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MAPPING THE 2D MATTER DISTRIBUTION IN THE LOCAL UNIVERSE USING
BLUE GALAXIES FROM SDSS DR12

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BLUE GALAXIES FROM SDSS DR12

por

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MARIA VITORIA LAZARIN

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Lança o barco contra o mar

Venha o vento que houver

E se puder, voa.

— Rubel, O Velho e o Mar

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MAPPING THE 2D MATTER DISTRIBUTION IN THE LOCAL UNIVERSE USING BLUE GALAXIES FROM SDSS DR12

RESUMO

As Oscilações Acústicas Bariônicas (OABs), uma escala preferencial de aproximadamente 150 Mpc impressa na distribuição de matéria em larga escala por ondas acústicas no plasma primordial, constituem uma das réguas padrão mais robustas em cosmologia. Ao contrário da abordagem angular mais usual, baseada na função de correlação $\omega(\theta)$, neste trabalho as OABs são medidas no espectro de potência angular C_ℓ , no espaço harmônico, a partir de uma amostra de galáxias azuis com formação estelar em baixo redshift, da décima segunda liberação de dados do Sloan Digital Sky Survey (SDSS). Com isso, busca-se validar um pipeline de análise angular de OABs no espaço harmônico e aplicá-lo a essa amostra não convencional. A metodologia adotada depende apenas fracamente do modelo cosmológico: por situar-se no espaço harmônico transversal, dispensa a conversão de redshifts em distâncias radiais, de modo que a informação sobre a escala acústica decorre exclusivamente da posição da característica oscilatória no espaço de multipolos. Para as análises realizadas, a amostra foi dividida em dois intervalos de redshift: $0.15 \leq z \leq 0.25$ (medida principal) e $0.05 \leq z \leq 0.15$ (validação do pipeline). O espectro de ambas as amostras é estimado pelo formalismo pseudo- C_ℓ , e a covariância é construída a partir de 1 000 catálogos lognormais gerados com o código GLASS, recalibrados iterativamente. O parâmetro de deslocamento α é determinado por dois métodos independentes: o perfil de verossimilhança e cadeias de Markov via Monte Carlo (MCMC). Para o intervalo principal, obtêm-se $\alpha = 1.034^{+0.155}_{-0.162}$ e $\alpha = 0.974^{+0.247}_{-0.194}$, respectivamente, ambos consistentes, dentro das incertezas, com o valor $\langle \alpha \rangle = 1$ esperado no modelo Λ CDM. A preferência pelo modelo oscilatório é marginal ($\Delta\chi^2 = 1.38; 1.17\sigma$), consistente com a acurácia permitida pelo volume e pela cobertura de céu do levantamento ($f_{\text{sky}} \approx 0.18$); no intervalo de baixo redshift não há preferência significativa ($\Delta\chi^2 < 0.1$), como esperado na ausência de sinal detectável. A escala medida traduz-se na razão entre a distância comóvel transversal e o horizonte sonoro, $D_M/r_d = 5.606 \pm 1.276$ em $z_{\text{eff}} = 0.20$, sem que seja possível discriminar entre as previsões do Planck (2018) e do DESI (2025), e na escala angular acústica $\theta_{\text{BAO}} = 10.21^\circ \pm 2.326^\circ$. Combinada com compilações angulares da literatura, conduz a $\Omega_{m,0} = 0.269^{+0.018}_{-0.017}$ e $h r_d = 103.11^{+0.98}_{-0.99}$ Mpc. Embora a baixa significância reflita a dificuldade intrínseca da medida nesse regime, os resultados sugerem a viabilidade das galáxias azuis como traçadoras das OABs e validam o arcabouço de análise para aplicação em levantamentos futuros.

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ABSTRACT

Baryon Acoustic Oscillations (BAO), a preferred scale of approximately 150 Mpc imprinted on the large-scale matter distribution by acoustic waves in the primordial plasma, constitute one of the most robust standard rulers in cosmology. In contrast to the more common angular approach, based on the angular correlation function $\omega(\theta)$, in this work the BAO are measured through the angular power spectrum C_ℓ , in harmonic space, using a sample of star-forming blue galaxies at low redshift from the twelfth data release of the Sloan Digital Sky Survey (SDSS). The aim is to validate a pipeline for the angular analysis of BAO in harmonic space and to apply it to this unconventional sample. The methodology is only weakly dependent on the assumed cosmological model: by working in transverse harmonic space, it does not require the conversion of redshifts into radial distances, so that the information on the acoustic scale follows exclusively from the position of the oscillatory feature in multipole space. For the analysis, the sample was divided into two redshift bins: $0.15 \leq z \leq 0.25$ (the main measurement) and $0.05 \leq z \leq 0.15$ (pipeline validation). The angular power spectrum of both samples is estimated with the pseudo- C_ℓ formalism, and the covariance is built from 1000 lognormal catalogs generated with the GLASS code and iteratively recalibrated. The shift parameter α is determined by two independent methods: the profile likelihood and Markov Chain Monte Carlo (MCMC). For the main bin, we obtain $\alpha = 1.034^{+0.155}_{-0.162}$ and $\alpha = 0.974^{+0.247}_{-0.194}$, respectively, both consistent, within the uncertainties, with the value $\langle\alpha\rangle = 1$ expected in the Λ CDM model. The preference for the oscillatory model is marginal ($\Delta\chi^2 = 1.38; 1.17\sigma$), consistent with the accuracy allowed by the volume and sky coverage of the survey ($f_{\text{sky}} \approx 0.18$); in the low-redshift bin no significant preference is found ($\Delta\chi^2 < 0.1$), as expected in the absence of a detectable signal. The measured scale corresponds to a ratio between the transverse comoving distance and the sound horizon of $D_M/r_d = 5.606 \pm 1.276$ at $z_{\text{eff}} = 0.20$, without being able to discriminate between the predictions of Planck (2018) and DESI (2025), and to an acoustic angular scale $\theta_{\text{BAO}} = 10.21^\circ \pm 2.326^\circ$. Combined with angular compilations from the literature, it yields $\Omega_{m,0} = 0.269^{+0.018}_{-0.017}$ and $h r_d = 103.11^{+0.98}_{-0.99}$ Mpc. Although the low significance reflects the intrinsic difficulty of the measurement in this regime, the results suggest the viability of blue galaxies as BAO tracers and validate the analysis framework for application to future surveys.

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Chapter 1

Introduction

In the first moments after the Big Bang, the Universe was a plasma of free electrons, protons, and high-energy photons, under conditions so extreme that no stable atomic nucleus could survive [1, 2]. Matter and radiation were coupled, forming a single fluid, and the Universe was opaque: photons were incessantly scattered by free electrons, incapable of travelling more than a tiny fraction of a metre before being deflected [3]. Gravity drew this fluid into regions of small overdensity, seeded by quantum fluctuations during an early epoch of rapid expansion known as inflation [4, 5], in which dark matter had already begun to accumulate. But gravity did not act unopposed. As the fluid was compressed, the radiation pressure of the trapped photons resisted, preventing further collapse. The result was a competition between two forces playing out on a cosmic scale: gravitational attraction pulling the fluid inward, and radiation pressure pushing it back out. The outcome of this competition was not collapse, but sound: acoustic waves propagating outward through the photon-baryon plasma at speeds at a substantial fraction of the speed of light [6].

For approximately 380 000 years, the Universe expanded and cooled as these acoustic waves travelled outward from every primordial overdensity. Dark matter, which interacts gravitationally but is insensitive to radiation pressure, remained behind, deepening the potential wells at the original perturbation regions while the photon-baryon fluid oscillated around them. Then, at a temperature of roughly 3000 K, the Universe cooled enough for electrons and protons to combine into neutral hydrogen for the first time [7, 8]. The photons, no longer scattered by free electrons, flowed freely through space, and with them went the radiation pressure that had driven the acoustic wave. The baryons, however, did not decouple instantaneously: for a brief period after recombination they remained coupled to the photon field through residual Thomson scattering, and the oscillations persisted until the photon density dropped enough for radiation pressure to become negligible. This moment, known as the drag epoch, marks the freezing of the acoustic waves [9]. From this point on, baryons evolved only under the action of gravity, and the sound horizon, the maximum distance a wave could travel before freezing, became a fixed scale imprinted

on the distribution of matter.

The consequence of this freezing is a preferred scale in the clustering of matter. At the drag epoch, the baryonic matter sat in two preferred locations: at the original overdensity, where dark matter had accumulated, and at the wavefront of the acoustic wave, having been carried outward by radiation pressure. The distance between these two concentrations is the comoving sound horizon at the drag epoch, approximately 150 Mpc [9], which constitutes a cosmic standard ruler. As the Universe continued to expand and galaxies formed preferentially at both locations, this imprinted an excess of galaxy pairs separated by precisely this preferred distance, the Baryon Acoustic Oscillation (BAO) feature [9, 10].

The photons that decoupled at recombination did not simply disappear: they are observed today as the Cosmic Microwave Background (CMB), first detected by Penzias and Wilson in 1965 [11, 12] and mapped with increasing precision by the COBE [13], WMAP [14], and Planck [15] satellites. The temperature fluctuations imprinted on the CMB are a direct snapshot of the acoustic pattern at recombination: the peaks and troughs in the CMB power spectrum correspond to modes that were caught at maximum compression or maximum rarefaction when the oscillations froze [6]. COBE established that these fluctuations exist at the level of one part in 10^5 [13, 16], and Planck mapped them with sufficient precision to constrain cosmological parameters to sub-percent accuracy [15]. The relative heights of the acoustic peaks encode fundamental quantities such as the ratio of baryonic to dark matter [6], since the depth of the dark-matter potential wells modulates the balance between successive compressions and rarefactions.

