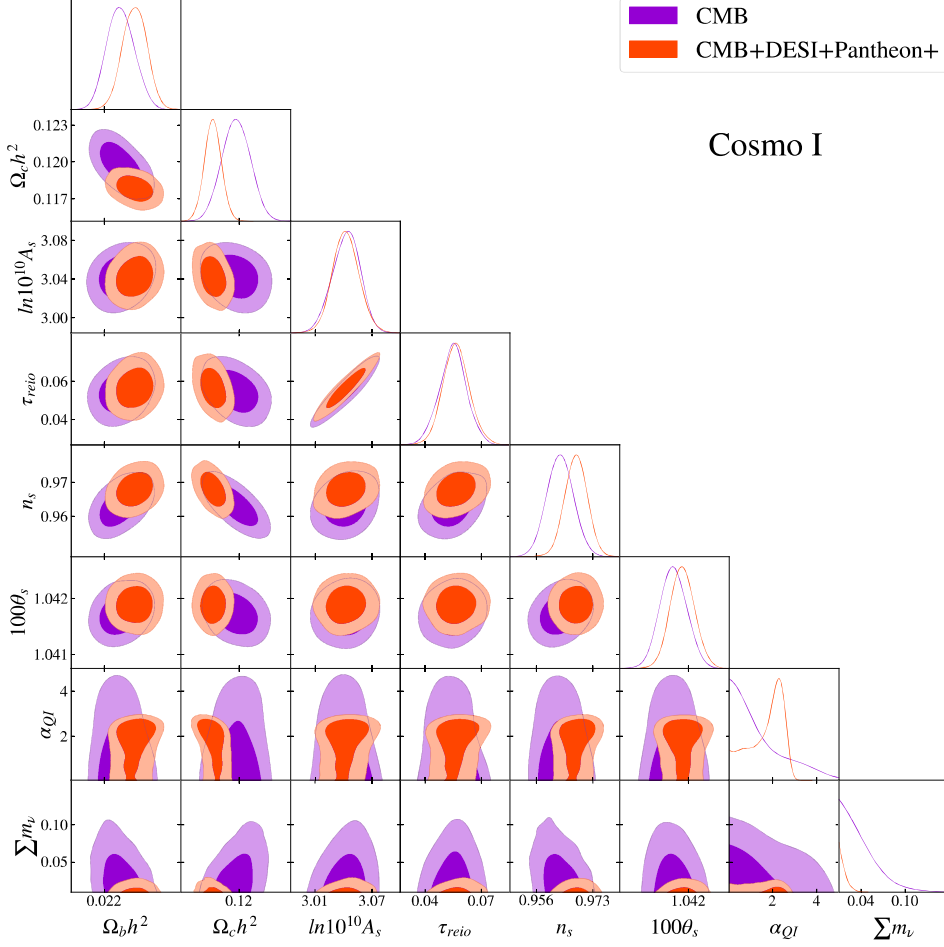


Figure 3.1: 68% and 95% contours and normalized posteriors on the quintessential inflation model given by Eq. (3.9).

degeneracy between $\sum m_\nu$, Ω_m , and H_0 [Loverde and Weiner \[2024\]](#). This degeneracy is partially broken when BAO distance measurements and supernova data are included in the analysis.

On the other hand, early-time observations alone are unable to constrain the curvature of the Kaller manifold, α_{QI} . The CMB-only data imposes merely an upper bound, $\alpha_{QI} < 3.45$, at the 95% C.L. This limitation is expected, since the constraining power of the CMB on this parameter originates from the absence of detected tensor modes, as described by Eq. (3.5).

The situation changes significantly when late-time data are included. As illustrated in Figure 2 of [Akrami et al. \[2021\]](#) and discussed therein, the parameter α_{QI} is directly linked to the dark energy equation of state at low redshift. As a result, tighter constraints are obtained when information from the local Universe is considered. In particular, the joint analysis of CMB + DESI + Pantheon+ data yields $\alpha_{QI} = 1.70^{+1.00}_{-0.41}$, at 68% C.L.⁴.

The combination of early- and late-time cosmological probes also improves the constraints on

⁴However, only an upper limit of $\alpha_{QI} < 2.64$ can be established at 95% C.L.

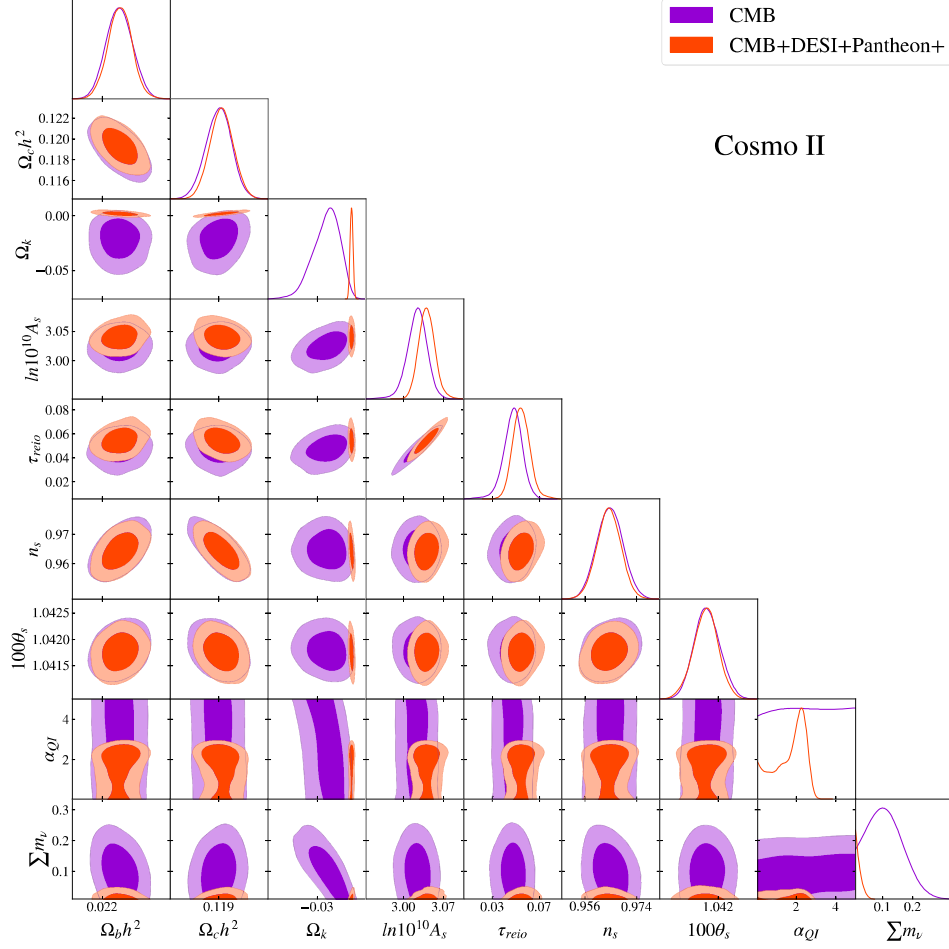
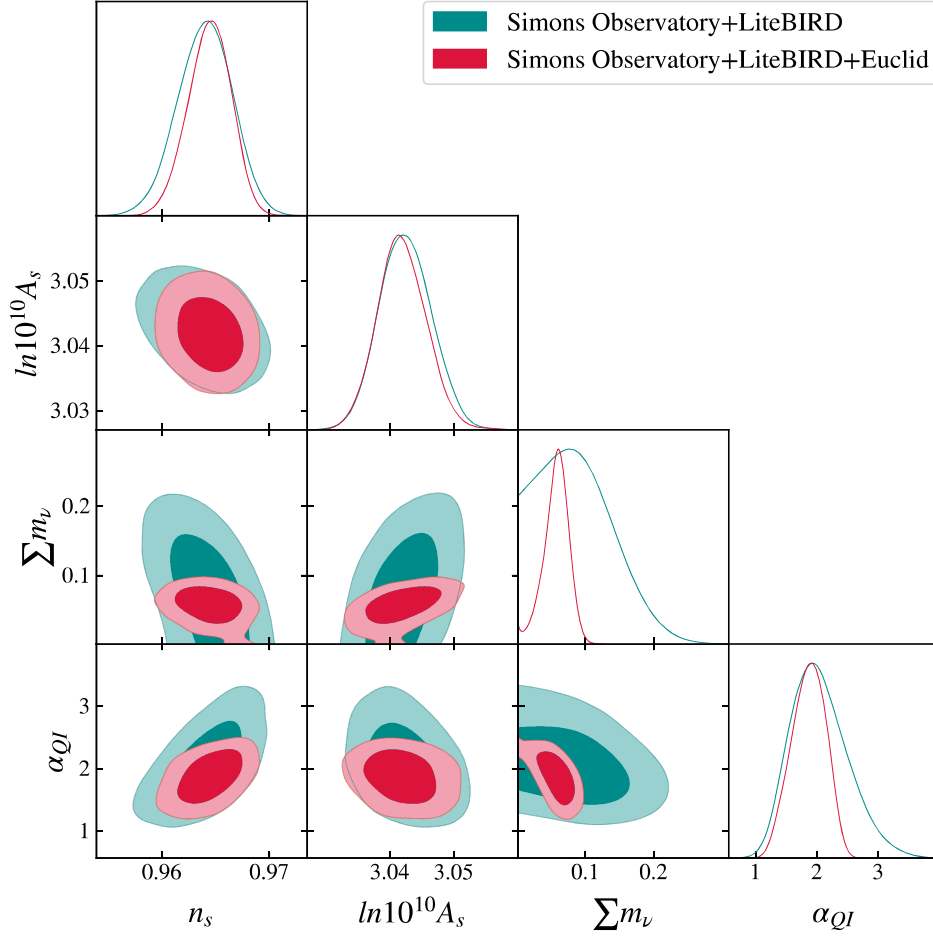


Figure 3.2: 68% and 95% contours and normalized posteriors on the quintessential inflation model given by Eq. (3.10).

the total neutrino mass, $\sum m_\nu$. In particular, the joint analysis of CMB, DESI, and Pantheon+ data yields an upper limit of $\sum m_\nu < 0.0670$ eV, which is approximately 73% more stringent than the constraint from CMB data alone, $\sum m_\nu < 0.252$ eV. As shown in the figure, the marginal posterior for $\sum m_\nu$ peaks near the lower boundary of the prior, indicating the presence of a prior-weight effect, extensively discussed in [Elbers et al. \[2025b\]](#).

The total neutrino mass affects the late-time cosmic expansion, modifying the transverse comoving distance to the last scattering surface and altering the angular scale of the acoustic peaks. Notably, the tension between the distance scales inferred from CMB and BAO observations can be alleviated by reducing $\sum m_\nu$ within the Λ CDM framework [Elbers et al. \[2025b\]](#). This mechanism contributes to the tighter constraints obtained from the combined dataset.

Previous analyses involving dynamical dark energy parameterizations suggest that such bounds can be relaxed in those models [Rodrigues et al. \[2025b\]](#), due to degeneracies between $\sum m_\nu$ and the dark energy equation of state, $\omega_\phi(z)$ [Hannestad \[2005\]](#), [Roy Choudhury and Hannestad](#)



Cosmo I

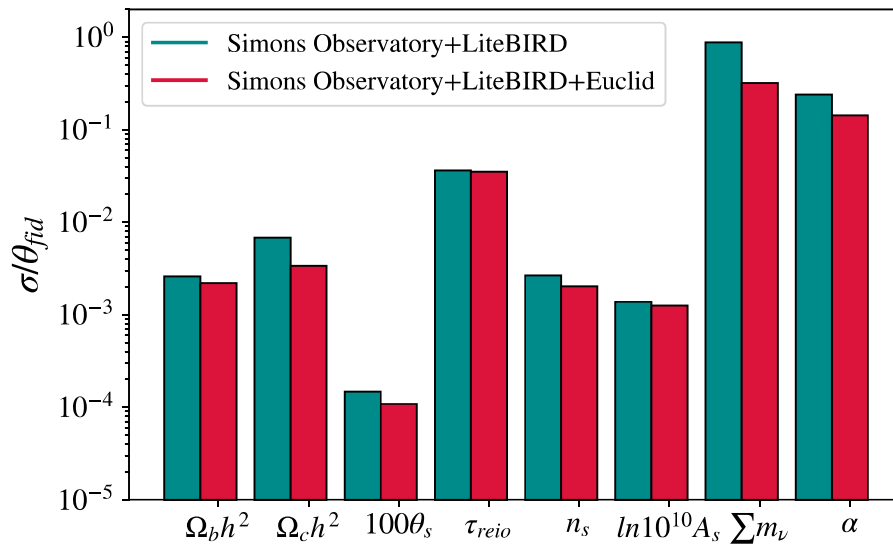


Figure 3.3: On the left panel, we show 68% and 95% contours and normalized posteriors on the forecasted sensitivity of the quintessential inflation ‘Cosmo I’ model for the Simons Observatory, LiteBIRD and Euclid configurations, while the 59% improvements on the uncertainties are shown on the right panel.

[2020]. This degeneracy is not present in the quintessential inflation model, where the unified description of early- and late-time expansion naturally breaks the $\sum m_\nu - \omega_\phi$ correlation, enabling bounds that are competitive with those from the most restrictive scenarios.

In order to further explore the effects of possible degeneracies, we also perform the statistical analysis of the CosmoII model, which considers the role of a non-zero space-time curvature in the background geometry. We present our results in Figure (5.2) and Table (3.2). The figure reveals a strong anti-correlation between Ω_k and $\sum m_\nu$ in the CMB-only contour. This degeneracy is sourced from the angular distance to the last scattering surface [Howlett et al. \[2012\]](#), being partially broken by the late-time data. The constraints are significantly improved when the BAO and supernovae data are included in the analysis. In particular, an improvement of 81.6% is obtained in the constraint of the total neutrino masses, and the 1σ interval is settled for the quintessential parameter, $\alpha_{QI} = 1.69^{+1.0}_{-0.44}$.

It is also possible to note, from Table (3.2), that late-time data displaces the constraints on the curvature to the positive range at 1σ , while still encompassing the zero point in the 95% confidence level interval, $\Omega_K = 0.0021^{+0.0027}_{-0.0025}$. Furthermore, the degeneracy in the background evolution of Ω_k and $\sum m_\nu$ is not entirely broken by late-time data, which results in a loosening of the upper limit for the total neutrino masses with respect to what is obtained for the CosmoI model. As a result, both the normal and inverted hierarchies of the neutrino masses are possible in the CosmoII scenario. A similar result is reported in Ref. [Chen and Zaldarriaga \[2025\]](#), using CMB + DESI BAO data. Allowing the spatial curvature to deviate from zero significantly relaxes the constraints on $\sum m_\nu$ within the Λ CDM model, while revealing a preference for negative Ω_k . Only in the analysis of the CosmoII model with exclusively CMB data did the posterior of $\sum m_\nu$ peak in the positive parameter region, while the inclusion of the BAO data in the analysis seems to shift the posterior to the negative range⁵. One may interpret this result as symptomatic of the discrepancy between the distance scales obtained from the DESI-BAO measurements and those from the CMB data, as discussed in [Elbers et al. \[2025b\]](#).

Finally, we test the models against the configurations of future cosmological surveys. Once again, we perform the MCMC analysis for simulated data with the settings of the Simons Observatory and LiteBIRD experiments (early-time data, CMB) and a joint analysis considering the configurations of the Euclid satellite (late-time data, galaxy survey). In Fig. 5.3, we present the 2D contour regions for a representative subset of parameters of the Cosmo I model (left panel) and the corresponding improvement in their uncertainties (right panel), represented by the ratio of the uncertainties shown in Table 4.5 with the fiducial parameters chosen for the analysis, being $(\Omega_b h^2, \Omega_c h^2, 100\theta_s, \tau_{reio}, n_s, \ln(10^{10} A_s), \sum m_\nu, \alpha) = (0.02277, 0.12, 1.04110, 0.055, 0.965, 3.045, 0.06, 2)$.

At first glance, one may note the improvement in precision when considering early- and late-time simulated data jointly, especially for α_{QI} and $\sum m_\nu$ parameters. Not surprisingly, the precision has also increased in relation to the previous MCMC analysis using current data. In

⁵Although we do not consider a negative prior for the total neutrino mass, we consider the peak of the posterior at 0 to indicate a preference of the models for negative values of this parameter.

particular, for the α -attractor parameter, α_{QI} , we obtain the forecasted 1σ uncertainty of 0.286 (Simons + LiteBIRD + Euclid), which represents an improvement in precision of approximately 72% with respect to the most constraining current case, i.e, the flat model with CMB + DESI + Pantheon+. For the total neutrino mass, we also find a significant increase in sensitivity, with the expected error of 0.0192 for the combined configurations of the future galaxy survey and CMB observations, corresponding to an improvement of roughly 8.6% from the constraining power obtained with the analysis using real data⁶. We also note that our results for $\sum m_\nu$ are competitive with those obtained in similar analyses [Rodrigues et al. \[2025d\]](#), [Qin et al. \[2026\]](#), [Chudaykin and Ivanov \[2019\]](#). In particular, using a Fisher matrix approach, Ref. [Rodrigues et al. \[2025d\]](#) reported a 2σ sensitivity of 0.033 eV for the total neutrino mass within the Λ CDM + $\sum m_\nu$ framework, considering the J-PAS galaxy survey configuration combined with CMB and SNe Ia observations. This constraint is approximately 15% tighter than ours, a difference that can be naturally attributed to the distinct data sets and analysis strategies employed to derive the limits on the sum of neutrino masses.