The excess of pairs at the sound-horizon scale manifests in the galaxy distribution as a bump in the two-point correlation function, or equivalently as a series of oscillations in the power spectrum [2, 17]. Because the physical size of this scale is determined by well-understood early-Universe physics, measuring its apparent angular or radial extent in a galaxy survey directly constrains the comoving distance to the observed galaxies and, through it, the expansion history of the Universe. This is why the BAO feature is often described as one of the cleanest probes in cosmology: it relies on a single preferred scale set before the epoch of galaxy formation, and its measurement requires no assumption about the intrinsic properties of galaxies beyond their spatial distribution.

These properties made the BAO feature a primary target for large galaxy surveys, and in 2005 the signal was detected for the first time in the galaxy distribution, simultaneously in the two-point correlation function of luminous red galaxies from the Sloan Digital Sky Survey [18] and in the power spectrum of the 2dF Galaxy Redshift Survey [19]. These detections confirmed a key prediction of the standard cosmological model and opened a new chapter in precision cosmology. In the two decades since, the BAO feature has been measured across a wide range of redshifts and tracer populations, including galaxies, quasars, and the Lyman- α forest, by surveys such as BOSS [20], eBOSS [21], and most recently DESI [22]. Combined with CMB temperature fluctuations and Type Ia super-

novae distances, these measurements have been crucial in establishing the concordance Λ CDM model and in constraining departures from it, including dynamical dark energy and modifications of general relativity [23–30].

Despite this progress, most BAO analyses rely on three-dimensional clustering statistics, using spectroscopic redshifts to reconstruct the full spatial distribution of galaxies. A complementary route uses only the projected positions of galaxies on the sky, through two-point statistics measured in tomographic redshift shells. Such projected analyses can be carried out either in configuration space, through the angular correlation function $\omega(\theta)$, or in harmonic space, through the angular power spectrum C_ℓ ; the latter is adopted in this work. Because the quantity it constrains (the angular position of the BAO feature) is a direct observable rather than a derived distance, this approach depends only weakly on a fiducial cosmology, avoiding the redshift-to-distance conversion required in three-dimensional analyses.

Furthermore, traditionally, BAO analyses have favored luminous red galaxies as tracers, owing to their high luminosity and strong clustering signal. Restricting the analysis to a single population, however, leaves cosmological information unexplored. Blue, star-forming galaxies are the most abundant population at low redshifts and, by virtue of their smaller linear bias, trace the underlying dark-matter distribution more closely than their red counterparts. Measuring the BAO feature in blue galaxy samples, therefore, not only tests whether the acoustic scale can be recovered from a tracer population with distinct astrophysical properties and systematics, but also contributes to extracting the maximum cosmological information from existing survey data. Combining this with a purely angular analysis yields a methodology that is weakly dependent on a cosmological model and has seen comparatively few applications at BAO scales.

Within this context, this dissertation presents an angular BAO analysis using a sample of spectroscopic data from star-forming blue galaxies extracted from the twelfth data release of the SDSS. The central goal is to validate a complete analysis pipeline, from covariance estimation and angular power spectrum measurement through template fitting and cosmological inference, in a realistic setting with a non-standard tracer population, and to assess the viability of blue galaxies as BAO tracers at low redshift.

This dissertation is organized as follows. Chapter 2 reviews the theoretical foundations of standard cosmology and structure formation, from the Friedmann-Lemaître-Robertson-Walker framework and linear perturbation theory to the origin of acoustic oscillations and the matter power spectrum. Chapter 3 presents the statistical tools used to characterize the galaxy distribution: the two-point correlation function, the angular power spectrum, and the pseudo- C_ℓ estimator. Chapter 4 describes the observational data and the construction of the blue galaxy sample from the Sloan Digital Sky Survey. Chapter 5 details the methodology: sample characterization, angular power spectrum measurement, mock catalog generation, covariance estimation, and model fitting for parameter infer-

ence. Chapter 6 presents the main results, including the measured angular power spectra and the BAO model fits. Finally, Chapter 7 summarizes the conclusions and outlines perspectives for future work, with supplementary material and robustness tests provided in the appendices.

Chapter 2

Standard Cosmology and Structure Formation

In the standard cosmological model, the dynamics of the Universe is well described by the theory of General Relativity (GR) [31], formulated by Albert Einstein at the beginning of the 20th century. On sufficiently large scales, the Universe shows good concordance with the hypotheses of spatial homogeneity and isotropy [2, 32]. These two hypotheses, known as the Cosmological Principle, help to simplify Einstein equations to suitably describe the expansion history of the Universe. The initial conditions of the Universe evolution are provided by the inflationary paradigm, which generates the primordial density fluctuations that later seed the formation of cosmic structures through the gravitational growth of perturbations [4, 5, 33, 34]. Together, these three ingredients, namely the theory of General Relativity, the Cosmological Principle and the inflationary paradigm, establish the theoretical framework underlying the Universe dynamics and the origin and evolution of its large-scale structure.

This chapter presents the theoretical framework necessary for understanding the basis of modern cosmology, which serves as the foundation for the analysis developed in this master's dissertation. It is organized as follows. In Section 2.1, a brief review of the main aspects of General Relativity relevant to cosmology is presented. Section 2.2 introduces the Friedmann-Lemaître-Robertson-Walker (FLRW) metric, while Section 2.3 derives the Friedmann equations from Einstein's field equations and discusses the dynamical evolution of the homogeneous and isotropic background spacetime. Distance measurements in an expanding Universe are then discussed in Section 2.4. Finally, Section 2.5 provides a brief introduction to linear perturbation theory, which forms the basis for the study of large-scale structure formation and baryon acoustic oscillations developed in the subsequent chapters.

2.1 General Relativity and Cosmology

General Relativity provides the theoretical framework for describing gravity as a geometric property of spacetime. In this theory, the presence of matter and energy determines the curvature of spacetime, which, in turn, governs the motion of particles and light [32, 35, 36]. The formulation of GR is based on two fundamental principles: the equivalence principle and the general covariance principle.

The **equivalence principle** states that, locally, the effects of gravity are indistinguishable from those of acceleration. More precisely, in a sufficiently small region of spacetime, it is always possible to choose a reference frame in which the laws of physics reduce to those of special relativity [35]. This implies that freely falling particles follow inertial trajectories, which are generalized in curved spacetime as *geodesics* [36].

The second fundamental principle is the **principle of general covariance**, which states that the laws of physics must assume the same form in all coordinate systems. This requirement implies that physical equations must be expressed in a tensorial form [37], ensuring invariance under arbitrary coordinate transformations. As a consequence, the spacetime is described by the metric tensor $g_{\mu\nu}$, which defines the invariant line element [38]

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \quad (2.1)$$

where ds^2 is the spacetime interval between two infinitesimally separated events, with dx^μ and dx^ν the infinitesimal coordinate shifts in spacetime.

The metric tensor $g_{\mu\nu}$ and its inverse $g^{\mu\nu}$, defined such that $g^{\mu\alpha}g_{\alpha\nu} = \delta^\mu_\nu$, are used to lower and raise tensor indices, for example $V_\mu = g_{\mu\nu}V^\nu$ and $V^\mu = g^{\mu\nu}V_\nu$, and thereby relate the covariant and contravariant components of any tensor. The indices $\mu, \nu = 0, 1, 2, 3$ label the spacetime coordinates, conventionally written as $x^\mu = (t, x, y, z)$ [35, 38], where t is the time coordinate and (x, y, z) are the spatial coordinates. The sign of the interval determines the causal structure of spacetime. Events connected by trajectories with $ds^2 < 0$ are *timelike* separated, while those with $ds^2 > 0$ are *spacelike* separated [35, 36]. The boundary between these regions is defined by null trajectories satisfying

$$ds^2 = 0, \quad (2.2)$$

which corresponds to the propagation of massless particles, such as photons [35, 36]. These null line elements define the *light cone* associated with each spacetime event, separating causally connected regions from those that cannot be linked by signals traveling at or below the speed of light.

The field equations can be derived from the variational principle, in which the dynamics of the gravitational field arise from an action S . The requirement that S be a

generally covariant scalar constructed from the metric $g_{\mu\nu}$ and its curvature, as required by the equivalence and general covariance principles, highlights the Einstein-Hilbert action [35],

$$S = \int d^n x \sqrt{-g} (R - 2\Lambda), \quad (2.3)$$

as the simplest choice, where g is the determinant of the metric tensor, R is the Ricci scalar, Λ is the cosmological constant, and n is the number of spacetime dimensions. Its variation with respect to the metric, through the least action principle, $\delta S = 0$, yields the Einstein field equations, which relate the geometry of spacetime to its matter-energy content,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \quad (2.4)$$

where $G_{\mu\nu}$ is the Einstein tensor, defined as

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R. \quad (2.5)$$

Here, $R_{\mu\nu}$ is the Ricci tensor, obtained by contracting the Riemann curvature tensor,

$$R_{\mu\nu} = R^\lambda_{\mu\lambda\nu}, \quad (2.6)$$

and it can be expressed explicitly in terms of the Christoffel symbols $\Gamma^\lambda_{\mu\nu}$ as

$$R_{\mu\nu} = \partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial_\nu \Gamma^\lambda_{\mu\lambda} + \Gamma^\lambda_{\lambda\rho} \Gamma^\rho_{\mu\nu} - \Gamma^\lambda_{\nu\rho} \Gamma^\rho_{\mu\lambda}, \quad (2.7)$$

with

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\alpha} (\partial_\mu g_{\alpha\nu} + \partial_\nu g_{\alpha\mu} - \partial_\alpha g_{\mu\nu}) \quad (2.8)$$

determining how vectors are parallel transported in curved spacetime [35]. The Ricci scalar R is obtained by contracting the Ricci tensor with the metric,

$$R = g^{\mu\nu} R_{\mu\nu}, \quad (2.9)$$

and provides a scalar measure of spacetime curvature.

On the right-hand side of Einstein's equations, the stress-energy tensor $T_{\mu\nu}$ encodes the distribution and flow of momentum and energy, as well as the pressure of the fluid and the shear stress [35, 36, 39] (Figure 2.1); its form depends on the physical system

under consideration.