3.5 Conclusions

The ever-increasing volume of high-quality data has enabled cosmological surveys to probe both the early- and late-time Universe with unprecedented precision, placing stringent constraints on some of the most fundamental theories in physics. Examples include the inflationary paradigm and neutrino physics. In the latter case, cosmological upper bounds on the sum of neutrino masses have progressively approached the lower limits inferred from neutrino oscillation experiments. Although no significant tension is currently observed among existing data sets, forthcoming survey configurations are expected to substantially improve these constraints, potentially determining the mass scale of active neutrinos or ruling out some of the standard scenarios within the cosmological model.

In this work, we tested the total neutrino mass in the framework of a quintessential inflationary model based on the α -attractor scenario. Data from early- and late-time Universe were considered in order to break some degeneracies in the model parameters. We focus on two configurations of the cosmological model. The best constraint for the neutrino masses is obtained for the CosmoI model, $\sum m_\nu < 0.067$ eV, which seems to exclude the inverted ordering while still being in agreement with the normal one. On the other hand, the limits for the CosmoII model are consistent with both orderings, $\sum m_\nu < 0.116$ eV. Also, 68% C.L. constraints for α were obtained on both models (see (5.1) and (3.2)), while only upper limits were obtained at 95% C.L. In what concerns curvature, the CMB measurements returned negative values for Ω_k , with the late-time data displacing this parameter into the positive range. Although the 95% C.L. constraints on both models are still consistent with the flat Universe, which is an important consistency test

⁶The analysis of the Cosmo I model with CMB + DESI + Pantheon+ data has returned an integrated 1σ posterior uncertainty of 0.021 for the total mass of neutrinos.

for inflation itself, the apparent tension between these constraints may point to an underlying discrepancy between the scales measured from early and late Universe.

We also test the model against the configurations of future cosmological surveys. The results project a notable improvement in parameter sensitivity. For the α -attractor parameter, we obtain an improvement in precision of approximately 72% with respect to the most constraining current case. For the total neutrino mass, we also find a significant increase in sensitivity, corresponding to an improvement of roughly 8.6% in constraining power. It is worth noting that this increase in precision may help determine the scale of neutrino masses and, ultimately, establish the neutrino hierarchy.

Although current experiments have not yet provided a clear determination of the origin of neutrino masses, future cosmological surveys hold significant potential to address long-standing questions regarding the fundamental nature of these particles. Meanwhile, forecast constraints on the sum of neutrino masses suggest that this parameter may act as a powerful probe of viable quintessence scenarios.

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J-PAS: FORECASTING CONSTRAINTS ON NEUTRINO MASSES

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The large-scale structure survey J-PAS is taking data since October 2023. In this work, we present a forecast based on the Fisher matrix method to establish its sensitivity to the sum of the neutrino masses. We adapt the Fisher Galaxy Survey Code (FARO) to account for the neutrino mass under various configurations applied to galaxy clustering measurements. This approach allows us to test the sensitivity of J-PAS to the neutrino mass across different tracers, with and without non-linear corrections, and under varying sky coverage. We perform our forecast for two cosmological models: $\Lambda\text{CDM} + \sum m_\nu$ and $w_0 w_a \text{CDM} + \sum m_\nu$. We combine our J-PAS forecast with Cosmic Microwave Background (CMB) data from the Planck Collaboration and Type Ia supernova (SN) data from Pantheon Plus. Our analysis shows that, for a sky coverage of 8,500 square degrees, J-PAS galaxy clustering data alone will constrain the sum of the neutrino masses to an upper limit at 95% C.L of $\sum m_\nu < 0.32$ eV for the $\Lambda\text{CDM} + \sum m_\nu$ model, and $\sum m_\nu < 0.36$ eV for the $w_0 w_a \text{CDM} + \sum m_\nu$ model. When combined with Planck data, the upper limit improves significantly. For J-PAS+Planck at 95% C.L, we find $\sum m_\nu < 0.061$ eV for the $\Lambda\text{CDM} + \sum m_\nu$ model, and for J-PAS+Planck+Pantheon Plus, we obtain $\sum m_\nu < 0.12$ eV for the $w_0 w_a \text{CDM} + \sum m_\nu$ model. These results demonstrate that J-PAS clustering measurements can play a crucial role in addressing challenges in the neutrino sector, including potential tensions between cosmological and terrestrial measurements of the neutrino mass, as well as in determining the mass ordering.

4.1 Introduction

The mechanism behind the origin of particle masses was revealed after the award-winning work of Englert and Higgs [Englert and Brout \[1964\]](#), [Higgs \[1964b,a\]](#). The key ingredient in this mechanism is the Higgs field, a scalar field with a non-zero vacuum expectation value that provides a mass term to the particles in the standard model of particle physics (SM). This mechanism was supported by the discovery of a Higgs-like boson at the Large Hadron Collider (LHC) in 2012 [Aad et al. \[2012\]](#), [Chatrchyan et al. \[2012\]](#). Neutrinos, however, are the exception. Only left-handed (active) neutrinos participate in weak interactions, resulting in massless neutrinos for the SM particle content. The appearance of neutrino masses requires the addition of new fields whose mass is not protected by the SM symmetry, or of higher-dimensional operators that break lepton number [Weinberg \[1979\]](#). Therefore, the mechanisms usually used to describe the smallness of neutrino masses require new particles above the electroweak scale.

The scale of neutrino masses was apparent in the anomalies found in solar and atmospheric neutrinos, which point to an oscillatory behavior between the three flavors explained by the nondiagonal structure of their mass matrix. In particular, the set of parameters [Esteban et al.](#)

[2020], de Salas et al. [2021], Navas et al. [2024],

$$\begin{aligned}
 \sin^2 \theta_{12} &= 3.07_{-0.11}^{+0.12} \times 10^{-1}, \quad \sin^2 \theta_{23} = 5.61_{-0.15}^{+0.12} \times 10^{-1}, \\
 \sin^2 \theta_{13} &= 2.195_{-0.058}^{+0.054} \times 10^{-2}, \\
 (4.1) \quad \Delta m_{21}^2 &= 7.49_{-0.19}^{+0.19} \times 10^{-5}, \quad \Delta m_{32}^2 = 2.534_{-0.023}^{+0.025} \times 10^{-3}
 \end{aligned}$$

with the quadratic mass differences $m_{ij}^2 = m_i^2 - m_j^2$ evaluated in eV^2 , provide the best fit for the data. A lower limit for the sum of neutrino masses is then obtained from this information. For the Normal Hierarchy spectrum (NH), $m_1 < m_2 \ll m_3$, one obtains $\sum m_i \gtrsim 0.057 \text{ eV}$, while for the Inverted Hierarchical scheme (IH), $m_3 \ll m_1 < m_2$, one may infer $\sum m_i \gtrsim 0.099 \text{ eV}$, where the sum runs over the different mass states.

On the other hand, laboratory experiments such as KATRIN Osipowicz et al. [2001] (tritium beta decays) are rapidly improving the constraints on the absolute value of the effective electron neutrino mass, placing an upper limit of 0.45 eV on the effective electron anti-neutrino mass at the 90% confidence level Aker et al. [2024]. Combined with searches for neutrinoless double beta decays Agostini et al. [2020], they could establish the absolute value and the Dirac or Majorana nature of this mass in the near future.

Complementary to oscillation and laboratory experiments, cosmological observations may place stringent constraints on neutrino masses. The presence of a light, stable, weakly interacting particle in the cosmological fluid can affect its evolution, both at the background and perturbative level. The main effect of the neutrinos on the evolution of cosmological perturbations is dictated by their free-streaming, which acts to dampen the metric perturbations at small length scales. For neutrinos transitioning to the non-relativistic regime in a matter-dominated universe, the comoving wavenumber associated with the free-streaming scale reaches a minimum at about $k_{nr} \simeq 0.018 \Omega_m^{1/2} (\sum m_\nu / \text{eV})^{1/2} h \text{ Mpc}^{-1}$ Lesgourgues and Pastor [2012], Lesgourgues et al. [2013], setting the scale above which perturbations are suppressed.

The resulting scale-dependent suppression of the matter power spectrum makes cosmological surveys particularly useful in constraining the sum of the neutrino masses (see e.g. Cuesta et al. [2016]). This is especially true when early- and late-time probes of cosmological evolution, such as Cosmic Microwave Background (CMB) data and galaxy distribution observations, are jointly analyzed. In particular, the angular power spectrum of Planck temperature and polarization data, including CMB lensing, constrains the total neutrino mass to 0.24 eV Aghanim et al. [2020c]. Consequently, a wide variety of cosmological data have recently been used to set an upper limit on the sum of the neutrino masses Palanque-Delabrouille et al. [2020], Di Valentino et al. [2021a, 2022], with potential implications for defining their mass hierarchy.

Recent analysis of the second-year data release from the Dark Energy Spectroscopic Instrument (DESI) Aghamousa et al. [2016], combined with cosmic microwave background (CMB) temperature, polarization and (Planck and ACT) lensing information, indicates a preference for the normal hierarchy (NH) spectrum over the inverted hierarchy, reducing the 95% confidence limit

on neutrino masses to $\sum m_\nu < 0.064$ eV, and showing a preference for a zero to negative values [Abdul Karim et al. \[2025a\]](#). The exact value of these bounds is model-dependent, with some of the most restrictive results arising from the standard cosmological model $\Lambda\text{CDM} + \sum m_\nu$. On the other hand, dynamical dark energy (DDE) models, specifically those based on phenomenological parameterizations of the dark energy equation of state (EoS), such as the Chevallier–Polarski–Linder (CPL) model, relax this constraint to the point where there is no tension, and both neutrino mass hierarchies are consistent. More recently, physically motivated DDE models, such as thawing quintessence, have been shown to constrain the total neutrino mass at the same level as the standard model $\sum m_\nu < 0.07$ eV [Rodrigues et al. \[2025a\]](#).

Nevertheless, there is a clear trend toward decreasing the upper limit on neutrino masses as new datasets are considered. The authors in [Jiang et al. \[2024\]](#), by assuming an $\Lambda\text{CDM} + \sum m_\nu$ model, placed the upper limit at $\sum m_\nu < 0.042$ eV considering the same CMB data together with DESI DR1, galaxy cluster angular diameter distance observations, and a H_0 prior from SH0ES. The implications of this latest analysis extend beyond the hierarchy discussion (see, e.g. [Jimenez et al. \[2022b\]](#), [Gariazzo et al. \[2022b\]](#)) and highlight the potential for an emerging tension between cosmological observations and oscillation data. It is imperative to obtain new and accurate data to either dismiss or confirm this tension.

The Javalambre-Physics of the Accelerated Universe Astrophysical Survey (J-PAS) [Benitez et al. \[2014\]](#) is currently collecting positions and spectrophotometric information of galaxies up to redshift $z \lesssim 1.0$. The miniJPAS survey [Bonoli et al. \[2021\]](#), which serves as a proof of concept for the full survey, using the J-PAS Pathfinder camera and the 56 J-PAS filters (54 narrow-band, FWHM 145Å, and two broader filters extending to the UV and the near-infrared, complemented by the u, g, r, i SDSS broad-band filters) already carried out a ~ 1 deg² survey on the AEGIS field (along the Extended Groth Strip).

This work presents a standalone forecast for the J-PAS galaxy clustering probe, serving as a baseline quantification of its intrinsic constraining power on the sum of neutrino masses. We explore J-PAS’s potential for cosmological parameter estimation, particularly its sensitivity to the sum of neutrino masses, within the standard ΛCDM as well as the dynamical dark energy framework. These Fisher forecasts are important for assessing the survey’s expected constraining power and scientific return. The paper is organized as follows. In Section 2, we describe the methodology and the survey specifications considered to forecast the results. In Section 3, we present the results, both for the forecasts alone and for the combination of J-PAS with other datasets. In Section 4, we summarize our findings and discuss future perspectives.

4.2 Methodology

4.2.1 A review on the Fisher matrix formalism

One of the most straightforward ways to forecast future constraints of an experiment is the Fisher matrix formalism [Tegmark et al. \[1997\]](#), [Tegmark \[1997\]](#), [Coe \[2009\]](#), [Amendola and Tsujikawa \[2015\]](#). The main advantage of this approach is that we do not need to make mock data, so it is a fast formalism.

Given a likelihood $\mathcal{L}(\mathbf{x}|\boldsymbol{\theta})$ for data vector $\mathbf{x}$ and model parameters $\boldsymbol{\theta}$, the Fisher matrix is defined as the expectation value of the curvature of the log-likelihood,

$$(4.2) \quad F_{\alpha\beta} \equiv - \left\langle \frac{\partial^2 \ln \mathcal{L}}{\partial \theta_\alpha \partial \theta_\beta} \right\rangle.$$

Assuming a Gaussian distribution for the data, the likelihood can be written as

$$(4.3) \quad \mathcal{L} \propto e^{-\frac{\chi^2}{2}} \equiv \exp \left[-\frac{1}{2} \sum_{ij} (x_i - \tilde{x}_i) C_{ij}^{-1} (x_j - \tilde{x}_j) \right],$$

where x_i are the observables, $\tilde{x}_i$ is the prediction of a model with parameters θ_k and C_{ij} is the covariance matrix of the data. Although the likelihood, defined as (4.3), is Gaussian in the observables x_i , the likelihood of the parameters θ_k is not necessarily Gaussian. The Fisher matrix formalism approximates the likelihood of the parameters θ_k as Gaussian around fiducial values $\tilde{\theta}_k$. Given this approximation, a linear change of variable can be made from the Fisher matrix of the data to the Fisher matrix of the parameters:

$$(4.4) \quad F_{kl}^\theta = \left. \frac{\partial x_i}{\partial \theta_k} \right|_{\tilde{\theta}} F_{ij}^x \left. \frac{\partial x_j}{\partial \theta_l} \right|_{\tilde{\theta}}.$$

4.2.2 The FARO code

To perform this forecast, we use the Fisher gAlaxy suRvey cOde (FARO)¹ [Resco and Maroto \[2021\]](#), [Aparicio Resco et al. \[2020\]](#). FARO is a Python code that computes the Fisher matrix for galaxy survey observables. The main approach of the code is to perform a tomographic analysis in a model-independent way. In the case of galaxy map observables, we will focus only on the multi-tracer galaxy distribution power spectra [Abramo \[2012\]](#), [Abramo and Amendola \[2019\]](#).

The main parameters of the code for the multitracer power spectra are: $A_i(z) \equiv \sigma_8(z) b_i(z)$, $R(z) \equiv \sigma_8(z) f(z)$ and $E(z) \equiv H(z)/H_0$. Where $b_i(z)$ is the bias for the tracers, $H(z)$ is the Hubble parameter, and $\sigma_8(z) = \sigma_8 D(z)$, where $D(z)$ is the linear growth factor and σ_8 is the normalization of the matter power spectrum on scales of $8 h^{-1}$ Mpc today. The growth function $f(z)$ is defined as $D(z) = \exp \left(- \int_0^z \frac{f(z')}{1+z'} dz' \right)$. It is indeed true that the linear growth factor D and the growth rate f acquire a weak scale dependence in the presence of massive neutrinos. However, as pointed out in

¹<https://www.ucm.es/iparcos/faro>

section 2.5 of [Blanchard et al. \[2020\]](#), this effect is only sub-percent. This approximation is fully adequate for the sole purpose of projecting the survey's sensitivity. We note also that the linear growth factor can exhibit a stronger scale dependence at early times, while at the low redshifts relevant to J-PAS this dependence remains very weak.

Also, a model-independent parametrization is performed for $P(k)$ as a function of a_n parameters that modify the fiducial $P(k)$ in each different k -bin.

We have considered three effects for the power spectrum: the Kaiser effect for redshift space distortions [Kaiser \[1987\]](#), the convolution of the redshift error term [Amendola and Tsujikawa \[2015\]](#), and the Alcock-Paczynski effect [Alcock and Paczynski \[1979\]](#), with the multi-tracer power spectra given by,

$$(4.5) \quad P_{ab}^{\delta\delta}(z, \hat{\mu}_r, k_r) = \frac{D_{A,r}^2 E}{D_A^2 E_r} (A_a + R\hat{\mu}^2)(A_b + R\hat{\mu}^2) \hat{P}(k) \exp\left(-k_r^2 \hat{\mu}_r^2 \frac{\sigma_a^2}{2}\right) \exp\left(-k_r^2 \hat{\mu}_r^2 \frac{\sigma_b^2}{2}\right).$$

Here, the subscript r indicates that the corresponding quantity is evaluated in the fiducial model, and $\hat{\mu}$ is the angle between the wavevector $\mathbf{k}$ and the line of sight. $D_A(z)$ is the angular diameter distance for a flat universe, given by $D_A(z) = (1+z)^{-1} \chi(z)$, where,

$$(4.6) \quad \chi(z) = H_0^{-1} \int_0^z \frac{dz'}{E(z')}.$$

The term $\sigma_{ab} = \frac{\delta z_{ab}^C (1+z)}{H(z)}$ represents the radial error for the tracers, where the redshift error in the clustering measurements is defined as $\delta z_{ab}^C (1+z)$. The Fisher matrix for the 3D multitracers power spectrum is given by:

$$F_{\alpha\beta}^{\delta\delta} = \sum_{i,c,m} V_i \Delta\hat{\mu}_m \Delta\log k_c k_c^3 \frac{3c}{8\pi^2} \frac{\partial P_{ab}^{\delta\delta}(z_i, \hat{\mu}_m, k_c)}{\partial p_\alpha} C_{ba'}^{-1} \\ \times \frac{\partial P_{a'b'}^{\delta\delta}(z_i, \hat{\mu}_m, k_c)}{\partial p_\beta} C_{b'a}^{-1} \exp\left(-k_c^2 \Sigma_\perp^2 - k_c^2 \hat{\mu}_m^2 (\Sigma_\parallel^2 - \Sigma_\perp^2)\right),$$

The integrals in $\hat{\mu}$ have an interval from -1 to 1 , and k ranges from $k_{\min}$ to ∞ . The code sets the value $k_{\min} = 7 \times 10^{-3} h/\text{Mpc}$. The quantities $\Sigma_\perp(z) = 0.785 D(z) \Sigma_0$ and $\Sigma_\parallel(z) = 0.785 D(z) (1+f(z)) \Sigma_0$ represent the exponential cut-off, which removes the contribution from non-linear scales [Seo and Eisenstein \[2007\]](#). The covariance matrix C_{ab} is given by

$$(4.7) \quad C_{ab}(z_i, \hat{\mu}_m, k_c) = P_{ab}^{\delta\delta}(z_i, \hat{\mu}_m, k_c) + \frac{\delta_{ab}}{\bar{n}_a(z_i)},$$

where $\bar{n}_a(z_i)$ is the galaxy density of the tracers in the bins of redshift z and, V_i is the total volume of the i -th bin, given by,

$$(4.8) \quad V_i = \frac{4\pi f_{\text{sky}}}{3} [\chi(\bar{z}_i)^3 - \chi(\bar{z}_{i-1})^3],$$

f_{sky} is the sky fraction of the survey and $\bar{z}_i$ is the upper limit of the i -th bin. The Kronecker delta δ_{ab} ensures that shot noise contributes only to the auto-correlation of each tracer, effectively expressing the absence of shot noise in cross-correlations between different tracer types. The FARO code will then project the Fisher matrix for the observable power spectra into the parameters we want to constrain.

The standard FARO code also has the possibility to project the information of each tomographic parameter into another set of parameters like, for example, a simple $w_0 w_a$ Λ CDM dark energy model. Given the initial Fisher matrix for model-independent parameters $\{p_\alpha\}$, we can obtain the Fisher matrix for a new set of parameters $\{q_\alpha\}$ as follows:

$$(4.9) \quad \mathbf{F}^q = \mathbf{P}^t \mathbf{F}^p \mathbf{P},$$

where $P_{\alpha\beta} = \partial p_\alpha / \partial q_\beta$, evaluated on the fiducial model. Even though this approach is straightforward to implement for redshift dependent parameters, it can be somewhat complicated for the $P(k)$ model-independent parametrization. To solve this inconvenience, we have modified the code to ensure that this change of variables can be done in the simplest way. Instead of obtaining the Fisher matrix for tomographic parameters and then doing the projection, we implement the projection inside the code to directly obtain the Fisher matrix for the desired cosmological parameters. This allows us to calculate $\partial P(k) / \partial q_\alpha$ in a straightforward way, solving the issue of computing how the model-independent parametrization of $P(k)$ depends on the chosen cosmological parameters. Most derivatives inside the code are done analytically, except for the derivatives of the matter power spectrum with respect to the cosmological parameters. These derivatives are computed numerically. We also assume that for dynamical dark energy models, $\partial \hat{P} / \partial w_0 = \partial \hat{P} / \partial w_a = 0$, where $\hat{P} \equiv P / \sigma_8^2$.

In our template the effect of neutrino masses is constrained to the normalized power spectrum $\hat{P}(k)$. Besides, at the redshift values relevant for large scale structure observations, background effects due to non-zero Σm_ν are already accounted for in Ω_m . Therefore, the only explicit dependence of our model on the sum of neutrino masses is in the derivative $\partial \hat{P} / \partial \Sigma m_\nu$. Moreover, the fact that Σm_ν is physically bounded from below to non-negative values is dealt with at the posterior level (i.e. not in the Fisher matrix), including a step function prior that removes negative values when necessary.

4.2.3 Convergence and consistency tests

We use the vibration matrix method [Yahia-Cherif et al. \[2021\]](#) to check the stability of the Fisher matrix with respect to the individual numerical derivatives. In particular, we find that a 5-point stencil is stable with an epsilon value of $\varepsilon = 10^{-2}$. This is the optimal choice, as it is the smallest possible value that does not introduce artificial numerical noise. In addition, the new version of FARO was tested against the forecast for the $w_0 w_a$ Λ CDM model using EUCLID [Casas et al. \[2024a\]](#), [Blanchard et al. \[2020\]](#) and obtained consistent results. For the EUCLID consistency check, we

assume a sky area of 15000 deg^2 , along with the number densities, redshift bins, and redshift errors shown in Tables I and II of [Resco and Maroto \[2021\]](#). We have also compared the results from the Fisher matrix directly using the parameter space of the cosmological parameters, and indirectly using the model-independent parameter space of FARO, and then projecting this Fisher matrix on the cosmological parameter space, using the change of variable equation (Eq. 4.9). For our constraints, we have used a modified version of FARO (FARO 2.0) which works on the cosmological parameter space directly and also can run using Python 3.0.

A critical test here is that, to perform the change of variables, the model-independent parameters $E_i \equiv E(z_i) = H(z_i)/H_0$ have to be replaced by $H_i \equiv H(z_i)$ so that the numerical derivative $\partial E/\partial h$ was non-zero. The derivative $\partial P/\partial h$ was performed numerically to avoid using the chain rule given the h dependence of k (see discussion at Section 4.5.1 in [Blanchard et al. \[2020\]](#)).

4.2.4 Forecast settings

To compute the derivatives and cosmological quantities that are used in the Fisher matrices, we need to choose fiducial values for the parameters. Here, we explore two different cosmological models, $\Lambda\text{CDM} + \sum m_\nu$ and $w_0 w_a \text{CDM} + \sum m_\nu$. Thus, we set fiducial values for the Hubble constant h , the matter density parameter Ω_m , the baryon density parameter Ω_b , the spectral index n_s , the amplitude of perturbations σ_8 , the total neutrino mass $\sum m_\nu$, and the dark energy equation of state parameters w_0 and w_a . Table 5.1 presents the values that have been used. These are the fiducial values defined by the J-PAS collaboration, they are the same as those used by the Planck collaboration [Planck Collaboration et al. \[2014\]](#) and also used by the project Universe N-body simulations for the Investigation of Theoretical models from galaxy surveys (UNIT) [Angulo and Pontzen \[2016\]](#).

Parameter	Value
h	0.6774
Ω_m	0.3089
Ω_b	0.0223
n_s	0.9667
σ_8	0.8161
$\sum m_\nu \text{ (eV)}$	0.06
w_0	-1
w_a	0.0

Table 4.1: Fiducial values for the cosmological parameters.

In addition to the fiducial values, we need to define the survey specifications. Here, we consider two different tracers: Luminous Red Galaxies (LRGs) and Emission Line Galaxies (ELGs). The

fiducial bias for these is given by:

$$(4.10) \quad b(z) = \frac{b_0}{D(z)},$$

where $D(z)$ is the growth function, with $b_0 = 0.84$ for ELGs and $b_0 = 1.7$ for LRGs [Aghamousa et al. \[2016\]](#). In order to account for a possible residual redshift dependence, we have also tested a bias function with an additional degree of freedom, $b(z) = \frac{b_0 + c_0 z}{D(z)}$. In this case, c_0 is treated as a nuisance parameter, and we marginalize over it. As a result, we observed only minor changes in the overall neutrino mass constraints. Therefore, we used the simplest bias function as our fiducial model. We divided our data into four redshift bins, ranging from $z = 0.1$ to $z = 0.7$. In Table 4.2, we present the $b(z)$ values for each redshift bin and tracer.

It should be noted that we do not use the galaxy number densities estimated pre-observations with JPCam mentioned in [Benitez et al. \[2014\]](#). Instead we use currently measured values based on real observations. For a similar forecast comparing values of galaxy number densities "pre-observation" J-PAS with PFS and future CMB experiments, see [Qin et al. \[2026\]](#). In the forecasts presented in this work, we make use of galaxy number density estimates in redshifts and odds bins obtained from J-PAS Internal Data Release (IDR) of June 2024. In particular, we use about 27 sq.deg. with full filter coverage, and where the i -band is used as detection band under exposure times that are deemed as representative for the full J-PAS survey. This data set incorporates the latest calibration pipeline, including correction for the temporal instability of the bias. The photometric redshifts (photo- z s) have been obtained with the J-PAS implementation of LePhare [Hernán-Caballero et al. \[2021\]](#), [Hernán-Caballero et al. \[2023\]](#). We tested three different star-galaxy separation codes, independently developed within the collaboration and since the differences among them were not significant, we quote results for the one that is based exclusively upon morphological information of the sources, together with their i -band magnitude. We also rely on a red/blue classification provided as in [Díaz-García et al. \[2024\]](#) in order to provide estimates of the number density of each type of galaxy for different redshift and odds bins.

z	$b(z)$ for ELG	$b(z)$ for LRG
0.1	0.888	1.797
0.3	0.986	1.996
0.5	1.0953	2.21
0.7	1.208	2.446

Table 4.2: Bias function values for each redshift bin and tracer.

The probe considered in this work is galaxy clustering (GCs). The galaxy number densities and redshift errors were divided into three distinct subsamples based on their odds. The first subsample contains galaxies with the top 30% odds, the second includes galaxies with odds in the range of 50% to 70%, and the third comprises galaxies with the lower 50% odds. For blue galaxies,

we used all available odds, while for red galaxies, we restricted our analysis to the top 30%. This decision was made due to the low galaxy counts for LRGs in the remaining 20% and the lower 50% odds, which resulted in redshift errors that were too small to be reliable.

Our implementation of LePhare also provides PHOTOZ-ERR which is an estimate of the width of the photo- z probability density distribution (PDF), the details of which can be seen in [Hernández-Caballero et al. \[2023\]](#)). Table 4.3 and Table 4.4 show the redshift bins, the galaxy number densities in units of $h^3 \text{Mpc}^{-3}$, and the redshift errors for the clustering case, for LRGs and ELGs, respectively.

z	nLRG [$h^3 \text{Mpc}^{-3}$]	$\delta z^c \text{LRG}$
Top 30 %		
0.1	0.0036	0.0023
0.3	0.0027	0.0023
0.5	0.00072	0.0038
0.7	0.00037	0.0043

Table 4.3: Redshift bins, number densities and redshift errors of luminous red galaxies (LRG) for J-PAS.

z	nELG [$h^3 \text{Mpc}^{-3}$]	$\delta z^c \text{ELG}$
Top 30 %		
0.1	0.011	0.0019
0.3	0.0054	0.0019
0.5	0.0026	0.0058
0.7	0.0013	0.0054
Remaining 20 %		
0.1	0.0086	0.0026
0.3	0.0047	0.0043
0.5	0.0022	0.011
0.7	0.0011	0.013
Lower 50 %		
0.1	0.028	0.039
0.3	0.013	0.033
0.5	0.0057	0.039
0.7	0.0027	0.033

Table 4.4: Redshift bins, number densities and redshift errors of emission line galaxies (ELG) for J-PAS.

We have explored two sky coverages here: $f_{\text{sky}} = 1500 \text{ deg}^2$ for the forecast of constraints from the expected analysis of initial results, and $f_{\text{sky}} = 8500 \text{ deg}^2$ for the forecast of the full J-PAS operation. It is important to mention that the speed of J-PAS sky coverage has not been defined yet. In Fig. 5.1, we show the distribution of number densities for each sub-sample and each type of galaxy, as well as the redshift error for each redshift bin.

Due to the suppression that massive neutrinos add to the power spectrum, which is larger on small scales (large k -values), we are going to model the matter power spectrum into the non-linear regime. For this analysis, we considered scales that range from $k = 7 \times 10^{-3} h/\text{Mpc}$ to k_{max} . To model the non-linear power spectrum, we have used the Cosmic Linear Anisotropy Solving System code (CLASS) [Lesgourgues \[2011\]](#), [Blas et al. \[2011a\]](#) with the Halofit recipe [Smith et al. \[2003\]](#), [Bird et al. \[2012\]](#), [Takahashi et al. \[2012\]](#).

A key aspect of the Halofit prescription employed in this work is its capability to account for the effect of neutrino mass on the non-linear matter power spectrum, thereby correcting the lack of power on the smallest scales (see [Bird et al. \[2012\]](#)). Recently, it has been shown that this revisited version of Halofit, referred sometimes in the literature as "TakaBird", remains accurate in the non-linear regime at low redshifts, with deviations below 2% for $k \sim 0.3 h \text{ Mpc}^{-1}$ at $z = 1$ and $\sum m_\nu \sim 0.16 \text{ eV}$ when compared with DEMNUni simulations [Parimbelli et al. \[2022\]](#). Therefore, we do not expect significant effects from non-linear corrections to the power spectrum given our specific setup, redshift and the chosen k_{max} . Furthermore, we must recognize that, even though we model the matter power spectrum in the non-linear regime, this does not make our analysis fully non-linear. A complete treatment would require a consistent modeling of the galaxy bias and redshift-space distortions at those scales.

We have explored three scenarios: a pessimistic case (Pess), with $k_{\text{max}}(\text{GCs}) = 0.10 h \text{ Mpc}^{-1}$, an optimistic case (Opt), with $k_{\text{max}}(\text{GCs}) = 0.20 h \text{ Mpc}^{-1}$, and an only linear $P(k)$ case, with the maximum scale for the observed power spectrum $k_{\text{max}}(\text{GCs}) = 0.20 h \text{ Mpc}^{-1}$.

4.3 Results

4.3.1 Results for J-PAS

To obtain the sensitivity of galaxy clustering information from J-PAS to $\sum m_\nu$, we performed the forecast by combining the three subsamples with different odds. We compared the results for the optimistic and pessimistic cases, examined the impact of sky coverage and non-linear corrections on the constraints, and performed an analysis using the two tracers, LRG and ELG, both together and separately.

In Fig. 4.2, we show the comparison between the non-linear pessimistic and optimistic cases in the full survey forecast. A significant improvement in the constraints is observed when smaller scales are considered. This is expected since we are adding more information to the analysis.

Especially for neutrinos, the suppression on $P(k)$ becomes increasingly higher at smaller scales, meaning that we have more information about the neutrino mass.

We also compare the forecast with and without the non-linear corrections in the Opt case, as shown in Fig. 4.3. One can observe that when the non-linear effects are incorporated, they can provide a slight improvement in the constraints on the neutrino mass. However, the magnitude of the improvement depends on the non-linear cutoff scale set by Σ_0 . The cutoff scale Σ_0 determines the extent of suppression caused by nonlinear effects. A low Σ_0 means less suppression on non-linear effects, which in turn can improve the precision on neutrino mass constraints. On the other hand, a high Σ_0 leads to more suppression of non-linear effects, reducing the differences between the linear and non-linear cases and therefore making the non-linear corrections less relevant. Non-linear effects are then most beneficial when low Σ_0 and larger $k_{\text{max}}(\text{GCs})$ are considered. The value of Σ_0 is correlated with the amplitude of matter fluctuations σ_8 . For a cosmology with $\sigma_8 \approx 0.80$ the optimal value of Σ_0 is approximately $11 \ h^{-1} \text{ Mpc}$ Eisenstein et al. [2007], Seo and Eisenstein [2007].

In Fig. 4.4, we show the difference in the results between the two values of the sky coverage explored. We can observe that the power of the constraints scales with the sky coverage as expected. This shows that the sky fraction covered by the survey is an important piece in the constraining power of the experiment. In Fig. 4.5, we can see the results for each tracer separately, in comparison with the total case (LRG+ELG). We can observe a clear improvement in the constraints provided by the LRG+ELG combination. However, ELGs alone present better constraints than LRGs alone. Hereafter, we will assume that our J-PAS baseline case is: full survey with $f_{\text{sky}} = 8500 \text{ deg}^2$ and Opt case with $k_{\text{max}}(\text{GCs}) = 0.20 h \text{ Mpc}^{-1}$ with non-linear corrections.

We show the $w_0 w_a \text{CDM} + \sum m_\nu$ contours for the J-PAS baseline case in Fig. 4.6. The main thing to note here is that J-PAS GC_s alone will not provide a very tight bound on the dark energy equation of state parameters w_0 and w_a . We can also observe a moderate correlation between $\sum m_\nu$ and w_0 or w_a . The orientation of the resulting probability ellipses is in agreement with the results shown by Archidiacono et al. [2024] for the EUCLID survey.