The diagram shows the stress-energy tensor $T_{\mu\nu}$ as a 4x4 matrix. The components are color-coded and labeled with arrows indicating their physical interpretations:

- Energy density:** T_{00} (blue box, blue arrow pointing up).
- Energy flux:** T_{0i} (green boxes, green arrow pointing up).
- Momentum density:** T_{i0} (purple boxes, purple arrow pointing down).
- Momentum flux:** T_{ij} (red boxes, red arrow pointing down).
- Pressure:** Diagonal terms T_{11}, T_{22}, T_{33} (orange arrow pointing down).
- Shear stress:** Off-diagonal terms T_{12}, T_{13}, T_{23} (red arrow pointing right).

Figure 2.1: Components of the stress-energy tensor $T_{\mu\nu}$ and their physical interpretation. The time-time component T_{00} gives the energy density; the time-space components T_{0i} the energy flux and the space-time components T_{i0} the momentum density (equal for a symmetric tensor). The space-space components T_{ij} gives the momentum flux, whose diagonal terms (T_{11}, T_{22}, T_{33}) represent the pressure and whose off-diagonal terms the shear (or anisotropic) stresses. Source: Prepared by the author (2026).

From a large-scale perspective, galaxies can be treated as effective constituents of a continuous medium that fills the Universe. Although galaxies possess internal structure and cluster into groups and clusters on small scales, these features can be neglected when describing the Universe on sufficiently large scales [39]. Within this approximation, the matter content is modeled as a perfect fluid, whose stress-energy tensor is

$$T_{\mu\nu} = \left(\rho + \frac{p}{c^2} \right) u_\mu u_\nu + p g_{\mu\nu}, \quad (2.10)$$

where ρ is the mass-energy density measured in the local rest frame, p the isotropic pressure, and u^μ the fluid four-velocity, representing the average motion of the matter distribution [37?]. Raising the indices with the metric, Eq. 2.10 can equivalently be written in contravariant form as

$$T^{\mu\nu} = \left(\rho + \frac{p}{c^2} \right) u^\mu u^\nu + p g^{\mu\nu}. \quad (2.11)$$

In both expressions, the energy density and pressure are written as a sum over individual components,

$$\rho(t) = \sum_i \rho_i(t), \quad p(t) = \sum_i p_i(t), \quad (2.12)$$

with the index i running over the different constituents of the Universe. In the following sections, these concepts will be applied within a cosmological context in order to describe the large-scale geometry and dynamical evolution of the Universe.

2.2 The Friedmann-Lemaître-Robertson-Walker Framework

In this section, we introduce the geometric framework that forms the basis of modern cosmology. We first discuss the metric tensor and its role in describing distances and time intervals in curved spacetime. We then present the symmetry assumptions of homogeneity and isotropy. Next, we examine the possible spatial geometries associated with different curvature regimes. Finally, by combining these geometrical and symmetry considerations, we present the Friedmann-Lemaître-Robertson-Walker (FLRW) metric, which describes an expanding homogeneous and isotropic Universe.

2.2.1 The Metric Tensor

As we saw in Section 2.1, the metric tensor (Eq. 2.1) is the fundamental geometric object that characterizes the structure of spacetime, from which the infinitesimal interval between neighbouring events is obtained [2]. To develop intuition, it is helpful to first consider a three-dimensional Euclidean space (Figure 2.2), where the spatial line element takes the familiar form

$$d\ell^2 = dx^2 + dy^2 + dz^2 = \delta_{ij} dx^i dx^j, \quad (2.13)$$

where δ_{ij} is the Kronecker delta, with $i, j = 1, 2, 3$. Because the cosmological setting is naturally described in terms of radial distances and angular positions on the sky, it is convenient to work in spherical coordinates. The line element in such a system follows from the transformation of the metric under a general change of coordinates $x^i \rightarrow q^\alpha(x^i)$. As a rank-2 tensor, it transforms according to [35]

$$g_{\alpha\beta}(q) = \frac{\partial x^i}{\partial q^\alpha} \frac{\partial x^j}{\partial q^\beta} g_{ij}, \quad (2.14)$$

where α, β label the new coordinate system and $g_{ij} = \delta_{ij}$ for Cartesian coordinates. To carry out this transformation explicitly for spherical coordinates $q^\alpha = (r, \theta, \phi)$, it is convenient to introduce the Jacobian matrix

$$J^i_\alpha = \frac{\partial x^i}{\partial q^\alpha}, \quad (2.15)$$

so that Eq. 2.14 can be written compactly as

$$g_{\alpha\beta} = g_{ij} J^i_\alpha J^j_\beta. \quad (2.16)$$

For the transformation from Cartesian coordinates (x, y, z) to spherical coordinates (r, θ, ϕ) , the Jacobian reads

$$J^i_{\alpha} = \begin{pmatrix} \sin \theta \cos \phi & r \cos \theta \cos \phi & -r \sin \theta \sin \phi \\ \sin \theta \sin \phi & r \cos \theta \sin \phi & r \sin \theta \cos \phi \\ \cos \theta & -r \sin \theta & 0 \end{pmatrix}, \quad (2.17)$$

where the columns of J correspond to the coordinate basis vectors associated with (r, θ, ϕ) . Computing the diagonal components ($\alpha = \beta$) gives

$$\begin{aligned} g_{rr} &= \sin^2 \theta \cos^2 \phi + \sin^2 \theta \sin^2 \phi + \cos^2 \theta = 1, \\ g_{\theta\theta} &= r^2 \cos^2 \theta (\cos^2 \phi + \sin^2 \phi) + r^2 \sin^2 \theta = r^2, \\ g_{\phi\phi} &= r^2 \sin^2 \theta (\sin^2 \phi + \cos^2 \phi) = r^2 \sin^2 \theta, \end{aligned}$$

while the off-diagonal components ($\alpha \neq \beta$) vanish. The spatial metric in spherical coordinates therefore takes the form

$$g_{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \sin^2 \theta \end{pmatrix},$$

and the corresponding line element is

$$d\ell^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2, \quad (2.18)$$

with the angular part conveniently expressed through the solid-angle element,

$$d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2. \quad (2.19)$$

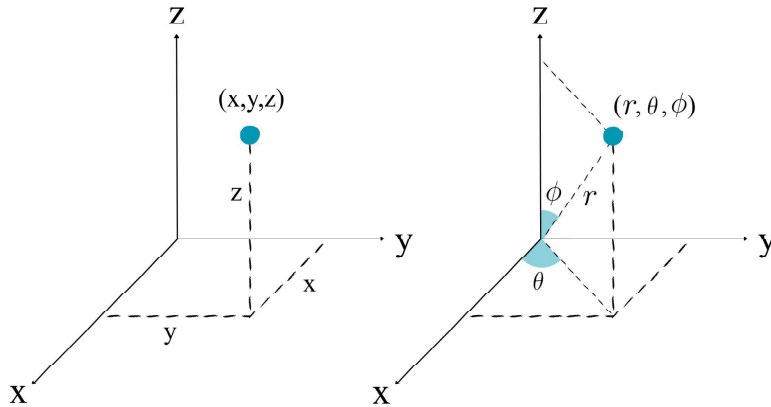


Figure 2.2: Euclidian (left) and spherical (right) coordinate systems. Source: Prepared by the author (2026).

Having derived the spatial line element, this result is now extended to a description of spacetime itself, where spatial and temporal intervals are treated within a unified frame-

work. In special relativity, the spacetime is the empty, static, flat Minkowski space [2, 37], whose line element is

$$ds^2 = -c^2 dt^2 + d\ell^2 = \eta_{\mu\nu} dx^\mu dx^\nu. \quad (2.20)$$

where dt is the differential of the time coordinate and c is the speed of light. The tensor $\eta_{\mu\nu}$ is the Minkowski metric, given by $\text{diag}(-c^2, 1, 1, 1)$ in Cartesian coordinates.

However, observations of extragalactic objects indicate that the separation between distant sources changes with time [40, 41]. This implies that spatial distances are not static, but evolve along the cosmic history, suggesting that the spacetime geometry must be dynamical. This requires generalizing the spatial line element obtained above (Eq. 2.18) to include an evolving rescaling of spatial distances. Thus, a time-dependent scaling factor $a(t)$ is introduced into the spatial part of the metric (Eq. 2.18), together with the temporal component, such that the line element can be written as

$$ds^2 = -c^2 dt^2 + a^2(t) d\ell^2. \quad (2.21)$$

The form of this rescaling, however, is not arbitrary, indeed, it is strongly constrained by symmetry considerations.

2.2.2 Symmetry assumptions: Homogeneity and Isotropy

The large-scale distribution of matter observed in the Universe suggests that, when averaged over sufficiently large distances and angles on the sky, no spatial position or direction is preferred [42–56]. This assumption, known as the Cosmological Principle, constitutes the fundamental symmetry hypothesis underlying modern cosmology. By construction, homogeneity and isotropy relate to the distribution of matter at a fixed instant and, therefore, are properties of the spatial sections of geometry [35].

In General Relativity, these symmetries are characterized through the isometries of the metric tensor. An isometry is a transformation that preserves the spacetime interval and therefore leaves the metric invariant [36]. Spatial **homogeneity** requires that every point on a spatial hypersurface be geometrically equivalent [39]. More precisely, given any two points $p, q \in \Sigma_t$, where Σ_t denotes a hypersurface of constant cosmic time t , there exists an isometry that maps p into q [35, 36]. This condition ensures that spatial geometry does not allow for preferred locations.