In Table 4.5, we present our forecast for the $\Lambda\text{CDM} + \sum m_\nu$ and $w_0 w_a \text{CDM} + \sum m_\nu$ models using the J-PAS baseline case. For neutrino mass, J-PAS GC_s alone will provide us with an upper limit (one-tail) $\sum m_\nu < 0.32 \text{ eV}$ at 95% C.L. for the $\Lambda\text{CDM} + \sum m_\nu$ model. This constraint is less tight when considering the $w_0 w_a \text{CDM} + \sum m_\nu$, with an upper limit of $\sum m_\nu < 0.36 \text{ eV}$ at 95% C.L. (for comparison, see Adame et al. [2024a], Du et al. [2024], Shao et al. [2024], Adame et al. [2024b] for recent constraints on this cosmological model). Given the value of the upper limit for the neutrino mass, this shows that there is no conclusive evidence in favor of normal or inverted ordering for either model with J-PAS GC_s alone, which is consistent with expectations from other future stage IV galaxy surveys. This places our results for the sum of the neutrino mass in a highly competitive position.

Parameter	$\Lambda\text{CDM} + \sum m_\nu$	$w_0 w_a \text{CDM} + \sum m_\nu$
J-PAS baseline		
h	0.0079	0.0098
Ω_m	0.010	0.028
Ω_b	0.0046	0.0095
n_s	0.022	0.038
σ_8	0.0095	0.034
$\sum m_\nu (\text{eV})$	0.26 (2σ)	0.30 (2σ)
w_0	–	0.24
w_a	–	0.77

Table 4.5: Forecasted cosmological constraints at 1σ confidence level (2σ constraints for the total neutrino mass) for the J-PAS baseline dataset (GC_s non-linear/Opt and 8500 deg^2).

4.3.2 Results for the combination J-PAS+Planck+Pantheon Plus

To perform the combined analysis, we used Planck 2018 low and high multipole data for temperature, polarization, and lensing ($plikHM + TTTEEE + lowl + lowE + lensing$) [Aghanim et al. \[2020c\]](#), along with Type Ia supernovae data from Pantheon Plus (SN) [Scolnic et al. \[2022\]](#), [Brout et al. \[2022\]](#). Given that the public chains from the Planck collaboration remove negative values for the total neutrino mass (and thus the full probability distribution function could not be recovered), we computed the CMB Fisher matrix directly from the cosmological parameter covariance matrix obtained through the official Planck likelihood. We interfaced the *CLASS* code [Lesgourgues \[2011\]](#), [Blas et al. \[2011a\]](#) with *MontePython* [Audren et al. \[2013a\]](#), [Brinckmann and Lesgourgues \[2018\]](#) to perform a Monte Carlo Markov Chain (MCMC) numerical analysis. This analysis was applied to the $\Lambda\text{CDM} + \sum m_\nu$ and $w_0 w_a \text{CDM} + \sum m_\nu$ models using Planck 2018 data, both with and without Pantheon Plus data. We then used the covariance matrix from these analyses along with the J-PAS covariance matrix from the Fisher forecast to obtain the combined likelihood function. This function is defined as the product of the probability distribution function from the CMB data covariance matrix (centered at their bestfit values of the cosmological parameters) and the J-PAS Gaussian likelihood (using the Fisher matrix for the J-PAS baseline case), centered at our fiducial cosmology. We note that centering CMB and J-PAS likelihoods at different bestfit values would account for the effect of possible tensions between these two datasets. We emphasize that this combination should be interpreted with caution: when datasets are partially in tension, the apparent tightening of constraints may be influenced by such inconsistencies rather than by genuine complementarity. Therefore, the combined analysis is presented only as an illustration of possible sensitivity improvements, and we do not draw physical conclusions. In this case we used the *emcee* library to perform MCMC sampling and explore the parameter space from the combined posterior distribution [Foreman-Mackey et al.](#)

[2013]. The one-tail 95% C.L. limits reported hereafter on the sum of neutrino masses Σm_ν are computed on the truncated posterior distribution, in which the negative values for this parameter have been removed.

In Fig. 4.7, we present the results for Planck 2018 combined with J-PAS, with and without Pantheon Plus supernovae, for the $\Lambda\text{CDM} + \Sigma m_\nu$ model. We observe that the inclusion of SN makes little difference to the result. In Table 4.6, we present the cosmological constraints obtained for both combinations. We have an upper 95% one-tail limit of $\Sigma m_\nu < 0.061$ eV for the case without supernovae. When we include Pantheon Plus data, this puts our upper limit at $\Sigma m_\nu < 0.065$ eV (95% C.L.) This demonstrates that J-PAS, when combined with Planck, can significantly improve the constraints on neutrino mass.

We also performed the analysis for the $w_0w_a\text{CDM} + \Sigma m_\nu$ model. On Fig. 4.8, we show the $w_0w_a\text{CDM} + \Sigma m_\nu$ probability contours for J-PAS combined with Planck 2018 and Pantheon Plus SN data, and on Table 4.7 we show the cosmological parameter constraints. When comparing with the $\Lambda\text{CDM} + \Sigma m_\nu$ model, we see a significant loosening of neutrino mass constraints. We obtain an upper limit of $\Sigma m_\nu < 0.12$ eV at 95% C.L for the combination with J-PAS.

Parameter	$\Lambda\text{CDM} + \Sigma m_\nu$
J-PAS baseline + Planck 2018	
h	0.6785 ± 0.0023
Ω_m	0.3080 ± 0.0032
Ω_b	0.04826 ± 0.00034
n_s	0.9669 ± 0.0033
σ_8	0.8164 ± 0.0018
$\Sigma m_\nu(\text{eV})$	$0.028^{+0.033}_{-0.028} (2\sigma)$
J-PAS baseline + Planck 2018 + Pantheon Plus	
h	0.6767 ± 0.0022
Ω_m	0.3093 ± 0.0032
Ω_b	0.04829 ± 0.00034
n_s	0.9638 ± 0.0035
σ_8	0.8169 ± 0.0018
$\Sigma m_\nu(\text{eV})$	$0.032^{+0.033}_{-0.032} (2\sigma)$

Table 4.6: Forecasted cosmological constraints at 1σ confidence level for the $\Lambda\text{CDM} + \Sigma m_\nu$ model, and 2σ constraints for the total neutrino mass for J-PAS baseline + Planck (including CMB lensing) and J-PAS baseline + Planck (including CMB lensing) + Pantheon Plus.

It is worth noting that on all the combined analyses where J-PAS is combined with external data, we have excluded the non-physical regions for the sum of the neutrino masses, i.e., regions for $\Sigma m_\nu < 0$. The results show that the excluded regions are not a significant part of the

Parameter	$w_0 w_a CDM + \sum m_\nu$
J-PAS baseline + Planck 2018 + Pantheon Plus	
h	0.6750 ± 0.0030
Ω_m	0.3114 ± 0.0033
Ω_b	0.04801 ± 0.00050
n_s	0.9659 ± 0.0039
σ_8	0.8134 ± 0.0036
$\sum m_\nu (\text{eV})$	$0.064^{+0.059}_{-0.064} (2\sigma)$
w_0	-0.940 ± 0.057
w_a	-0.22 ± 0.23

Table 4.7: Forecasted cosmological constraints at 1σ confidence level for the $w_0 w_a CDM + \sum m_\nu$ model, and 2σ constraints for the total neutrino mass for J-PAS baseline + Planck (including CMB lensing) and J-PAS baseline + Planck (including CMB lensing) + Pantheon Plus.

constraint, indicating that any preference for a negative neutrino mass is thinned for this data configuration. Therefore, J-PAS, when combined with external data, mainly CMB, will achieve excellent sensitivity in measuring the sum of the neutrino masses, comparable to the measurements expected from stage IV surveys. In Fig. 4.9, we present a summary of our predicted results for J-PAS alone and in combination with external data. Our constraints on the total neutrino mass are competitive for both the $\Lambda CDM + \sum m_\nu$ and $w_0 w_a CDM + \sum m_\nu$ models when compared with the latest results reported by the DESI Collaboration [Elbers et al. \[2025a\]](#).

4.4 Discussion and conclusion

In this paper we have presented a Fisher matrix forecast on neutrino mass constraints from J-PAS galaxy power spectrum measurements and their combination with CMB measurements from the Planck Collaboration and SN from Pantheon Plus. We conducted a series of tests to see how sensitive our forecasted constraints are to different choices for the survey settings as well as for the cosmological analysis, before including the information from cosmic microwave background and type Ia supernovae. Besides the trivial scaling with survey area (comparing, for instance, a $1,500 \text{ deg}^2$ versus a $8,500 \text{ deg}^2$ survey), one can choose to include clustering data at small scales, relying on the modeling of non-linear scales given by Halofit. Increasing the maximum wavenumber of the power spectrum from a pessimistic value of $0.10 h \text{ Mpc}^{-1}$ to an optimistic value of $0.20 h \text{ Mpc}^{-1}$ significantly improves the survey's constraining power. The EUCLID collaboration finds only a small difference when extending the maximum scale from $0.25 h \text{ Mpc}^{-1}$ to $0.30 h \text{ Mpc}^{-1}$, except for theoretical errors and systematic effects that may have been overlooked in the modeling of the observables (see [Archidiacono et al. \[2024\]](#)). Similarly, applying a linear theory treatment of clustering at such small scales results in only a minor

difference that can be safely neglected, although other cosmological parameters, such as n_s and σ_8 , are more sensitive to this choice. This shows that beyond a certain scale, the incremental gain in constraining power becomes marginal, particularly when theoretical uncertainties are taken into account. Interestingly, splitting the galaxy sample by type reveals that neutrino mass constraints from emission line galaxies (ELGs), which have a higher number density, are stronger than those from luminous red galaxies (LRGs). In the multitracers case using both LRGs and ELGs, the improvement in the total neutrino mass bounds is negligible, but the inclusion of LRGs still enhances constraints on other cosmological parameters. This makes ELGs particularly valuable tracers for obtaining competitive constraints on the sum of neutrino masses.

We have shown that an 8,500 sq. deg. survey from J-PAS, given the galaxy number density over the redshift range covered, can place competitive neutrino mass constraints. The constraints from density fluctuations as extracted by galaxy clustering information from the galaxy power spectrum break the degeneracies present in background measurements such as those traced by baryon acoustic oscillations or the cosmic microwave background (which is itself mostly a background measurement, [Bertólez-Martínez et al. \[2024\]](#)). In particular, we have combined the clustering Fisher matrix from J-PAS with the actual measurements from Planck temperature, polarization and lensing power spectra at low and high multipoles, and Type Ia Supernovae from Pantheon Plus. We have also tested the numerical stability of the resulting Fisher matrix by checking the sensitivity of the numerical derivatives used when computing the clustering Fisher matrix. If J-PAS future data shows to be consistent with the assumed fiducial parameter values and forecast uncertainties as reported in this work, the combination with Planck for the $\Lambda\text{CDM} + \sum m_\nu$ model would return an upper limit of $\sum m_\nu < 0.061$ eV (95% C.L.). Recently, other works have shown tighter constraints; however, those use a larger number of cosmological probes than this forecast. Although it is well known that EUCLID will have a sensitivity of 0.02 eV when combined with Planck data [Laureijs et al. \[2011\]](#), [Audren et al. \[2013b\]](#), [Sprenger et al. \[2019\]](#), [Archidiacono et al. \[2024\]](#), those forecasts assume a combination of clustering and lensing information (Euclid primary probes) with Planck measurements, while in our baseline dataset we did not include J-PAS galaxy lensing information to separate more cleanly large-scale structure from cosmic microwave background datasets, since the latter includes CMB lensing. In this sense, our constraints are conservative and, if J-PAS cosmic shear measurements are included, they can result in further improvement of these cosmological constraints. In order to compare on equal footing the forecasts from J-PAS and EUCLID for the $\Lambda\text{CDM} + \sum m_\nu$ model, we should compare the results displayed in Table 4.5 using J-PAS baseline, with the ones provided in Table 5 of [Archidiacono et al. \[2024\]](#) using galaxy clustering (GC_{sp}) data. The error bars estimated with J-PAS (EUCLID) in each case are $\delta h = 0.0079$ (0.033), $\delta\Omega_m = 0.010$ (0.0068), $\delta\Omega_b = 0.0046$ (0.0037), $\delta n_s = 0.022$ (0.029) and $\delta\sigma_8 = 0.0095$ (0.0077). Therefore, for the h and n_s parameters with J-PAS baseline we obtain error bars 76% and 20% smaller than with EUCLID GC_{sp} data, whereas for the parameters Ω_m , Ω_b and σ_8 EUCLID gets respectively, 31%, 20% and 19% smaller

error bars. As for the sum of the neutrino mass parameter, both J-PAS and EUCLID GC_{sp} forecast a 95% upper bound of 0.32 eV on $\sum m_\nu$.

We also performed this analysis for the $w_0w_a\text{CDM}+\sum m_\nu$ model, where we found a relaxation of the neutrino mass constraints. Specifically, combining J-PAS with CMB and SN data, we obtained an upper bound of $\sum m_\nu < 0.12$ eV (95% C.L.). This clearly demonstrates that the cosmological model will play a central role in future discussions about a potential tension between neutrino mass measurements from cosmological observables and neutrino oscillation experiments. Additionally, the J-PAS dataset will allow us to explore different models that modify the neutrino sector, potentially resolving important issues in the standard model of cosmology, such as the Hubble tension.

Our results show that the constraints expected from J-PAS will be comparable to those forecasted for other forthcoming large-scale structure surveys, such as Euclid and LSST, and will provide a valuable cross-check for space-based clustering constraints from other Stage IV galaxy surveys. Our constraints assume a neutrino mass best-fit value from J-PAS equal to the minimum allowed by neutrino oscillation data assuming normal hierarchy mass ordering, which is about 0.06 eV. Recent claims, especially since the DESI survey first year results [Craig et al. \[2024\]](#), [Green and Meyers \[2024\]](#), [Elbers et al. \[2024\]](#), [Naredo-Tuero et al. \[2024\]](#), [Jiang et al. \[2024\]](#), have shown a preference for zero or negative neutrino mass which might signal an inconsistency in the assumed cosmological model. However, the results from this forecast show that the negative neutrino mass will show tension with neutrino oscillation measurements and that J-PAS will provide a useful cosmological dataset to evaluate the significance of these issues. These results show the forecasted constraining power of the clustering measurements from the J-PAS survey, which in the case of the neutrino mass data are particularly promising. The need of independent datasets that confirm or rule out claims from other large galaxy surveys such as DESI, EUCLID, or LSST, makes the J-PAS dataset essential to address the forthcoming challenges that the current cosmological model is facing.

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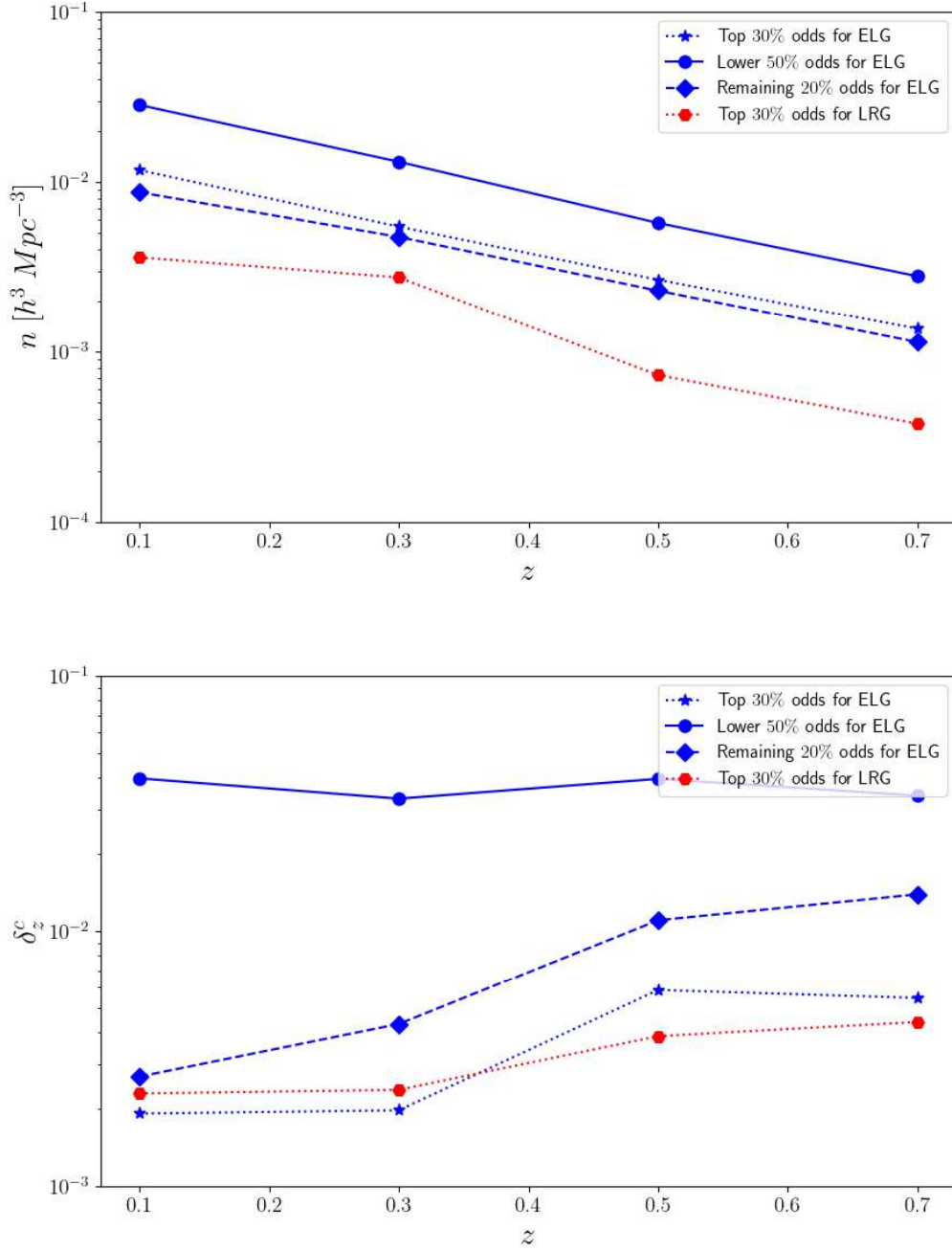


Figure 4.1: Galaxy number density in units of $h^3 \text{ Mpc}^{-3}$ and redshift errors for each divided sub-sample and each tracer as a function of redshift.