Spatial **isotropy** requires that the Universe appear identical in every spatial direction [35, 36, 39]. Let u^a denote the four-velocity field of these observers, defining a local rest frame at each spacetime point. Any vector orthogonal to u^a therefore lies entirely within the spatial hypersurface associated with that observer [36]. More precisely, consider two unit spatial vectors s_1^a and s_2^a satisfying

$$u_a s_1^a = 0, \quad u_a s_2^a = 0, \quad (2.22)$$

so that both vectors belong to the spatial subspace orthogonal to u^a [36]. Isotropy implies that there exists an isometry of the spacetime metric that leaves the spacetime point p and the four-velocity u^a fixed while rotating s_1^a into s_2^a [36]. Consequently, it is not possible to define preferred spatial directions. Figure 2.3 shows a representation of both isotropy and homogeneity concepts.

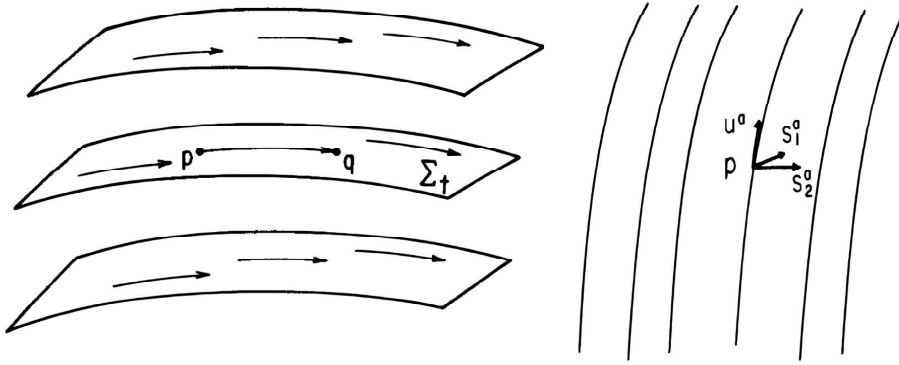


Figure 2.3: Schematic representation of the symmetry assumptions underlying the cosmological principle. Left: spatial homogeneity. Right: spatial isotropy. Source: [36].

More generally, cosmological spacetimes can be classified according to their symmetry properties, ranging from completely non-homogeneous solutions, such as Lemaître-Tolman-Bondi (LTB) models [57–59], to spatially homogeneous but anisotropic geometries, such as the Bianchi class [60]. In this context, the homogeneity and isotropy hypotheses select a particularly restricted subclass of solutions in which the spatial hypersurfaces are maximally symmetric, corresponding to the FLRW geometry [2, 35, 36].

2.2.3 Spatial Geometry and Three-Dimensional Curvature

In the last two sections, the temporal dependence of the metric was determined, and the scaling factor $a(t)$ was introduced to account for the universe expansion. But what geometries could a homogeneous and isotropic spacetime admit? The symmetry hypotheses discussed in the last section restrict spatial sections to maximally symmetric spaces. These spaces can be represented as three-dimensional hypersurfaces embedded in $\mathbb{R}^4$ [36], satisfying a constraint determined by the curvature. A two-dimensional representation of the three possible geometries can be seen in Figure 2.4.

Flat space: in the case of zero curvature, the spatial geometry reduces to Euclidean space. The spatial line element is given by Eq. 2.18.

Positive curvature: this geometry corresponds to spatial hypersurfaces described by spherical geometry [2, 35],

$$x^2 + y^2 + z^2 + w^2 = R_0^2,$$

where R_0 denotes the radius of curvature of the spatial sections, and

$$w = \sqrt{R_0^2 - r^2}, \quad dw = -\frac{r}{\sqrt{R_0^2 - r^2}} dr.$$

Using

$$d\ell^2 = dx^2 + dy^2 + dz^2 + dw^2,$$

we obtain

$$d\ell^2 = \frac{dr^2}{1 - \frac{r^2}{R_0^2}} + r^2 d\Omega^2. \quad (2.23)$$

Negative curvature: corresponds to spatial hypersurfaces described by hyperbolic geometry [2, 35],

$$x^2 + y^2 + z^2 - w^2 = -R_0^2,$$

with

$$w = \sqrt{r^2 + R_0^2}, \quad dw = \frac{r}{\sqrt{r^2 + R_0^2}} dr.$$

In this case, the line element is

$$d\ell^2 = dx^2 + dy^2 + dz^2 - dw^2,$$

which leads to

$$d\ell^2 = \frac{dr^2}{1 + \frac{r^2}{R_0^2}} + r^2 d\Omega^2. \quad (2.24)$$

These three cases (flat, positive, and negative curvatures) can be written in a unified form by introducing the normalized curvature parameter k , defined as [2]

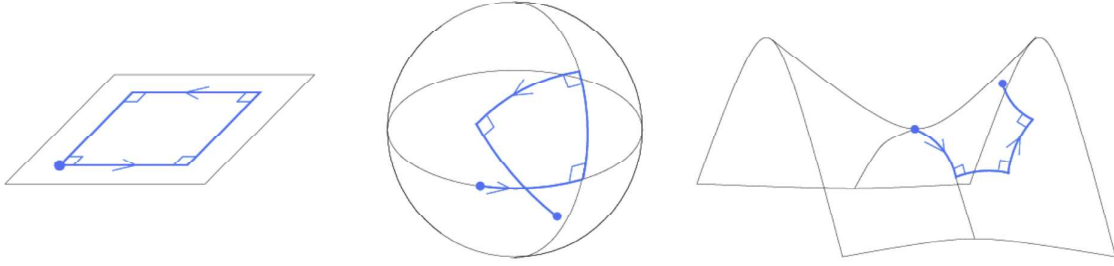


Figure 2.4: Two-dimensional spaces with constant curvature: flat (Euclidean), positively curved (spherical), and negatively curved (hyperbolic geometries), respectively. The figure illustrates how closed paths behave differently in each geometry, with geodesic squares not returning to the same coordinates in curved spaces. Adapted from [61].

$$k \equiv \begin{cases} +1, & \text{positive curvature,} \\ 0, & \text{zero curvature,} \\ -1, & \text{negative curvature.} \end{cases} \quad (2.25)$$

The spatial line element can then be written compactly as

$$d\ell^2 = \frac{dr^2}{1 - \frac{kr^2}{R_0^2}} + r^2 d\Omega^2. \quad (2.26)$$

This compact formulation takes into account all three possible constant curvature geometries for space hypersurfaces in a homogeneous and isotropic frame.

2.2.4 Derivation of the FLRW Metric

Given the homogeneity and isotropy symmetry hypotheses, we can now obtain the most general cosmological metric. Substituting the result from Eq. 2.26 into the general spacetime line element in Eq. 2.21, we obtain the Friedmann-Lemaître-Robertson-Walker (FLRW) metric [2, 35],

$$ds^2 = -c^2 dt^2 + a^2(t) \left[\frac{dr^2}{1 - k \frac{r^2}{R_0^2}} + r^2 d\Omega^2 \right]. \quad (2.27)$$

This metric was first obtained by Alexander A. Friedmann in 1922 [62, 63], with Georges H. J. E. Lemaître independently deriving similar solutions and providing their physical interpretation in terms of an expanding Universe, in 1927¹ [64]. Later (1935-1937), it was derived from the assumptions of spatial homogeneity and isotropy by Howard P. Robertson and Arthur G. Walker [65–67].

In these coordinates, the metric tensor takes the form

¹The original paper was translated to english only in 1931.

$$g_{\mu\nu} = \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & \frac{a^2(t)}{1 - kr^2/R_0^2} & 0 & 0 \\ 0 & 0 & a^2(t)r^2 & 0 \\ 0 & 0 & 0 & a^2(t)r^2 \sin^2 \theta \end{pmatrix}. \quad (2.28)$$

We can also rewrite Eq. 2.27 as a function of the comoving distance χ [2],

$$d\chi^2 = \frac{dr^2}{1 - kr^2/R_0^2} \Rightarrow d\chi = \frac{dr}{\sqrt{1 - kr^2/R_0^2}}. \quad (2.29)$$

Integrating this expression, we obtain the relation between the radial coordinate r and the comoving coordinate χ (described later in Section 2.4). The result depends on the value of the curvature parameter k . For a universe with Euclidean geometry (flat curvature: $k = 0$), the relation is

$$d\chi = dr \quad \Rightarrow \quad \chi = r, \quad (2.30)$$

while for a universe with spherical geometry (positive curvature: $k = +1$),

$$\chi = R_0 \arcsin\left(\frac{r}{R_0}\right), \quad r = R_0 \sin\left(\frac{\chi}{R_0}\right), \quad (2.31)$$

and for a universe with hyperbolic geometry (negative curvature: $k = -1$),

$$\chi = R_0 \operatorname{arcsinh}\left(\frac{r}{R_0}\right), \quad r = R_0 \sinh\left(\frac{\chi}{R_0}\right). \quad (2.32)$$

These three cases can be written in a unified form as [2]

$$r = S_k(\chi),$$

where

$$S_k(\chi) = \begin{cases} R_0 \sin\left(\frac{\chi}{R_0}\right), & \text{if } k = +1, \\ \chi, & \text{if } k = 0, \\ R_0 \sinh\left(\frac{\chi}{R_0}\right), & \text{if } k = -1. \end{cases} \quad (2.33)$$

Substituting these relations into the spatial line element, the metric can be expressed in terms of the comoving coordinate,

$$ds^2 = -c^2 dt^2 + a^2(t) [d\chi^2 + S_k^2(\chi) d\Omega^2]. \quad (2.34)$$

Here, χ remains fixed for comoving observers, while the physical distance between them scales with the scale factor $a(t)$. By convention, the present-day value of the scale factor

is chosen to be unity, i.e.,

$$a(t_0) \equiv 1. \quad (2.35)$$

Therefore, the FLRW metric provides the geometric basis for describing an expanding Universe and serves as a starting point for the dynamical analysis developed in the following section.

2.3 Background Cosmological Dynamics

The previous section established the geometrical framework of cosmology through the FLRW metric. We now proceed to determine the dynamical evolution of the Universe.