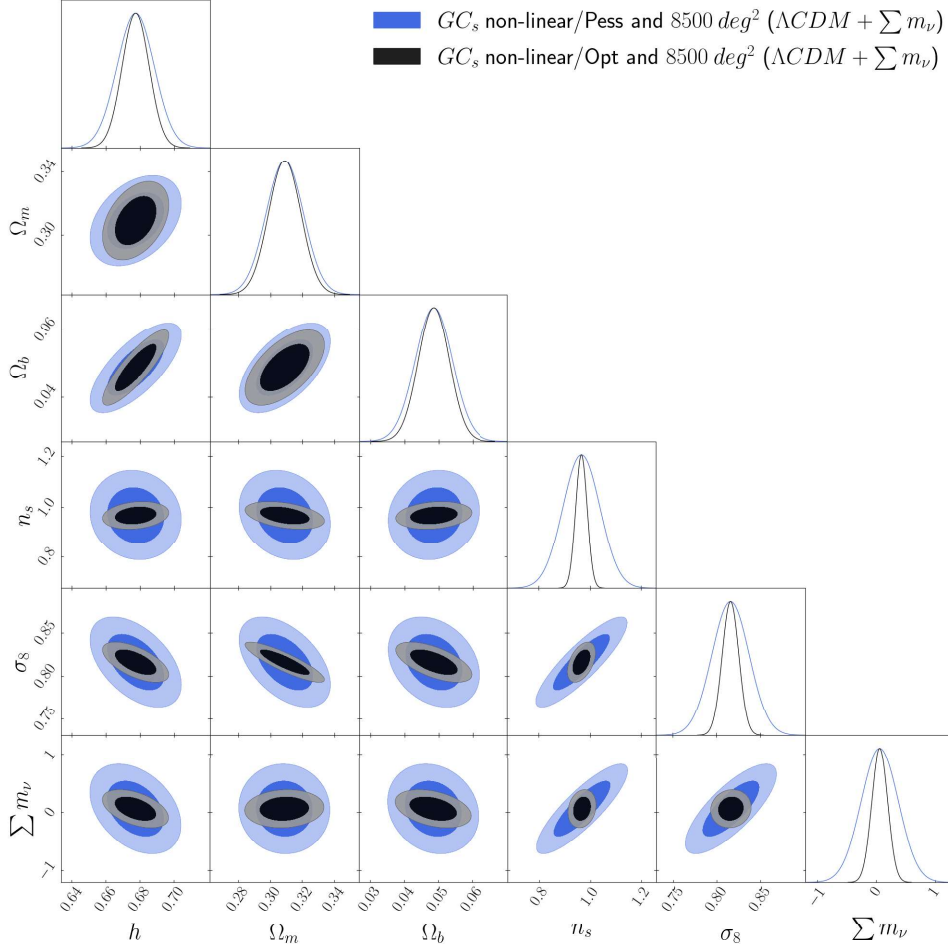


Figure 4.2: Confidence contours at 68% and 95% C.L. for the full survey with $f_{\text{sky}} = 8500 \text{ deg}^2$ comparing an Optimistic case with $k_{\text{max}}(\text{GCs}) = 0.20 h \text{ Mpc}^{-1}$, and a Pessimistic case with $k_{\text{max}}(\text{GCs}) = 0.10 h \text{ Mpc}^{-1}$, both with non-linear corrections.

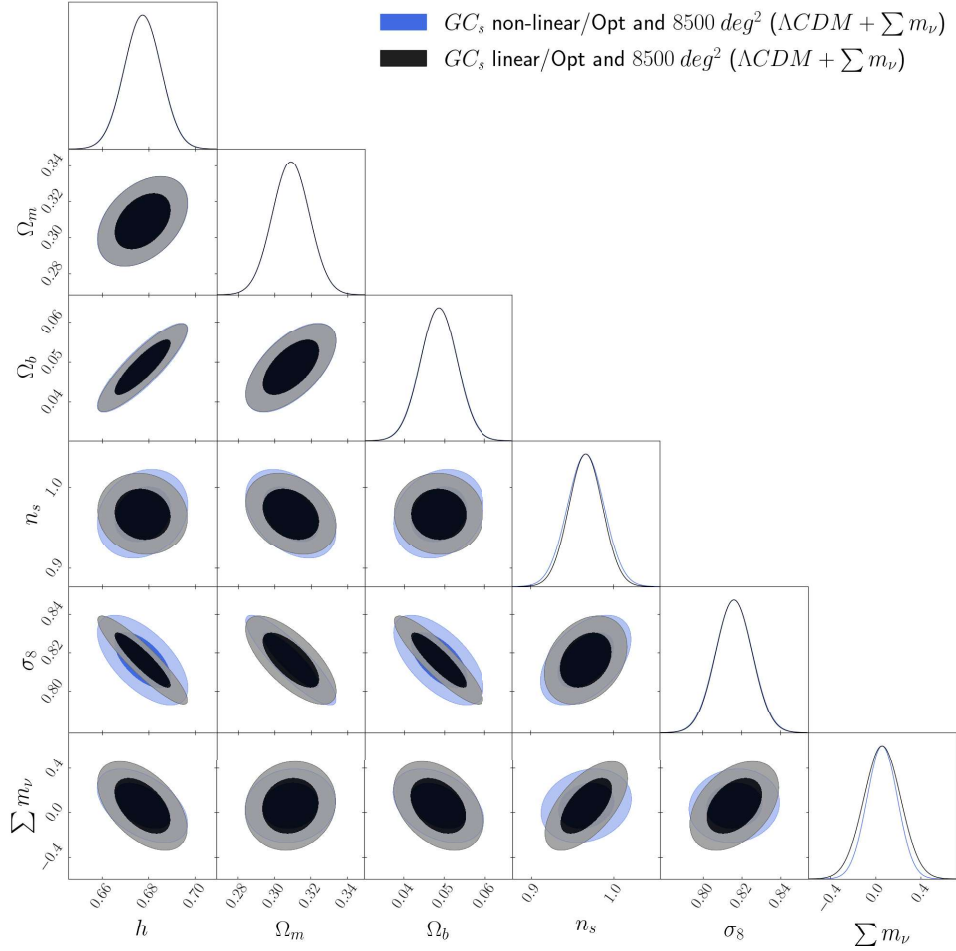


Figure 4.3: Confidence contours at 68% and 95% C.L. for the full survey with $f_{\text{sky}} = 8500 \text{ deg}^2$ in an Optimistic case with $k_{\text{max}}(\text{GCs}) = 0.20 h \text{ Mpc}^{-1}$, comparing an analysis assuming linear theory to the analysis including non-linear corrections.

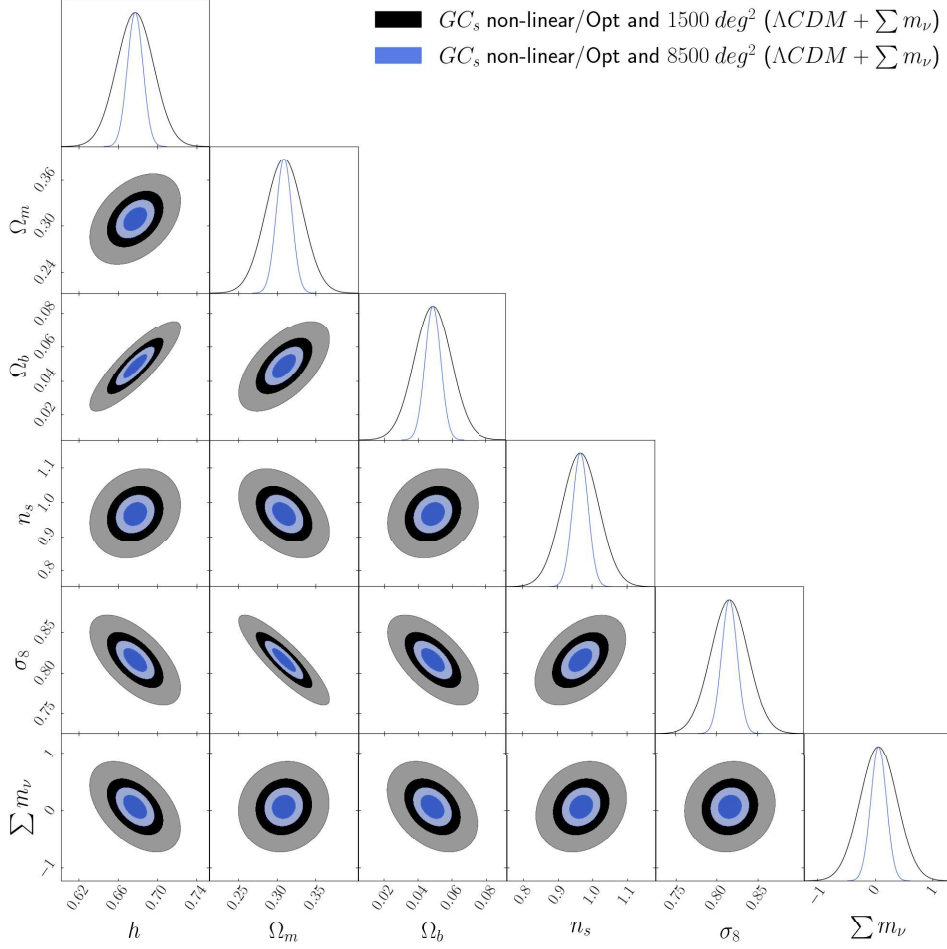


Figure 4.4: Confidence contours at 68% and 95% C.L. comparing a partial survey with $f_{\text{sky}} = 1500 \text{ deg}^2$ and a full survey with $f_{\text{sky}} = 8500 \text{ deg}^2$, both for an Optimistic case with $k_{\text{max}}(\text{GCs}) = 0.20 h \text{ Mpc}^{-1}$, and including non-linear corrections.

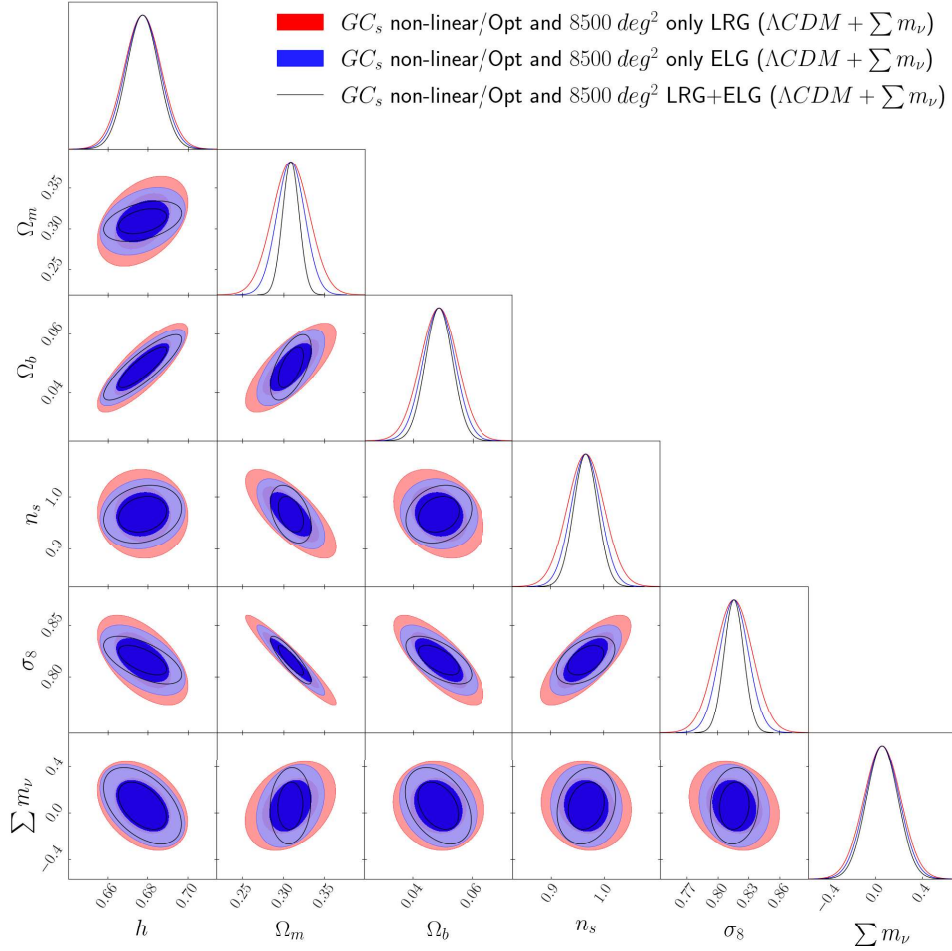


Figure 4.5: Confidence contours at 68% and 95% C.L. for the full survey with $f_{\text{sky}} = 8500 \text{ deg}^2$ in an Optimistic case with $k_{\text{max}}(\text{GCs}) = 0.20 h \text{ Mpc}^{-1}$, comparing the cosmological constraints from the LRG and ELG samples, and both samples combined.

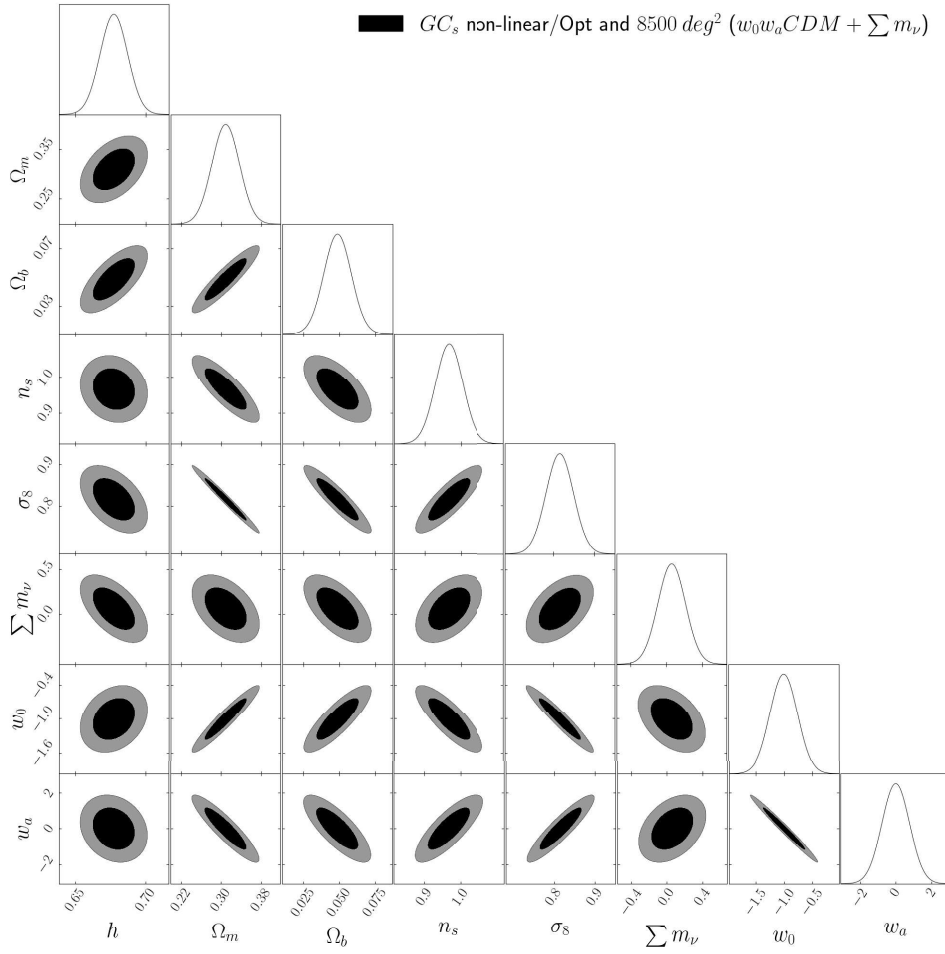


Figure 4.6: Confidence contours at 68% and 95% C.L. for the $w_0 w_a CDM + \sum m_\nu$ with full survey with $f_{\text{sky}} = 8500 \text{ deg}^2$ in an Optimistic case with $k_{\text{max}}(\text{GCs}) = 0.20 h \text{ Mpc}^{-1}$.

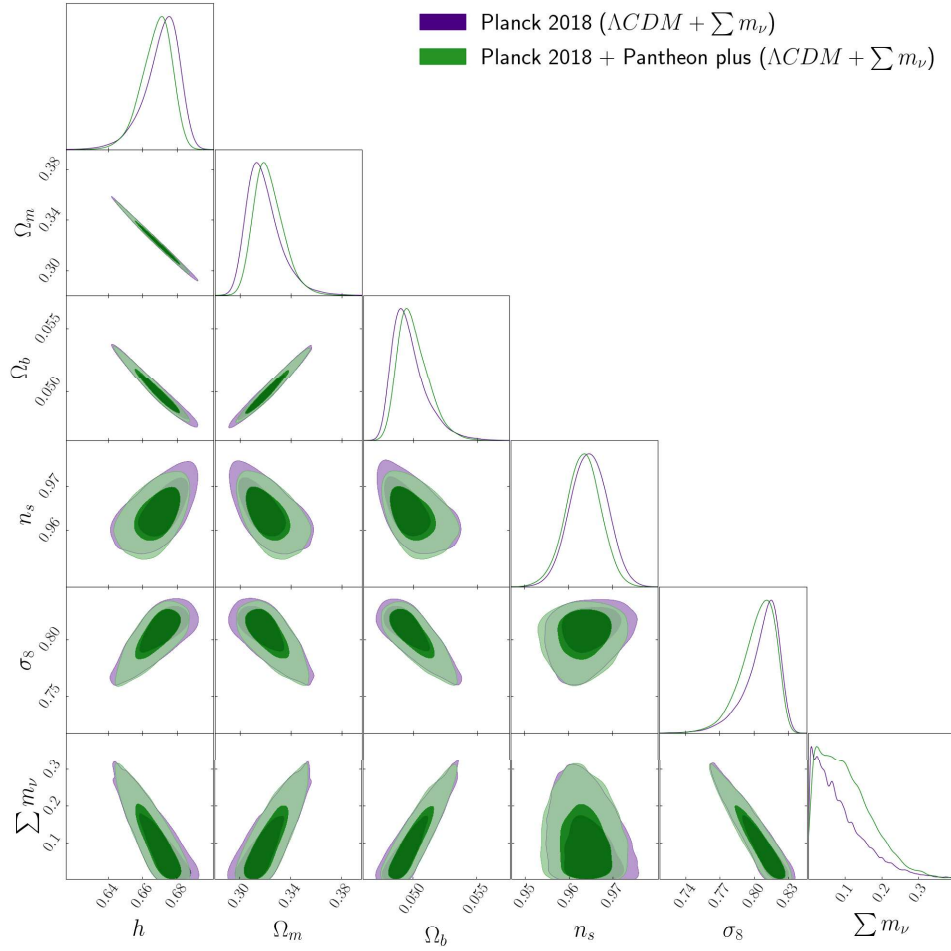


Figure 4.7: Confidence contours at 68% and 95% C.L. for the $\Lambda\text{CDM} + \sum m_\nu$ model with the J-PAS forecast with the full survey $f_{\text{sky}} = 8500 \text{ deg}^2$ in an Optimistic case with $k_{\text{max}}(\text{GCs}) = 0.20 h \text{ Mpc}^{-1}$ combined with Planck 2018 temperature, polarization and lensing with and without Pantheon Plus SN data.

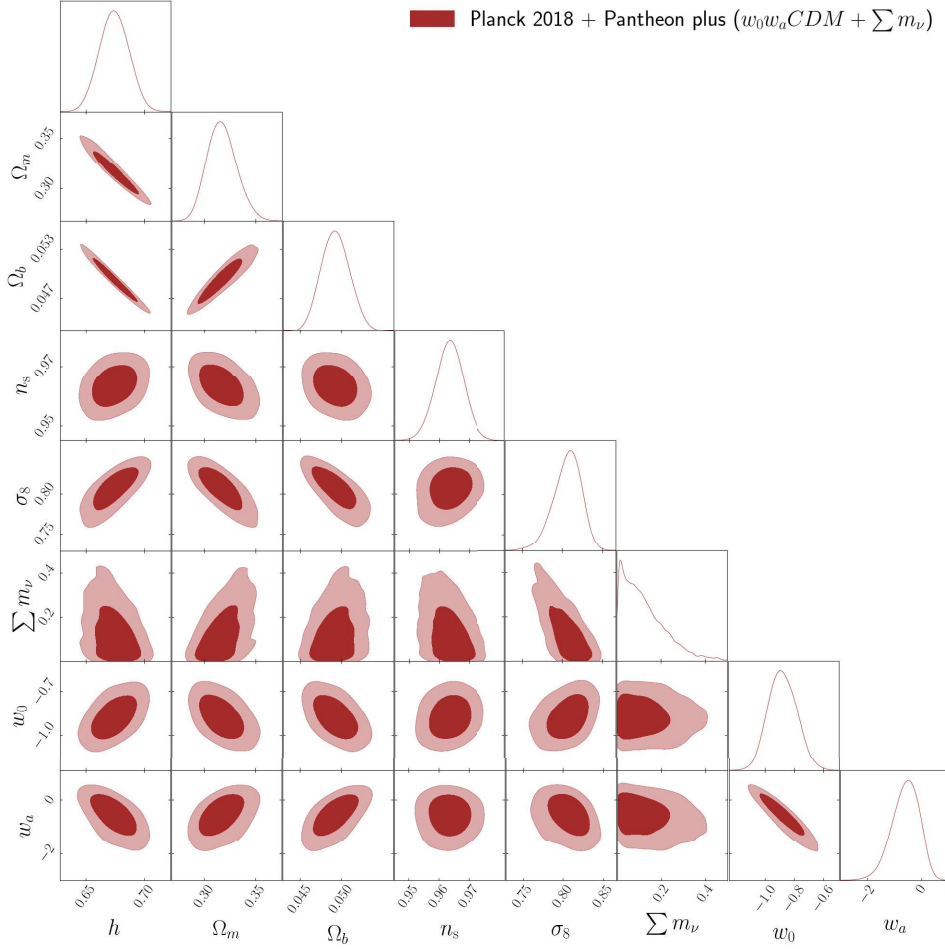


Figure 4.8: Confidence contours at 68% and 95% C.L. for the $w_0w_a\Lambda\text{CDM} + \sum m_\nu$ model with the J-PAS forecast with the full survey $f_{\text{sky}} = 8500 \text{ deg}^2$ in an Optimistic case with $k_{\text{max}}(\text{GCs}) = 0.20 h \text{ Mpc}^{-1}$ combined with Planck 2018 temperature, polarization and lensing and Pantheon Plus SN data.

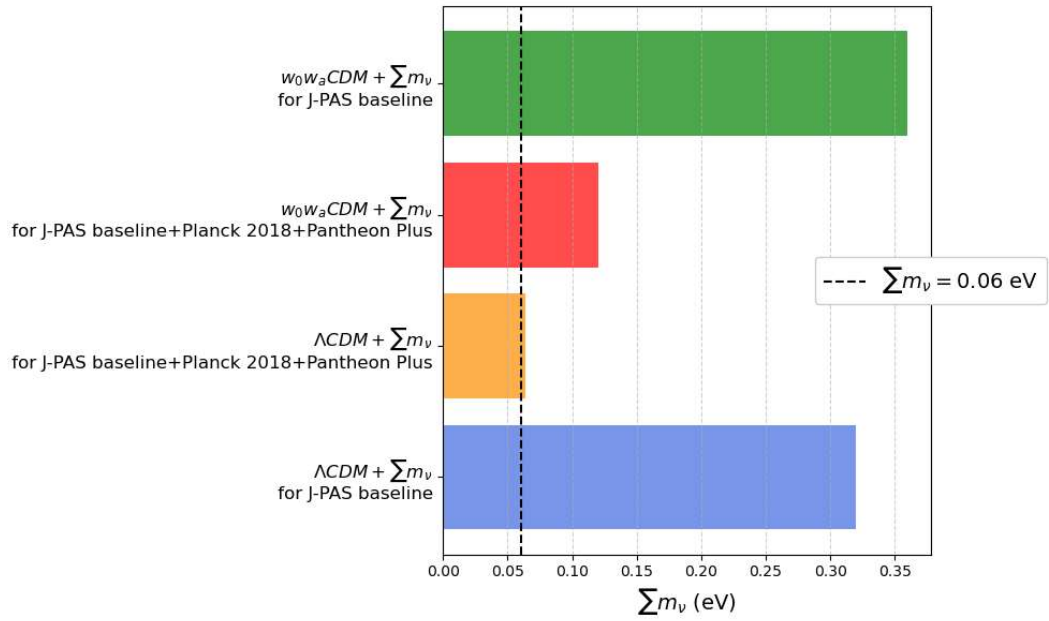


Figure 4.9: Neutrino mass upper limits for $\Lambda CDM + \Sigma m_\nu$ and w_0w_aCDM . The green and blue bars represent J-PAS alone for $w_0w_aCDM + \Sigma m_\nu$ and $\Lambda CDM + \Sigma m_\nu$, respectively. The red and yellow bars is J-PAS combined with CMB and SN. The black dashed line indicates the normal hierarchy limit for the neutrino mass.

THAWING DARK ENERGY AND MASSIVE NEUTRINOS IN LIGHT OF DESI

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Recent analyses have shown that a dynamic dark energy modeled by the CPL parameterization of the dark energy equation of state (EoS) can ease constraints on the total neutrino mass compared to the standard Λ CDM model. This helps reconcile cosmological and particle physics measurements of Σm_ν . In this study, we investigate the robustness of this effect by assessing the extent to which the CPL assumption influences the results. We examine how alternative EoS parameterizations - such as Barboza-Alcaniz (BA), Jassal-Bagla-Padmanabhan (JBP), and a physically motivated thawing parameterization that reproduces the behavior of various scalar field models - affect estimates of Σm_ν . Although both the BA and JBP parameterizations relax the constraints similarly to the CPL model, the JBP parameterization still excludes the inverted neutrino mass hierarchy at $\sim 2.1\sigma$ with $\Sigma m_\nu < 0.096$ eV. The thawing parameterization excludes the inverted hierarchy at $\sim 3.3\sigma$ and yields tighter constraints, comparable to those of the Λ CDM model, with $\Sigma m_\nu < 0.071$ eV. Finally, we show that the thawing model can be mapped into the BA and JBP w_0 - w_a parameter space, with the apparent preference for the phantom regime actually supporting quintessence (non-phantom) models.

5.1 Introduction

In the standard cosmology, a dark energy (DE) component drives the current cosmic acceleration. This component possesses negative pressure and is characterized by an Equation of State (EoS) parameter defined as $w = p/\rho$, where p is the dark energy pressure and ρ its energy density. A natural theoretical representation of DE is the vacuum energy density, also known as the cosmological constant Λ , for which $w = -1$. However, despite being able to explain most of the present-day observational data, the standard Λ -Cold Dark Matter (Λ CDM) model faces tensions in measurements of some cosmological parameters. The most prominent of these discrepancies is a tension of $\sim 5\sigma$ between the Hubble Constant H_0 measurements in the late- and early-time Universe. Furthermore, Λ is associated with well-known fundamental problems in theoretical physics [Weinberg \[1989, 2000\]](#), [Padmanabhan \[2003\]](#), [Alcaniz \[2006\]](#).

In the context of General Relativity, two major frameworks have emerged as alternatives to model the Universe's accelerated expansion (see e.g. [Copeland et al. \[1998\]](#), [Amendola \[2000\]](#), [Lima and Alcaniz \[2000\]](#), [Dev et al. \[2003\]](#), [Bagla et al. \[2003\]](#), [Sahni and Shtanov \[2003\]](#), [Carvalho et al. \[2006b\]](#)): the scalar field approach, exemplified by quintessence models, and phenomenological parameterizations of the dark energy equation of state (EoS), such as the widely used Chevallier-Polarski-Linder (CPL), Barboza-Alcaniz (BA), and Jassal-Bagla-Padmanabhan (JBP) forms [Chevallier and Polarski \[2001\]](#), [Linder \[2003\]](#), [Jassal et al. \[2005\]](#), [Barboza and Alcaniz \[2008\]](#), [Barboza et al. \[2009\]](#). In both frameworks, the dark energy EoS is a dynamical quantity, $w(z)$.

Quintessence models describe dark energy as a slowly rolling scalar field with a canonical kinetic term and a potential $V(\phi)$ that drives its evolution. A key feature of these models is that

the EoS remains strictly above the phantom divide, $w(z) > -1$, unless the theory is extended to include exotic components. On the other hand, phenomenological parameterizations do not assume an underlying physical origin for DE but instead provide flexible, model-independent forms for $w(z)$. These can allow the EoS to cross the phantom divide and are useful for fitting cosmological data. However, as discussed in [Shlivko and Steinhardt \[2024\]](#), [Wolf and Ferreira \[2023\]](#), [Wolf et al. \[2024\]](#), [Shlivko et al. \[2025\]](#), [de Souza et al. \[2025\]](#), a phantom-like behavior in parameterized EoS models — such as $w(z) < -1$ in the CPL framework — does not necessarily imply the existence of or even preference for phantom dark energy. When mapping physically motivated quintessence models into the CPL w_0 - w_a parameter space, many fall within the so-called phantom region, indicating that such behavior can emerge from standard scalar field dynamics. Therefore, while these parameterizations should not be dismissed — as they are useful in searching for deviations from Λ — they are limited in their ability to reveal the physical nature of DE.

To evaluate the observational viability of dynamical dark energy (DDE) models, various probes from both the early and local Universe have been employed [Wang \[2022\]](#), [Peracaula et al. \[2019\]](#), [Pan et al. \[2019\]](#), [Escamilla-Rivera and Nájera \[2022\]](#), [Valentino et al. \[2020\]](#), [Yang et al. \[2019, 2020\]](#). CMB data alone cannot strongly constrain these models due to a geometric degeneracy between late-time parameters and the angular diameter distance to the last scattering surface. Breaking this degeneracy requires the inclusion of local Universe observables. Although combining CMB with SN data improves constraints, no clear preference for DDE emerged initially. This remained the case until very recently, when BAO observations from the Dark Energy Spectroscopic Instrument (DESI), combined with Planck CMB and Pantheon+ SN data, showed a significant preference for DDE using the CPL EoS parameterization, $w(a) = w_0 + w_a(1 - a)$ [Abdul Karim et al. \[2025b\]](#). Even more recently, it has been shown that this preference is not limited to the CPL but extends to other parameterizations as well. This result further reinforces the preference indicated by parametric analyses of the DESI and SNe data for DDE models [Giarè et al. \[2024\]](#).

Beyond the DDE discussion, the late-time universe is plagued by further uncertainties related to the background and perturbations evolution, and the neutrino physics is of particular interest in this picture. In the standard scenario, outlined by the Standard Model of fundamental particles (SM), neutrinos are massless particles. Such a conclusion is challenged by the number density measurements of the solar and atmospheric neutrinos, pointing to an oscillatory behavior of the three flavors. Oscillation experiments are insensitive to the absolute mass scale of the neutrinos, but useful to probe their squared mass differences. The latter results from atmospheric neutrinos point to $|\Delta m_{32}^2| \approx 2.4 \times 10^{-3} \text{ eV}^2$, while from solar neutrinos one obtains $|\Delta m_{21}^2| \approx 7.4 \times 10^{-5} \text{ eV}^2$ [Navas et al. \[2024\]](#). This configuration enables at least two non-relativistic species of neutrinos. Lower limits on the sum of their masses can also be estimated: $\sum m_\nu > 0.06 \text{ eV}$ for the Normal Ordering schemes (NO), while the Inverted Ordering (IO) requires $\sum m_\nu > 0.10 \text{ eV}$.

Back to the cosmological perspective, neutrinos have a well-marked role in the expansion of the cosmos. They act as radiation in the very early universe, later changing to a hot dark matter behavior after the non-relativistic transition. At the perturbative level, the neutrinos' dispersion acts to suppress the growth of perturbations at small scales, characterized by their free-streaming and, ultimately, by the sum of their masses. Increasing $\sum m_\nu$ also changes the amplitude of the distance measurements in late-time Universe. In particular, the discrepancy between the values of $H_0 r_d$ measured by CMB observations and BAO measurements can be mitigated by reducing $\sum m_\nu$ in the context of Λ CDM [Elbers et al. \[2025b\]](#). One of the current tightest constraints, $\sum m_\nu < 0.07$ eV at 95% confidence level (C.L), comes from the combination of CMB and DESI BAO data within the Λ CDM framework, showing a clear preference for the NO [Abdul Karim et al. \[2025b\]](#), [Jimenez et al. \[2022a\]](#) (see also [Gariazzo et al. \[2022a\]](#)). More recently, the upper limit $\sum m_\nu < 0.042$ eV was obtained using CMB combined with DESI BAO, galaxy cluster measurements, and a H_0 prior from SH0ES [Jiang et al. \[2025\]](#). Therefore, the current cosmological limits for the Λ CDM are on the verge of excluding both inverted and normal ordering, creating tension with the results of particle physics experiments [Gariazzo et al. \[2023\]](#), [Jiang et al. \[2025\]](#).