2.3.1 Friedmann Equations

The dynamical equations governing the expansion of the Universe follow from solving Einstein's field equations 2.4 for the FLRW metric. The procedure is simple, but can be lengthy: starting from the metric, one computes the Christoffel symbols (Eq. 2.8), contracts them into the Ricci tensor (Eq. 2.7) and the Ricci scalar, and finally assembles the Einstein tensor (Eq. 2.5). Due to homogeneity and isotropy, only the diagonal components survive.

To close the system, the geometric side is equated to the matter content, described by the perfect-fluid stress-energy tensor (Eq. 2.10). The complete development of the equations can be found in the Appendix A.1. Substituting both sides into Eq. 2.4, the time-time component ($\mu = \nu = 0$) yields

$$3 \left(\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2 R_0^2} \right) - \Lambda c^2 = 8\pi G \rho,$$

which can be rearranged into the form

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{kc^2}{a^2 R_0^2} + \frac{\Lambda c^2}{3}. \quad (2.36)$$

This relation is known as the **first Friedmann equation**, and it determines how the expansion rate depends on the energy density, spatial curvature, and cosmological constant. The spatial components of Einstein's equations, while distinct, give the same evolution equation, which together with the first Friedmann equation leads to

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left(\rho + \frac{3p}{c^2} \right) + \frac{\Lambda c^2}{3}, \quad (2.37)$$

known as the **second Friedmann equation**, or acceleration equation. This expression shows explicitly how both the energy density and the pressure contribute to the acceleration (or deceleration) of the expansion. Under the assumption that the Universe is filled

with a perfect fluid, as introduced in Section 2.1, the evolution is governed entirely by the energy density ρ and pressure p of its components. Together, both Friedmann equations fully determine the background evolution of the Universe once the matter content is specified.

Specifying the matter content, in turn, requires knowing how the energy density evolves as the Universe expands. This relation is provided by the continuity equation, which follows most directly from the conservation of energy and momentum, expressed by the vanishing covariant divergence of the stress-energy tensor [35, 38],

$$\nabla_\mu T^{\mu\nu} = 0. \quad (2.38)$$

Evaluating its time component ($\nu = 0$) with the FLRW connection coefficients gives the **continuity equation** (the full derivation is also presented in Appendix A.1),

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0, \quad (2.39)$$

which describes how energy density evolves with the expansion of the universe. This same relation can also be obtained by differentiating the first Friedmann equation (Eq. 2.36) with respect to time and combining it with the acceleration equation (Eq. 2.37).

Furthermore, it is convenient to express the total energy density as a sum of fluid components (Eq. 2.12), each parameterized using an equation of state of the form [3, 35]

$$p = w\rho c^2, \quad (2.40)$$

where w is a constant. In this case, the continuity equation can be integrated to give

$$\frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a}. \quad (2.41)$$

Integrating both sides yields

$$\rho = \rho_0 a^{-3(1+w)}, \quad (2.42)$$

showing explicitly how different components dilute with expansion [2]. For example, non-relativistic matter ($w = 0$) scales as a^{-3} , while radiation ($w = 1/3$) scales as a^{-4} [35]. For a cosmological constant ($w = -1$), the energy density remains constant, $\rho_\Lambda = \text{const}$ [2]. These different scaling behaviors determine the relative evolution of each component of the universe and lead to distinct epochs of dominance. The resulting evolution of energy density is shown in Figure 2.5.

These results provide the basis for understanding the dynamical evolution of the Universe and will be used in the following sections to determine the temporal dependence of the scale factor in different cosmological regimes.

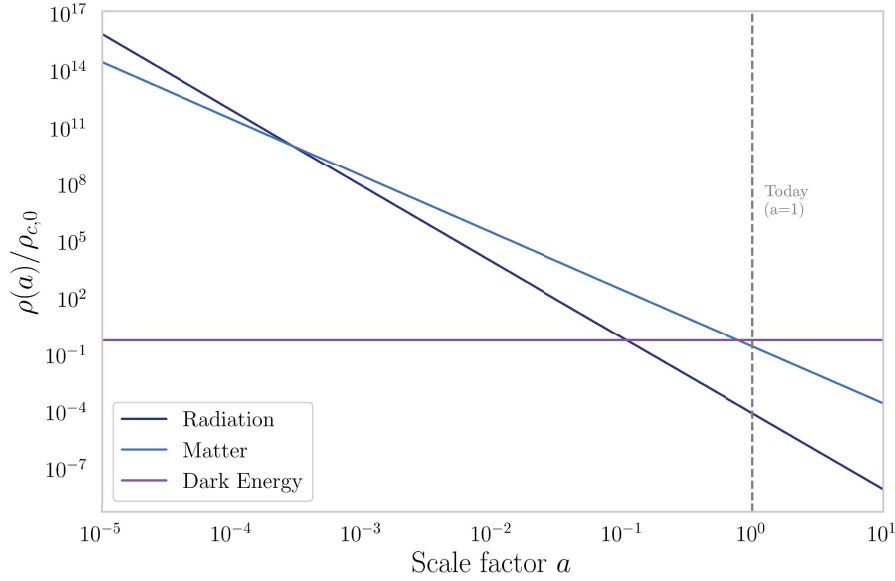


Figure 2.5: Evolution of the normalized energy density components of the Universe as a function of scale factor, showing radiation domination at early times, followed by a matter-dominated epoch, and the recent transition to dark-energy domination. Source: Prepared by the author (2026).

2.3.2 Hubble Expansion and Density Parameters

It is often convenient to express the Friedmann equation in terms of dimensionless quantities that characterize the relative contribution of each component to the total energy density. A central role is played by the **Hubble parameter**, defined as [2, 3, 35]

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)}, \quad (2.43)$$

which measures the relative expansion rate of the Universe at a given time. Its present-day value,

$$H_0 \equiv H(t_0), \quad (2.44)$$

is known as the **Hubble constant**. Associated with the expansion rate is a characteristic length scale, the **Hubble radius** [2],

$$R_H(t) \equiv \frac{c}{H(t)}, \quad (2.45)$$

the distance light travels in a Hubble time H^{-1} , which sets the scale over which physical processes can be causally connected at a given epoch.

In terms of $H(t)$, the first Friedmann equation can be written as

$$H^2(t) = \frac{8\pi G}{3}\rho(t) - \frac{kc^2}{a(t)^2 R_0^2} + \frac{\Lambda c^2}{3}. \quad (2.46)$$

A particularly important quantity is the **critical density** [35], defined as the energy density required for the universe to be spatially flat,

$$\rho_c(t) \equiv \frac{3H^2(t)}{8\pi G}. \quad (2.47)$$

Using this definition, one can introduce the dimensionless **density parameters** [35],

$$\Omega_i(t) \equiv \frac{\rho_i(t)}{\rho_c(t)}, \quad (2.48)$$

which quantify the fractional contribution of each component to the total energy density ρ . Similarly, one defines the curvature and cosmological constant contributions as [2, 35]

$$\Omega_k(t) \equiv -\frac{kc^2}{a(t)^2 H(t)^2 R_0^2}, \quad \Omega_\Lambda(t) \equiv \frac{\Lambda c^2}{3H(t)^2}. \quad (2.49)$$

Dividing Eq. 2.46 by $H^2(t)$, and substituting the above definitions, one obtains the constraint equation

$$\Omega_m(t) + \Omega_r(t) + \Omega_k(t) + \Omega_\Lambda(t) = 1. \quad (2.50)$$

where Ω_m and Ω_r are the matter and radiation density parameters, defined using Eq. 2.48. Evaluating Eq. 2.46 at the present time t_0 (with $a(t_0) = 1$) defines the critical density today,

$$\rho_{c,0} \equiv \frac{3H_0^2}{8\pi G}, \quad (2.51)$$

so we can define the present-day density parameter of each component, analogously to Eq. 2.48, as its density relative to this critical density,

$$\Omega_{i,0} \equiv \frac{\rho_{i,0}}{\rho_{c,0}}, \quad (2.52)$$

with $\rho_{i,0} \equiv \rho_i(t_0)$ the present-day density of component i , and the analogous curvature and cosmological-constant parameters

$$\Omega_{k,0} \equiv -\frac{kc^2}{a_0^2 H_0^2}, \quad \Omega_{\Lambda,0} \equiv \frac{\Lambda c^2}{3H_0^2}.$$

Now, considering how different components evolve with the scale factor (Eq. 2.42), the Eq. 2.46 can be rewritten as

$$H^2 = \frac{8\pi G}{3} \sum_i \rho_{i,0} a^{-3(1+w_i)} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}.$$

Dividing it by H_0^2 , and using Eqs. 2.51 and 2.52, we finally obtain

$$\frac{H^2}{H_0^2} = \sum_i \Omega_{i,0} a^{-3(1+w_i)} + \Omega_{k,0} a^{-2} + \Omega_{\Lambda,0}. \quad (2.53)$$

Accounting for radiation, matter, spatial curvature and the cosmological constant, this expression reduces to

$$\frac{H^2}{H_0^2} = \Omega_{r,0} a^{-4} + \Omega_{m,0} a^{-3} + \Omega_{k,0} a^{-2} + \Omega_{\Lambda,0}. \quad (2.54)$$

This form of the Friedmann equation explicitly shows how the expansion rate depends on the relative contributions of radiation, matter, curvature, and dark energy. Each component exhibits a distinct scaling with the scale factor, as shown in Figure 2.6, reflecting its underlying equation of state.