These bounds are substantially relaxed under a DDE framework modeled by CPL, yielding $\sum m_\nu < 0.11$ eV at 95% C.L [Abdul Karim et al. \[2025b\]](#) (see also [Roy Choudhury and Okumura \[2024\]](#)). When expanding the cosmological model to a dynamical dark energy scenario, larger values of $\sum m_\nu$ are allowed by decreasing the value of $w(z)$, since these two parameters are anti-correlated [Hannestad \[2005\]](#), [Roy Choudhury and Hannestad \[2020\]](#). The degeneracy of $\sum m_\nu$ and $w(z)$ is partially broken when CMB, BAO and supernova data are considered concomitantly.

This work examines how other well-established EoS phenomenological parameterizations, such as Barboza-Alcaniz (BA) and Jassal-Bagla-Padmanabhan (JBP), affect the $\sum m_\nu$ constraints. We also consider a $w(z)$ parameterization that describes the overall behavior of several thawing quintessence models [de Souza et al. \[2025\]](#). In our work, we performed a Markov Chain Monte Carlo (MCMC) analysis combining the latest CMB, SN, and DESI BAO data. We also perform a Bayesian analysis to compare the results of the above parameterizations with respect to the CPL parameterization. In Section 2, we detail the functional forms and the motivation of the DDE models considered. Section 3 presents the methodology adopted in our analysis and the results obtained. In Section 4, we present and discuss our main conclusions.

5.2 Dynamical Dark Energy Models

DDE models affect the evolution of the Universe in two primary ways: through the expansion rate and through the cosmological perturbations that give rise to structures. In the case of the Universe's expansion rate, assuming a statistically homogeneous and isotropic Universe that

follows the Friedmann-Lemaître-Robertson-Walker (FLRW) metric, we have:

$$(5.1) \quad H^2 = H_0^2 [\rho_r a^{-4} + \rho_m a^{-3} + \rho_{de} a^{-3} \exp[\Upsilon(a)]] ,$$

where $w(a)$ represents the EoS as a function of the scale factor and $\Upsilon(a) = \left[\int_1^a -3 \left(\frac{w(a)}{a} \right) da \right]$. For the cosmological perturbations the first and most obvious impact of a time-evolving EoS is on the density perturbation and on the fluid velocity. If we assume a Newtonian gauge,

$$(5.2) \quad ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1-2\Phi)\delta_{ij}dx^i dx^j ,$$

then, the Continuity and Euler equations are written as

$$(5.3a) \quad \dot{\delta} + 3\mathcal{H}(c_s^2 - w)\delta + (1+w)(\theta - 3\dot{\Phi}) = 0 ,$$

$$(5.3b) \quad \dot{\theta} + \mathcal{H}(1-3w)\theta - \frac{k^2 c_s^2 \delta}{1+w} - k^2 \Psi = 0 .$$

The dark energy also affects the adiabatic sound speed of the fluid c_s^2 . To investigate the impact of DDE models on the sum of neutrino mass, we will use the following functional forms for the EoS,

- The Jassal-Bagla-Padmanabhan parameterization [Jassal et al. \[2005\]](#)

$$(5.4) \quad w(a) = w_0 + w_a a(1-a) .$$

The parameterization above consists of a combination of linear and quadratic terms in the scale factor. In the current epoch, the term $-w_a a^2$ becomes comparable to $w_a a$, leading to differences at low-redshifts compared to the CPL model.

- The Barboza-Alcaniz parameterization [Barboza and Alcaniz \[2008\]](#)

$$(5.5) \quad w(a) = w_0 + w_a \frac{1-a}{a^2 + (1-a)^2} .$$

This parameterization exhibits a linear behavior at low-redshifts. Unlike the CPL parameterization, which blows up exponentially as the scale factor approaches infinity (for $w_a > 0$), the BA parameterization remains well-behaved at all times, while still allowing for deviations from the CPL scenario.

- The thawing quintessence parameterization [Carvalho et al. \[2006b\]](#), [de Souza et al. \[2025\]](#)

$$(5.6) \quad w(a) = -1 + \alpha a^\beta .$$

Standard quintessence scenarios generally predict an EoS parameter satisfying $w > -1$, never crossing into the phantom regime. To generalize a class of quintessence potentials, [Carvalho et al. \[2006a\]](#) proposed the ansatz, $1/\rho_\phi(d\rho_\phi/da) = -3\alpha/a^{1+\beta}$, which results in (5.6) when combined with the continuity equation, $\dot{\rho}_\phi + 3H\rho_\phi[1+w(a)] = 0$. For $\alpha > 0$, Eq. (5.6) captures both thawing and freezing behaviors of scalar field models, with α setting the present-day EoS value and $\beta > 0$ controlling the rate of deviation from $w = -1$. When both $\alpha > 0$ and $\beta > 0$, the model reproduces the typical dynamics of thawing quintessence, where the field is initially frozen due to Hubble friction and behaves like a cosmological constant, later evolving to $w(z) > -1$. For a recent analysis of this generalized quintessence scenario in the absence of neutrinos, see [de Souza et al. \[2025\]](#).

5.3 Analysis and Results

To obtain constraints on the cosmological parameters for the DDE models above, we perform a Monte Carlo Markov Chain (MCMC) numerical analysis using Planck 2018 temperature and polarization data, including low- and high-multipole measurements of temperature and polarization, combined with lensing (plikHM+TTTEEE+lowl+lowE+lensing) [Planck Collaboration \[2020\]](#)¹, along with Pantheon+ catalog SN data [Brout et al. \[2022\]](#), [Scolnic et al. \[2022\]](#) and geometric BAO information from the DESI collaboration data release two [Abdul Karim et al. \[2025b\]](#). For this, we implement the parameterizations in the Boltzmann solver code *Cosmic Linear Anisotropy Solving System* (CLASS) [Lesgourgues \[2011\]](#), in which we interfaced with the *Cobaya* code to perform the MCMC statistical analysis [Blas et al. \[2011a\]](#), [Torrado and Lewis \[2021b\]](#). For the purpose of constraining the neutrino mass, we assume the standard degenerate mass spectrum, in which the three massive neutrino species each carry one third of the total mass, i.e., $\sum m_\nu = 3(m_{\nu,1} = m_{\nu,2} = m_{\nu,3})$. See [Herold and Kamionkowski \[2025\]](#) for a detailed discussion on the choice of neutrino mass hierarchy in cosmological analyses. In our analysis, we also imposed a prior that the total neutrino mass must satisfy $\sum m_\nu > 0$, with $\sum m_\nu \in [0, 5]$. For the EoS parameters our priors are, $w_0 \in [-3, 1]$, $w_\alpha \in [-3, 2]$, $\alpha \in [0, 1]$ and $\beta \in [0, 15]$.

Figure 5.1 shows the marginalized posterior distribution for $\sum m_\nu$ across the models explored in this work. For comparison, see Fig. 7 of [Elbers et al. \[2025c\]](#), which presents results from the DESI collaboration using DESI DR2 BAO + CMB + Pantheon+ data. In their analysis of the CPL model, the posterior peaks at the prior boundary $\sum m_\nu = 0$, suggesting that the data favor non-physical values of $\sum m_\nu < 0$. This is also true in our analysis: all models show a peak at $\sum m_\nu = 0$. This suggests that regardless of the DDE model, when including DESI data, there is still a preference outward $\sum m_\nu \leq 0$ values.

For the CPL parameterization, the DESI collaboration reports an upper bound of $\sum m_\nu < 0.11$ eV, a value compatible with both the normal and inverted mass hierarchies. In our analysis, the BA and CPL models give us nearly the same upper bound, $\sum m_\nu < 0.12$ eV, also consistent

¹It is important to note that DESI uses Planck combined with Data Release 6 of the Atacama Cosmology Telescope (ACT), while in our analysis we use CMB data from the Planck baseline likelihood only.

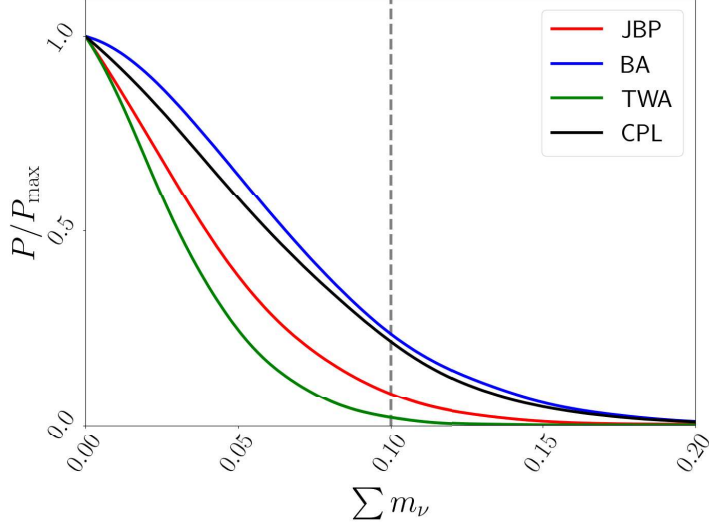


Figure 5.1: Upper bounds on Σm_ν from Planck 2018 temperature, polarization and lensing together with Pantheon+ SN and DESI DR2 data assuming JBP, BA, and Thawing parameterizations. The dashed vertical line represents the inverted hierarchy $\Sigma m_\nu > 0.10$ eV. For comparison, we also show the CPL prediction (black line).

with both mass orderings. The JBP, however, provides a tighter constraint, with a 2σ upper bound of $\Sigma m_\nu < 0.096$ eV, indicating a mild preference for the normal hierarchy. Interestingly, the thawing parameterization (5.6) imposes the tightest constraint, with $\Sigma m_\nu < 0.071$ eV. Not only does the thawing model constrain the total neutrino mass at a level comparable to that of the Λ CDM model, therefore excluding the inverted hierarchy by more than 3σ , while also weakens any preference for a non-physical negative neutrino mass within our data choice ².

In addition, we conducted a Bayesian statistical comparison of all models with respect to the CPL (for more details on this kind of analysis, we refer the reader to [de Souza et al. \[2025\]](#)). Table 5.1 presents the 2σ upper limits on Σm_ν for each model, along with the Bayes factor, $\Delta \ln B$. From this analysis, we find that the preference for both BA and JBP are statistically inconclusive compared to CPL ($|\Delta \ln B| < 1$) while the TWA model shows a moderate Bayesian preference over CPL, with $|\Delta \ln B| = 2.66$.

In Figure 5.2, we show the w_0 – w_a planes for JBP (upper panel) and BA parameterizations (bottom panel). When treating neutrino mass as a free parameter, the JBP model is consistent with the Λ CDM scenario within 2σ , while the BA model lies nearly outside the 2σ boundary. Following the procedure of [Shlivko et al. \[2025\]](#), we also project the (α, β) values from the thawing model into the (w_0, w_a) parameter space by matching the expansion history $H(z)$. The shaded regions in both panels of Fig. 5.2 represent the α – β values within the full prior range considered in the analysis. This shows that the phantom region in the w_0 – w_a plane for both models is fully

²We also performed the same analysis considering other SN samples, such as DESY5 [Collaboration \[2024a\]](#) and Union3 [Rubin et al. \[2024\]](#) compilations. Overall, we did not observe significant changes in the above conclusions.

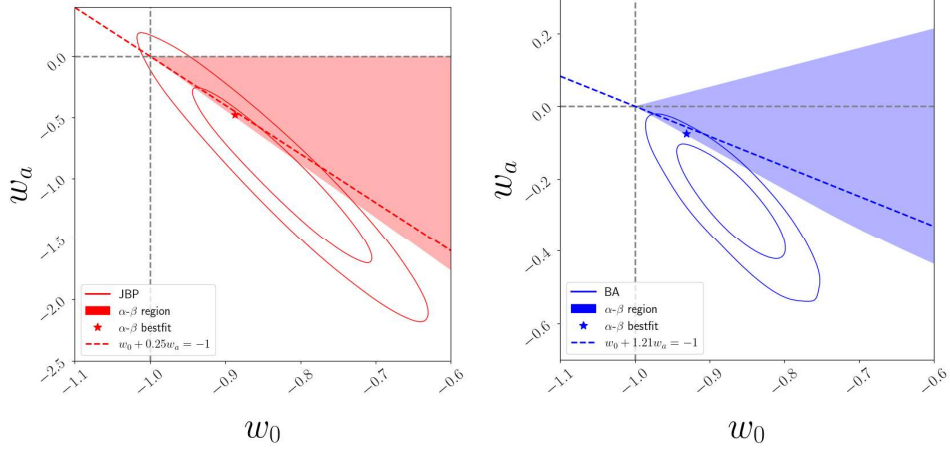


Figure 5.2: $w_0 - w_a$ plane for the JBP (upper) and BA (bottom) parameterizations obtained from Planck 2018 temperature, polarization and lensing together with Pantheon+ SN and DESI DR2 BAO data. The dashed line intersection represents the point, $w_a = 0$ and $w_0 = -1$. The shaded red (JBP) and blue (BA) regions represent the full range of $\alpha-\beta$ priors projected into the w_0-w_a space, while the star markers indicate the projected best-fit $\alpha-\beta$ values for each model. The red and blue dashed lines shows the dividing line between quintessence and phantom behavior for each EoS parameterizations.

Model	Σm_ν (2σ)	$\Delta \ln B$
CPL	< 0.123 eV	0.0
BA	< 0.127 eV	-0.64
JBP	< 0.096 eV	-0.60
TWA	< 0.0709 eV	2.66

Table 5.1: 2σ upper bounds on the sum of neutrino masses Σm_ν and the Bayes factor, $\Delta \ln B$, for all models discussed in the text. Negative values of $\Delta \ln B$ favors the reference model (CPL).

compatible with the scenario of thawing quintessence. The star markers correspond to the best-fit values, $\alpha = 0.15$ and $\beta = 7.84$, projected into the w_0-w_a parameter spaces for both models. These best-fit projections lie within the 1σ and 2σ confidence regions of the JBP and BA, respectively.

Figure 5.3 shows the EoS evolution with redshift for each model, using Planck CMB temperature, polarization, and lensing data, along with Pantheon+ SN and DESI BAO. The BA and JBP behave similarly at low redshifts, remaining in the quintessence region, i.e., $w(z) > -1$. The BA model crosses to phantom $w(z) < -1$ at a slightly lower redshift than the JBP, and remains in the phantom region near $w(z) = -1$. Interestingly, for the JBP model, it tends to cross $w(z) = -1$ twice, leaving the quintessence region and remaining very close to $w(z) = -1$ with a trend toward the

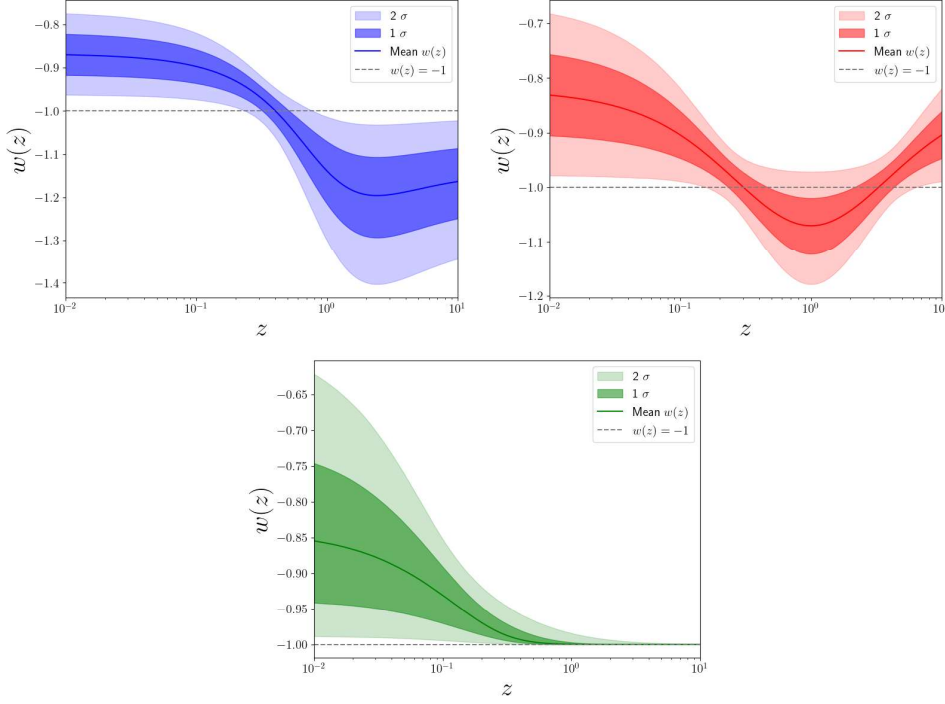


Figure 5.3: EoS evolution obtained for Planck+DESI+Pantheon+ data assuming the BA (blue), JBP (red) and Thawing (green) parameterizations.

phantom region until it returns to the quintessential region for high z values. The thawing EoS remains exactly equal to $w(z) = -1$ at high redshifts and begins to evolve toward the $w(z) > -1$ region near $z \approx 0$.