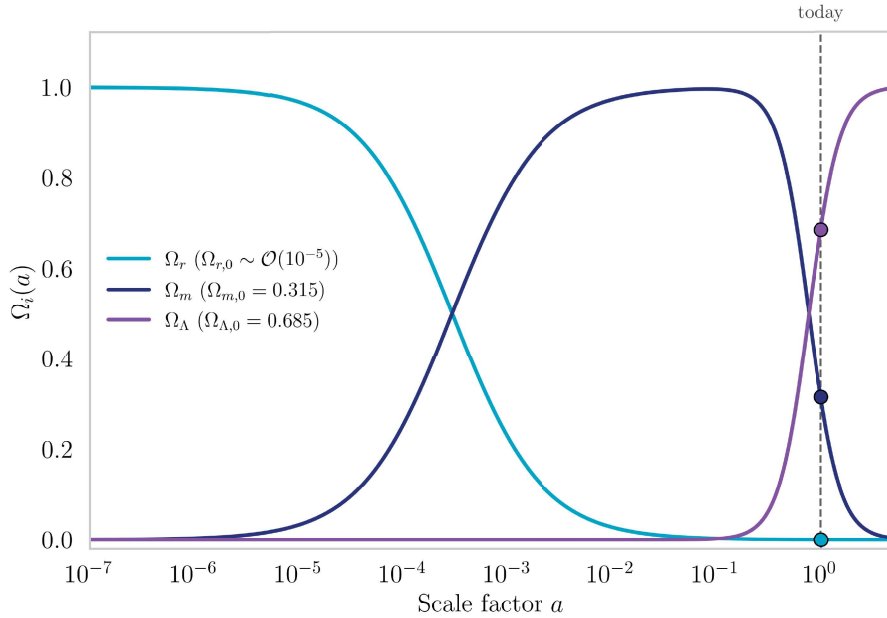


Figure 2.6: Evolution of the fractional density parameters $\Omega_i(a)$ as a function of scale factor, showing the relative contributions of radiation, matter, and dark energy to the total energy budget of the Universe over cosmic time. The dots mark the present-day ($a = 1$) values of each component. A flat universe ($\Omega_{k,0} = 0$) is assumed. Source: Prepared by the author (2026).

Cosmological models: A cosmological model is specified by a set of theoretical assumptions about the geometry, content, and initial conditions of the Universe, along with a minimal set of free parameters that, once fixed, determine the entire history of the background expansion and the statistics of linear perturbations. The standard framework adopted throughout modern cosmology is the so-called Λ CDM model [39, 68–70]. It assumes General Relativity as the theory of gravity, a spatially flat FLRW metric, and an energy content consisting of photons, three families of light neutrinos, baryons, cold dark matter (CDM), and a cosmological constant Λ playing the role of dark energy (with equation of state $w = -1$) [2, 71].

In addition to specifying the background dynamics, the model requires a description of the origin and statistical properties of the primordial perturbations that give rise to the cosmic structures. These initial fluctuations are naturally explained within the inflationary scenario, which corresponds to a phase of accelerated expansion in the very early Universe [5, 33, 34]. Inflation was originally proposed to solve the horizon, flatness, and monopole problems of the standard hot Big Bang, but its central importance in modern cosmology lies in providing a causal mechanism for the origin of cosmic structure. Quantum fluctuations of the inflaton field are stretched beyond the Hubble horizon during inflation and subsequently frozen as classical curvature perturbations, which later evolve under gravitational instability into the observed distribution of matter and galaxies [4, 72]. Within this context, these perturbations are assumed to be adiabatic, nearly scale-invariant and Gaussian, consistent with observations of the cosmic microwave background and large-scale structure [73–80].

Under these assumptions, the model is fully specified by six free parameters: the physical baryon and cold dark matter densities ($h^2\Omega_b$ and $h^2\Omega_c$, where $\Omega_b + \Omega_c = \Omega_m$), the angular scale of the sound horizon at the last scattering (θ_*), the reionization optical depth (τ), and the amplitude and tilt of the primordial scalar power spectrum (A_s and n_s) [15, 81, 82].

The numerical values of the cosmological parameters depend on the combination of data sets used to constrain them. The most precise measurements currently come from the Cosmic Microwave Background (CMB) temperature fluctuations measured by the *Planck* satellite, which provide high-precision constraints from the temperature (TT), E -mode polarisation, temperature-polarization cross-correlation (TE , EE) power spectra, the low-multipole polarization likelihood ($lowE$), and the reconstruction of the CMB *lensing* potential. These can also be combined with external probes such as BAO from galaxy surveys, which tighten the constraints by breaking degeneracies that persist in CMB-only analyses [15].

In this work we adopt as fiducial cosmology the standard Λ CDM model with parameters from the *Planck* 2018 baseline combination (TT,TE,EE+lowE+lensing) [15]. Under this choice, the present-day density parameters are $\Omega_m = 0.3153 \pm 0.0073$ and $\Omega_\Lambda = 0.6847 \pm 0.0073$ [15], and the Hubble constant is $H_0 = (67.36 \pm 0.54) \text{ km s}^{-1} \text{ Mpc}^{-1}$ [15]. The radiation density at the present epoch is negligible, $\Omega_r \sim \mathcal{O}(10^{-5})$, and the spatial curvature is consistent with zero, $\Omega_k = 0.001 \pm 0.002$ [15], supporting the flatness assumption adopted in the baseline model.

The Λ CDM model can be extended in several ways. One is through the introduction of a dynamic component of dark energy characterized by a time-dependent equation of state $w(a)$, often parameterized as $w(a) = w_0 + w_a(1 - a)$ [83, 84]. Such behavior arises in scalar field models such as quintessence or phantom dark energy [85, 86]. A conceptually different approach is to modify the theory of gravity itself. The most studied example is

gravity $f(R)$, in which the Einstein-Hilbert action is generalized by replacing the Ricci scalar R with a more general function [87, 88]. Current observations remain consistent with $w = -1$ and with general relativity on cosmological scales, leading to the adoption of the standard Λ CDM model as the fiducial cosmology in the analysis that follows.

Hubble tension: The present-day value of H_0 is a key observable in cosmology. Current measurements, however, show a significant discrepancy, namely the Hubble tension [89], depending on the method employed. Local determinations based on the distance ladder, such as those from Type Ia supernovae calibrated with Cepheid variables, yield values around [90]

$$H_0 \simeq 73.04 \pm 1.04 \text{ km s}^{-1} \text{ Mpc}^{-1}, \quad (2.55)$$

while estimates inferred from early-Universe observations, particularly from the cosmic microwave background assuming the Λ CDM model, give lower values in the order [15]

$$H_0 \simeq 67.36 \pm 0.54 \text{ km s}^{-1} \text{ Mpc}^{-1}. \quad (2.56)$$

These two classes of measurements are summarized in Figure 2.7. This discrepancy, with 4σ to 5σ disagreement depending on the datasets considered [91], has important implications for cosmology, as it may indicate either unresolved systematics or physics beyond the standard Λ CDM model.

2.3.3 Evolution of the Scale Factor

Having derived the Friedmann equations, we now investigate the time evolution of the scale factor for different dominant components of the cosmic energy density. In particular, the cosmological constant can be interpreted as an effective fluid associated with vacuum energy [2], with energy density defined through

$$\rho_\Lambda \equiv \frac{\Lambda c^2}{8\pi G}. \quad (2.57)$$

For simplicity, we restrict our analysis to a spatially flat universe ($k = 0$), for which the first Friedmann equation reduces to

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \Rightarrow \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3}\rho}. \quad (2.58)$$

Matter-dominated Universe.

For non-relativistic matter, the energy density scales as $\rho_m = \rho_{m,0}a^{-3}$. Substituting into Eq. 2.58 gives

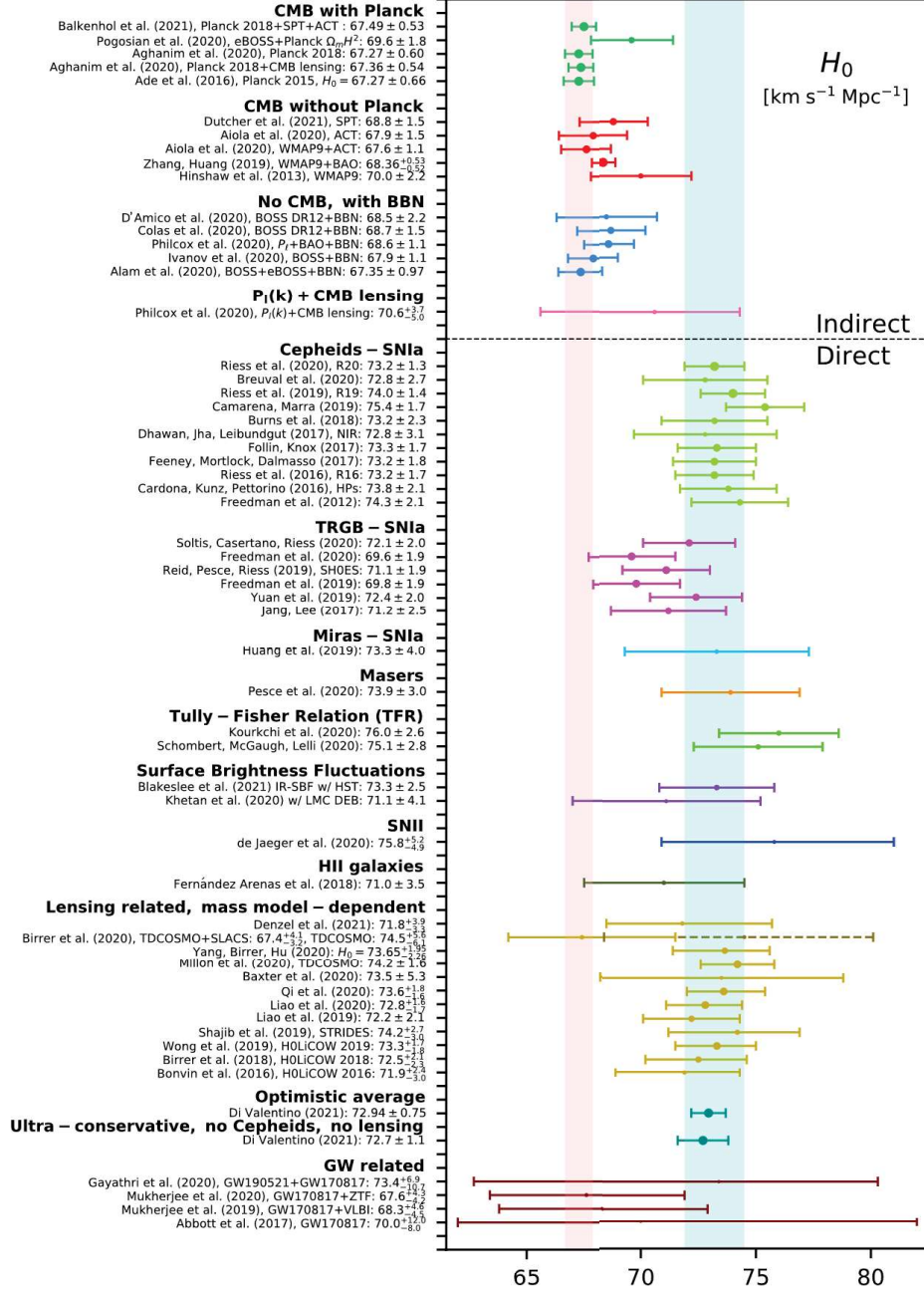


Figure 2.7: Whisker plot showing 68% confidence level constraints on the Hubble constant H_0 obtained through different direct and indirect measurements over the years. The cyan vertical band corresponds to the SH0ES measurement, while the light pink band indicates the value inferred from the Planck 2018 data within the Λ CDM model. Source: [91].

$$\frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3} \rho_{m,0} a^{-3}} \Rightarrow \dot{a} = \sqrt{\frac{8\pi G}{3} \rho_{m,0}} a^{-1/2}.$$

Separating variables and integrating, and normalizing so that $a = 1$ at a reference time t_0 , one obtains

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3}. \quad (2.59)$$

Radiation-dominated Universe.

For radiation, the energy density scales as $\rho_r = \rho_{r,0}a^{-4}$, so that

$$\frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3}\rho_{r,0}a^{-4}}. \quad (2.60)$$

Proceeding as before, with the same normalization $a(t_0) = 1$, one obtains

$$a(t) = \left(\frac{t}{t_0}\right)^{1/2}. \quad (2.61)$$

Cosmological constant-dominated Universe.

For a cosmological constant, the energy density remains constant in time, $\rho_\Lambda = \text{const.}$ Eq. 2.58 then becomes

$$\frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3}\rho_\Lambda}, \quad (2.62)$$

which is constant. Integrating directly, and again fixing $a(t_0) = 1$, we obtain

$$a(t) = \exp\left[\sqrt{\frac{8\pi G}{3}\rho_\Lambda}(t - t_0)\right]. \quad (2.63)$$

These solutions illustrate how the expansion history of the Universe depends on its dominant energy component.

2.4 Distances in Cosmology

In an expanding Universe, the notion of distance depends both on the geometry of space-time and on the observational method employed. This section will present the coordinate systems and time variables commonly used in cosmology and will define distance measures directly related to observable quantities.

2.4.1 Cosmological Coordinates and Time

The FLRW metric is naturally formulated in terms of comoving coordinates (Eq. 2.29) [2, 3], in which observers following the cosmic expansion remain in fixed spatial coordinates. The corresponding physical, or **proper distance** r [3], at a given cosmic time t is obtained by scaling the comoving separation with the scale factor (Figure 2.8),

$$\mathbf{r} = a(t) \chi. \quad (2.64)$$

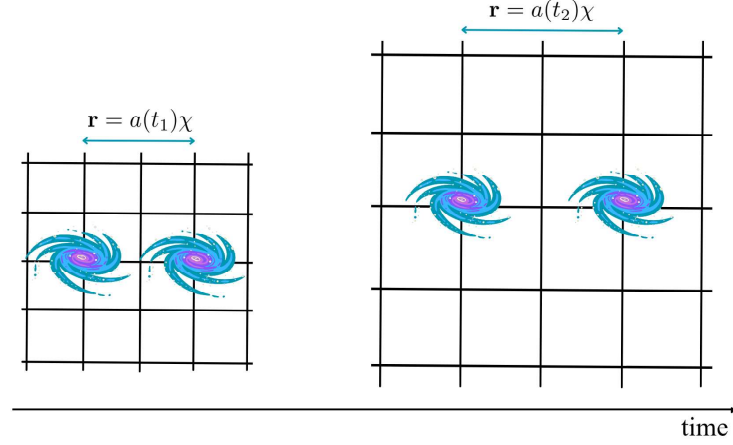


Figure 2.8: Representation of comoving versus proper distances in an expanding Universe. While the comoving separation between two objects remains constant, the proper distance increases with the scale factor due to cosmic expansion. Source: Prepared by the author (2026).

Differentiating this expression with respect to cosmic time, one obtains

$$\dot{\mathbf{r}} = \dot{a}(t) \chi = \frac{\dot{a}}{a} \mathbf{r}.$$

This relation leads directly to the **Hubble–Lemaître law** [40, 92] ,

$$\mathbf{v} = H(t) \mathbf{r}, \quad (2.65)$$

defining the **Hubble flow**, i.e., the large-scale motion of galaxies driven by the expansion of the Universe, for which comoving observers recede from each other with velocities $v = \dot{r}$ proportional to their separation (Figure 2.9). Evaluated at the present time, this becomes

$$\mathbf{v} = H_0 \mathbf{r}, \quad (2.66)$$

which provides the observational foundation for the expanding Universe.

Locally, galaxies do not follow the Hubble flow exactly. Their motion can be decomposed as

$$\mathbf{v} = H(t) \mathbf{r} + \mathbf{v}_{pec}, \quad (2.67)$$

where $\mathbf{v}_{pec}$ represents **peculiar velocities** [2], induced by local inhomogeneities in the matter distribution.

We can also estimate the **age of the Universe**, defined as the elapsed time between $a = 0$ and the present epoch ($a = a_0 = 1$), through the relation [2]

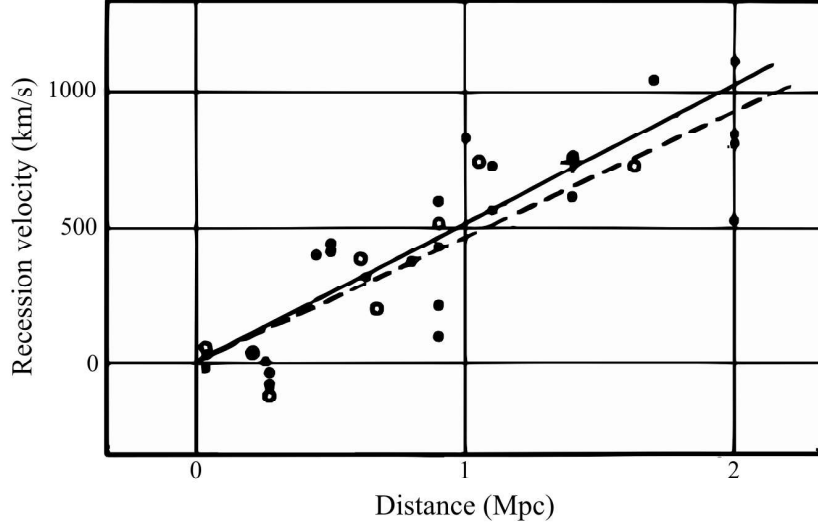


Figure 2.9: Historical Hubble diagram from Hubble (1929), illustrating the first empirical relation between the recessional velocity and distance for extragalactic nebulae. Adapted from: [40].

$$t = \int_0^1 \frac{da}{aH(a)}. \quad (2.68)$$

Evaluated today, one obtains $t_0 = 13.8 \pm 0.023$ Gyr [15].

Furthermore, it is often useful to introduce the **conformal time** η [17], defined through

$$d\eta \equiv \frac{dt}{a(t)}. \quad (2.69)$$

In terms of η , the FLRW metric can be written as

$$ds^2 = a^2(\eta) \left[-c^2 d\eta^2 + d\chi^2 + S_k^2(\chi) d\Omega^2 \right]. \quad (2.70)$$

Since null geodesics satisfy $ds^2 = 0$, this implies

$$c d\eta = d\chi, \quad (2.71)$$

so that the conformal time directly measures the comoving distance traveled by light.

Finally, it is important to highlight that, although physical coordinates and comoving systems provide a convenient theoretical description, they are not directly observable. Instead, cosmological observations rely on quantities such as redshift, flux, and angular positions on the celestial sphere, which must be related to the geometry of the underlying spacetime through the distance measurements introduced in the following subsections.

2.4.2 Cosmological Redshift

One of the most fundamental observables in cosmology is the **redshift** z , defined as the fractional difference between the observed and emitted wavelengths of light,

$$z \equiv \frac{\lambda_{\text{obs}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}}, \quad (2.72)$$

and measured directly from the displacement of spectral features. In an expanding Universe, the wavelength of a freely propagating photon is stretched in proportion to the scale factor. Although formally analogous to the Doppler shift produced by relative motion, this cosmological redshift originates from the dynamical evolution of spacetime itself [35].