5.4 Final Remarks

Our findings in this work highlight several key aspects. First, regardless of the underlying cosmological model, be it Λ CDM or a DDE scenario, the posterior distribution for $\sum m_\nu$, when DESI data are included, consistently peaks at the lower prior boundary, $\sum m_\nu = 0$. This behavior indicates that the data would prefer values of $\sum m_\nu$ that are negative, an unphysical regime excluded by the prior. Consequently, the derived upper bounds on $\sum m_\nu$ in this context are largely driven by the prior choice. The thawing parameterization provides the strongest constraint on the neutrino mass, $\sum m_\nu < 0.071$ eV at 95% C.L., comparable to that of Λ CDM and sufficient to exclude the inverted hierarchy at more than 3σ . The JBP model also shows a mild preference for the normal hierarchy over the inverted, with an upper bound of $\sum m_\nu < 0.096$ eV at 95% C.L. We find that the JBP model is consistent with Λ CDM within 2σ while the BA model sits near the outer limit of 2σ . Both models exhibit a shift toward Λ CDM when the neutrino mass is included in the analysis as a free parameter. This is significant, as it weakens the apparent preference for

DDE models seen in analyses where the total neutrino mass is fixed at $\sum m_\nu = 0.06$ eV.

By mapping the thawing model into the w_0 – w_a plane, we also demonstrate that this class of quintessence models is fully compatible with the regions occupied by the BA and JBP models, even extending into the phantom regime, which appears to be favored by the data. This suggests that, contrary to the claims of the DESI collaboration of phantom dark energy evidence, the data may in fact be suggesting evidence for quintessence scenarios. Notably, the thawing model also shows a moderate Bayesian preference over the CPL, with $\Delta \ln B = 2.66$.

Finally, we studied the redshift evolution of the dark energy equation of state. Although they all remain in the quintessence regime at low redshifts, their behaviors diverge at higher redshifts. The thawing model stands out because of its smooth transition from a cosmological constant-like state ($w = -1$) at high redshift to a quintessence-like behavior near the present epoch. Overall, our results provide evidence that thawing quintessence models are not only fully compatible with widely used phenomenological parameterizations when projected in the same parameter space, but also offer tighter bounds on the neutrino mass. The thawing model also shows a strong preference for the normal mass ordering.

Acknowledgements

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CONCLUSIONS

In this thesis, we have investigated the role of neutrinos in cosmology, focusing on their impact across different epochs of the Universe and on a variety of cosmological observables. By combining theoretical modeling, Bayesian statistical analyses, and forecasts for future surveys, we have explored how neutrino properties, particularly their masses, connection with inflation, dark energy, and the formation of LSS.

We began by revisiting the small-field double-well inflationary model, characterized by a potential of the form $V(\phi) = \frac{\lambda}{4}(\phi^2 - v_\phi^2)^2$, where λ is a dimensionless coupling constant and v_ϕ is the vacuum expectation value of the inflaton field. We considered both minimal and nonminimal couplings between the inflaton and gravity, and assessed the observational viability of the model using current CMB data from Planck and BICEP/Keck. Our analysis shows that, despite its theoretical appeal, the double-well inflationary scenario is statistically disfavored when compared to the Starobinsky model. From the Bayesian analysis, we obtained constraints on the inflaton vacuum expectation value, finding $\log_{10}(v_\phi/M_P) = 1.258^{+0.035}_{-0.048}$ in the minimal case, and $\log_{10}(v_\phi/M_P) = 1.247$ and $0.920^{+0.084}_{-0.26}$ for the nonminimal scenario at 1σ confidence level. We then explored the implications of this framework for neutrino physics within the canonical Type-I seesaw mechanism. In this context, the vacuum expectation value of the scalar field directly determines the mass scale of the heavy sterile neutrinos. Using the cosmologically constrained values of v_ϕ , together with neutrino oscillation data, we estimated the relevant Yukawa couplings and investigated possible signatures in lepton flavor violating processes. Focusing on the decay channel $\mu \rightarrow e\gamma$, we found that the predicted branching ratio is extremely suppressed, of order $\mathcal{O}(10^{-53})$, both in the minimal and nonminimal scenarios. This is many orders of magnitude below the current experimental upper bound, $B(\mu \rightarrow e\gamma) < 4.2 \times 10^{-13}$, showing that this process is unobservable in this framework. The lepton flavor violating processes would require extremely

small Yukawa couplings for the sterile neutrinos, $h' \lesssim 10^{-20}$, implying a high degree of fine-tuning. This result indicates that, within the double-well inflationary scenario, the heavy sterile neutrinos are effectively decoupled from low-energy phenomenology and remain hidden in the dark sector. These results present an important connection between inflationary dynamics and neutrino physics. Cosmological constraints on the inflaton sector can strongly limit the phenomenological viability of associated particle physics extensions. In particular, while the Type-I seesaw mechanism remains a compelling explanation for neutrino masses, its low-energy signatures may be highly suppressed in scenarios where the relevant mass scale is tied to inflationary physics.

We then explored quintessential inflation model, which presents a unified description of the early and late accelerated phases of the Universe. Within the context of α -attractor models, we implemented the scenario in a modified version of the Boltzmann solver CLASS, allowing for a consistent treatment of massive neutrinos in cosmological observables. By combining Planck CMB, DESI BAO, and Pantheon+ Type Ia supernova data, we showed that this model constrains the total neutrino mass to $\sum m_\nu < 0.0670$ eV at 2σ C.L assuming a flat Universe, which represents one of the most constrained limit available in the literature. In the case in which we allow curvature, the constraints are significantly relaxed to $\sum m_\nu < 0.116$ eV at 2σ C.L, presenting a important degeneracy between the curvature of the Universe and the neutrino mass. We also provide forecasts for future probes, showing that the Simons Observatory, LiteBIRD, and Euclid configurations may reduce the uncertainty on $\sum m_\nu$ by $\approx 9\%$ in this scenario, while the precision on the quintessential parameter α_{QI} is improved by $\approx 72\%$.

We also investigated the sensitivity of the J-PAS large-scale structure survey to the sum of neutrino masses. Using a Fisher matrix approach, we produced forecasts for different cosmological models and survey configurations. Considering galaxy clustering alone over a sky coverage of $8,500 \text{ deg}^2$, the projected 95% C.L. upper limits on $\sum m_\nu$ are 0.32 eV and 0.36 eV for the $\Lambda\text{CDM} + \sum m_\nu$ and CPL $w_0 w_a \text{CDM} + \sum m_\nu$ models, respectively. The inclusion of Planck data leads to a substantial tightening of these constraints: the combination J-PAS+Planck yields $\sum m_\nu < 0.061$ eV in the $\Lambda\text{CDM} + \sum m_\nu$ framework, while extending the dataset to include Pantheon Plus gives $\sum m_\nu < 0.12$ eV for the dynamical dark energy model. Taken together, these forecasts presents the potential of J-PAS clustering measurements to contribute on open questions in the neutrino sector, from possible tensions between cosmological and laboratory mass determinations to the yet undetermined neutrino mass ordering.

Lastly, we examined the impact of dynamical dark energy models on cosmological constraints on the total neutrino masses. Considering alternative equation of state parameterizations, such as Barboza-Alcaniz (BA), Jassal-Bagla-Padmanabhan (JBP), and a physically motivated thawing quintessence parameterization, we assessed how robust the relaxation of neutrino mass constraints, previously reported by the DESI collaboration for the CPL model, is to the choice of dark energy parameterization. While the BA and JBP models relax the constraints on $\sum m_\nu$

comparably to CPL, the JBP parameterization still excludes the inverted mass hierarchy at $\sim 2.1\sigma$, yielding $\sum m_\nu < 0.096$ eV at 2σ C.L. The thawing parameterization, in contrast, produces tighter constraints comparable to those of Λ CDM, with $\sum m_\nu < 0.071$ eV at 2σ C.L, and excludes the inverted hierarchy at $\sim 3.3\sigma$. Our results also shows that the thawing model maps onto the BA and JBP w_0-w_a parameter space in a way that reveals an apparent preference for the phantom regime present by the DESI collaboration, to be in fact consistent with quintessence models.

We have shown that current cosmological constraints on the sum of neutrino masses presents a clear preference for the normal ordering of neutrino masses. In some cases, these bounds approach, and may even come into mild tension with precise lower limits inferred from neutrino oscillation experiments. Interestingly, we also see an odd preference for zero, or even negative, values of $\sum m_\nu$. Although a negative $\sum m_\nu$ is not physically meaningful, it may indicate limitations in the underlying cosmological model. This suggests that the role of neutrinos in the evolution of the Universe may not yet be fully understood. The neutrino sector remains a key window into physics beyond the SCM through the possibility of non-standard interactions. Extensions of the neutrino sector have been shown to alleviate several current tensions in cosmology, such as the Hubble and S_8 discrepancy [Cuesta et al. \[2022\]](#), [Camarena and Cyr-Racine \[2025\]](#), [Du et al. \[2026\]](#), [Zu et al. \[2026\]](#), [Poudou et al. \[2025\]](#).

More recently, results from DESI have indicated a preference for a phantom equation of state for dark energy, $w < -1$, which would violate classical energy conditions. If confirmed, such a result would have profound implications for fundamental physics. One of the simplest frameworks to describe a dynamical dark energy component is provided by quintessence models [Copeland et al. \[1998\]](#), [Amendola \[2000\]](#), in which dark energy is modeled as a slowly evolving scalar field with a canonical kinetic term and a potential $V(\phi)$. In these scenarios, the equation of state is bounded by $w \geq -1$, and therefore cannot account for a phantom regime, provided the field is minimally coupled. A particularly compelling possibility to simultaneously address the nature of dark energy and the role of neutrinos in the evolution of the Universe is to consider a coupling between the dark energy scalar field and the neutrino sector. In these models, the neutrino mass becomes a dynamical quantity depending on the scalar field, $m_\nu = m_\nu(\phi)$ [Fardon et al. \[2004\]](#). In these scenarios the evolution of dark energy and the neutrino mass scale are intrinsically linked. As a consequence, the cosmological impact of neutrinos is modified, potentially reconciling the apparent tension between cosmological upper bounds on $\sum m_\nu$ and the lower limits inferred from particle physics experiments. Furthermore, the coupling induces an effective dark energy equation of state of the form

$$(6.1) \quad w_{\text{eff}} = w_\phi + \frac{d \ln m_\nu(\phi)}{d\phi} \frac{\dot{\phi} \rho_\nu}{3H \rho_\phi},$$

where $w_\phi \geq -1$ is the equation of state of the scalar field. Interestingly, this effective equation of state can enter the phantom regime, $w_{\text{eff}} < -1$, without requiring any fundamental violation of

energy conditions at the level of the scalar field. This could potentially explain DESIs preference for a phantom equation of state without the violation of any energy condition. Exploring such neutrinos-dark energy interacting scenarios represents a promising direction that we intend to pursue.

Finally, cosmology has long played a fundamental role in constraining particle physics beyond the reach of laboratory experiments. A notable example is the determination of the number of light neutrino species, N_{eff} . As discussed throughout this work, the primordial abundances of light elements are sensitive to the total radiation content of the Universe, thereby providing early constraints on the effective number of relativistic species. In particular, James Gunn, David Schramm, and Gary Steigman in 1977 [Steigman et al. \[1977\]](#) showed that the primordial helium abundance predicted by BBN implies an upper limit of fewer than seven light neutrino species, later refined to fewer than four by 1980. These results represent the first robust constraints on the number of neutrino species, at a time when particle physics experiments were not yet capable of probing this quantity. Only with the advent of precision measurements at CERN in 1989 did laboratory determinations become competitive, with improved analyses of LEP data today yielding $N_\nu = 2.9963 \pm 0.0074$ [Janot and Jadach \[2020\]](#), [Voutsinas et al. \[2020\]](#). Remarkably, modern cosmological observations continue to provide constraints of comparable precision, with current bounds of $N_{\text{eff}} = 2.990 \pm 0.070$ (68% C.L.) [Goldstein and Hill \[2026\]](#). As demonstrated throughout this work, cosmology continues to serve as a powerful laboratory linking fundamental particle physics to the dynamics and evolution of the Universe, while providing a complementary and increasingly precise avenue for probing particle properties.



APPENDIX: PUBLICATION LIST

During the course of this PhD, I have authored a total of 13 papers, of which I am the first author in 6. The full list of publications is presented below.

- DOUBLE-WELL INFLATION: OBSERVATIONAL CONSTRAINTS AND THEORETICAL IMPLICATIONS

G. Rodrigues, J.G. Rodrigues, F.B.M. dos Santos, and J.S. Alcaniz

Journal of Cosmology and Astroparticle Physics, JCAP 11 (2024) 055. e-Print: 2409.13918 [astro-ph.CO] **Presented in Chapter 2.**

- COSMOLOGICAL CONSTRAINTS ON NEUTRINO MASSES IN QUINTESSENTIAL INFLATION

Jamerson Rodrigues, **Gabriel Rodrigues**, Felipe B.M. dos Santos, Simony Santos da Costa, and Jailson Alcaniz

To be published . e-Print: 2602.20349 [astro-ph.CO] **Presented in Chapter 3.**

- J-PAS: FORECASTING CONSTRAINTS ON NEUTRINO MASSES

Gabriel Rodrigues, Antonio J. Cuesta, Jailson Alcaniz, Miguel Aparicio Resco, Antonio L. Maroto, et al.

Journal of Cosmology and Astroparticle Physics, JCAP 12 (2025) 047. e-Print: 2602.06027 [astro-ph.CO] **Presented in Chapter 4.**

- THAWING DARK ENERGY AND MASSIVE NEUTRINOS IN LIGHT OF DESI

Gabriel Rodrigues, Rayff de Souza, Jamerson Rodrigues, and Jailson Alcaniz

Journal of Cosmology and Astroparticle Physics, JCAP 08 (2025) 016. e-Print: 2503.00126 [astro-ph.CO] **Presented in Chapter 5.**

- FORECASTING CONSTRAINTS ON QUINTESSENTIAL INFLATION FROM FUTURE GENERATION OF GALAXY AND CMB SURVEYS
G. Rodrigues, F.B.M. dos Santos, S. Santos da Costa, J.G. Rodrigues, R. von Marttens, R. Silva, D.F. Mota, J.S. Alcaniz
 Journal of Cosmology and Astroparticle Physics, JCAP 11 (2025) 022. e-Print: 2506.22384 [astro-ph.CO]
- COSMOGRAPHY WITH DESI DR2 AND SN DATA
Gabriel Rodrigues, Rayff de Souza, and Jailson Alcaniz
 Physical Review D, 112 (2025) 10, 103519. e-Print: 2506.22373 [astro-ph.CO]
- ACT DR6+PLANCK IMPACT ON INFLATION WITH NON-ZERO VACUUM EXPECTATION VALUE AND THE POST-INFLATIONARY BEHAVIOR
 F.B.M. dos Santos, J.G. Rodrigues, **G. Rodrigues**, C. Siqueira, and J.S. Alcaniz
 To be published In Physical Review D.e-Print: 2602.06027 [astro-ph.CO]
- THAWING QUINTESSENCE AND TRANSIENT COSMIC ACCELERATION IN LIGHT OF DESI
 Rayff de Souza, **Gabriel Rodrigues**, and Jailson Alcaniz
 Physical Review D, 112 (2025) 8, 083533. e-Print: 2504.16337 [astro-ph.CO]
- J-PAS AND PFS SURVEYS IN THE ERA OF DARK ENERGY AND NEUTRINO MASS MEASUREMENTS
 Fuxing Qin, Yuting Wang, Gong-Bo Zhao, Antonio J. Cuesta, Jailson Alcaniz, **Gabriel Rodrigues** et al.
 The Astrophysical Journal 997 (2026) 2, 251. e-Print: 2505.04275 [astro-ph.CO]
- STAGE IV CMB FORECASTS FOR WARM INFLATION
 F.B.M. Dos Santos,**G. Rodrigues**, R. de Souza, and Jailson S. Alcaniz
 Journal of Cosmology and Astroparticle Physics JCAP 03 (2025) 062. e-Print: 2412.02696 [astro-ph.CO]
- IS NATURAL INFLATION IN AGREEMENT WITH CMB DATA?
 F.B.M. Dos Santos,**G. Rodrigues**, J.G. Rodrigues, R. de Souza, and Jailson S. Alcaniz
 Journal of Cosmology and Astroparticle Physics JCAP 03 (2024) 038. e-Print: 2312.12286 [astro-ph.CO]
- EXAMINING THE VALIDITY OF THE MINIMAL VARYING SPEED OF LIGHT MODEL THROUGH COSMOLOGICAL OBSERVATIONS: RELAXING THE NULL CURVATURE CONSTRAINT
 Purba Mukherjee, **Gabriel Rodrigues**, and Carlos Bengaly
 Physics of the Dark Universe, 43 (2024) 101380. e-Print: 2302.00867 [astro-ph.CO]

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- A MODEL-INDEPENDENT TEST OF SPEED OF LIGHT VARIABILITY WITH COSMOLOGICAL OBSERVATIONS

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