To derive the relation between redshift and the scale factor, consider a photon propagating along a radial null geodesic in the FLRW spacetime. As we saw on Section 2.1, light travels on null trajectories, so the spacetime interval satisfies Eq. 2.2. Furthermore, for radial propagation, the angular coordinates remain constant along the trajectory, implying $d\Omega = 0$. Under these conditions, the FLRW metric in comoving coordinates, Eq. 2.34, reduces to

$$0 = -c^2 dt^2 + a^2(t) d\chi^2,$$

which implies

$$d\chi = \pm \frac{c dt}{a(t)}.$$

Choosing the negative sign for a photon propagating toward the observer and integrating along the trajectory from emission to observation yields

$$\chi = \int_{t_e}^{t_0} \frac{c dt}{a(t)}. \quad (2.73)$$

Now, consider two successive wave crests emitted at times t_e and $t_e + \delta t_e$, and received at t_0 and $t_0 + \delta t_0$. Because the source and observer are comoving, the comoving distance χ between them is the same for both crests, so

$$\int_{t_e}^{t_0} \frac{dt}{a(t)} = \int_{t_e + \delta t_e}^{t_0 + \delta t_0} \frac{dt}{a(t)}.$$

To relate the emission and observation intervals, we can rewrite the above equation as

$$\begin{aligned} \int_{t_e}^{t_e + \delta t_e} \frac{dt}{a(t)} + \int_{t_e + \delta t_e}^{t_0} \frac{dt}{a(t)} &= \int_{t_e + \delta t_e}^{t_0} \frac{dt}{a(t)} + \int_{t_0}^{t_0 + \delta t_0} \frac{dt}{a(t)}, \\ \int_{t_e}^{t_e + \delta t_e} \frac{dt}{a(t)} &= \int_{t_0}^{t_0 + \delta t_0} \frac{dt}{a(t)} \end{aligned}$$

For sufficiently small intervals, the last two integrals can be approximated using the mean value theorem, so that

$$\int_{t_0}^{t_0+\delta t_0} \frac{dt}{a(t)} \simeq \frac{\delta t_0}{a(t_0)}, \quad \int_{t_e}^{t_e+\delta t_e} \frac{dt}{a(t)} \simeq \frac{\delta t_e}{a(t_e)}.$$

Substituting, and considering $a(t_0) \equiv 1$, one obtains

$$\delta t_0 = \frac{\delta t_e}{a(t_e)}.$$

Identifying the time intervals with the periods of the wave, $\delta t \propto \lambda$, it follows that

$$\frac{\lambda_0}{\lambda_e} = \frac{1}{a(t_e)}, \quad (2.74)$$

and therefore

$$1 + z = \frac{1}{a(t_e)}. \quad (2.75)$$

This result shows that cosmological redshift arises from the stretching of photon wavelengths due to the expansion of the three dimensional space: as the scaling factor increases, all physical length scales, including wavelengths, are stretched proportionally. Redshift, therefore, provides a direct observational probe of the expansion history of the Universe.

The relation between redshift and scale factor further allows the comoving distance to be rewritten directly as a function of redshift. Starting from the comoving distance to the source obtained in Eq. 2.73, all that remains is to convert the integration variable from cosmic time to redshift. Differentiating the redshift–scale-factor relation gives $da = -dz/(1+z)^2$, while the definition of the Hubble parameter (Eq. 2.43) yields Eq. 2.68. Substituting both into the comoving distance integral and using Eq. 2.75, with emission at redshift z and observation at $z = 0$, gives,

$$\chi = \int_{t_e}^{t_0} \frac{c dt}{a(t)} = \int_z^0 \left(-\frac{c dz'}{H(z')} \right) = \int_0^z \frac{c dz'}{H(z')}, \quad (2.76)$$

where reversing the integration limits absorbs the minus sign. This yields the comoving distance as an explicit function of redshift, where the Hubble parameter $H(z)$ is obtained from the Friedmann equation, together with the redshift scaling of each energy density component,

$$H(z) = H_0 \sqrt{\Omega_{r,0}(1+z)^4 + \Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{\Lambda,0}}. \quad (2.77)$$

In practice, redshifts are measured using different observational techniques: spectroscopy and photometry. Spectroscopic redshifts (z_{spec}) are obtained by identifying the displacement of spectral lines, such as emission or absorption, relative to their known

wavelengths in the rest frame. This method provides highly accurate redshift measurements, but requires substantially longer exposure times and observational resources [93], since only a single spectrum can be taken at a time.

In contrast, photometric redshifts (z_{phot}) are estimated from broadband photometric observations, comparing the observed colors of galaxies with spectral models or through machine learning techniques [94, 95]. Although significantly less precise than spectroscopic measurements, photometric redshifts can be obtained for much larger samples of galaxies, making them essential for modern wide-field cosmological surveys.

2.4.3 Luminosity Distance

One important way of measuring cosmological distances is through **standard candles**, i.e., astrophysical objects whose intrinsic luminosity is known, such as Type Ia supernovae and Cepheid variables [2, 3, 32]. The relationship between the measured flux of these objects and the distance to them is called the **luminosity distance**.

Consider a source located at comoving coordinate (χ, θ, ϕ) emitting light at cosmic time t_e , which is observed today at t_0 . As the photons propagate through the Universe, they spread over a spherical surface whose area depends on the spatial curvature. For a flat universe ($k = 0$), this area is simply given by the Euclidean relation $A = 4\pi\chi^2$. More generally, in a FLRW spacetime, of any geometry, the area becomes [2, 3]

$$A = 4\pi S_k^2(\chi), \quad (2.78)$$

where $S_k(\chi)$ is defined in Eq. 2.33. In positively curved spaces ($k = +1$), the photons are distributed over a smaller area than in the flat case, while in negatively curved spaces ($k = -1$) they spread over a larger area [3].

Besides this geometric effect, the expansion of the Universe also modifies the observed flux through cosmological redshift. As the Universe expands, the wavelength of each photon is stretched according to [3]

$$\lambda_0 = (1 + z)\lambda_e, \quad (2.79)$$

implying that the observed photon energy decreases as [17]

$$E_0 = \frac{E_e}{1 + z}. \quad (2.80)$$

In addition, the arrival rate of photons is reduced by another factor of $(1 + z)$ due to time dilation [3]. Together, these effects reduce the observed flux by a factor $(1 + z)^{-2}$.

If the source has intrinsic luminosity L and observed flux F , the luminosity distance d_L is defined through [2, 3]

$$F = \frac{L}{4\pi d_L^2}. \quad (2.81)$$

Combining the geometric and redshift contributions, the luminosity distance in an FLRW universe is given by

$$d_L = (1 + z) S_k(\chi). \quad (2.82)$$

Using Eq. 2.76, the luminosity distance can be expressed explicitly in terms of the redshift and the expansion history $H(z)$,

$$d_L(z) = (1 + z) S_k \left(c \int_0^z \frac{dz'}{H(z')} \right), \quad (2.83)$$

which, for a spatially flat universe ($k = 0$, $S_k(\chi) = \chi$), reduces to

$$d_L(z) = (1 + z) c \int_0^z \frac{dz'}{H(z')}. \quad (2.84)$$

The dependence of the luminosity distance on the cosmological parameters is illustrated in Figure 2.10, which shows $d_L(z)$ for different values of the matter density parameter Ω_m . In the spatially flat Λ CDM model adopted here, the density parameters are not independent, but satisfy Eq. 2.50, so that varying Ω_m simultaneously fixes Ω_Λ , since the contribution from radiation can be neglected. In this way, the matter density governs the shape of the distance–redshift relation, through its impact on the expansion history $H(z)$. The Hubble constant H_0 , in turn, sets only the overall normalization of $d_L(z)$, while spatial curvature, when present, would enter additionally through the geometric factor $S_k(\chi)$.

2.4.4 Angular Diameter Distance

Another important way of measuring cosmological distances is through **standard rulers**, i.e., objects or physical scales whose proper size is known a priori [2, 3]. The most important example in modern cosmology is the BAO scale, a characteristic comoving length imprinted in the large-scale distribution of matter, whose physical origin is discussed in detail in the next section. Since this scale is set by early-Universe physics and is therefore known independently, comparing its observed angular extent with its physical size provides a direct geometric probe of the expansion history, which makes the angular diameter distance the central distance measure for the analysis developed in this work.

Suppose an object of proper length ℓ is oriented perpendicular to the line of sight (Figure 2.11). If it subtends a small angle $\delta\theta \ll 1$ rad, its distance can be estimated through the small-angle approximation [2, 3],

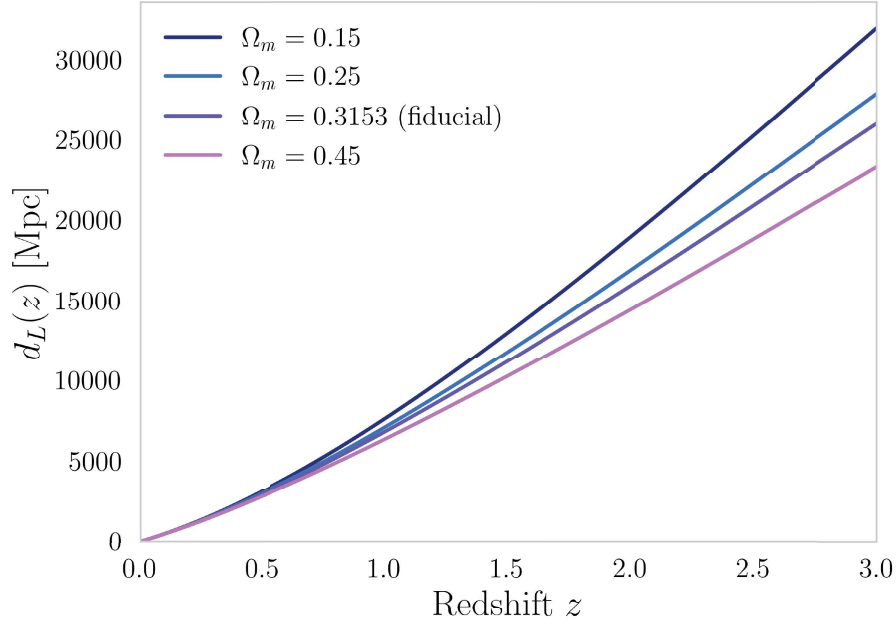


Figure 2.10: Luminosity distance $d_L(z)$ as a function of redshift for different values of the matter density parameter Ω_m , illustrating its influence on the distance-redshift relation. The fiducial model corresponds to the matter density parameter inferred from Planck [15] (TT,TE,EE+lowE+lensing). Source: Prepared by the author (2026).

$$d_A \equiv \frac{\ell}{\delta\theta}, \quad (2.85)$$

where d_A is the **angular diameter distance**. This expression holds in a static Euclidean geometry, in which distances do not evolve with time. In an expanding Universe, however, the relation between the physical size of an object and its observed angular size is modified by both the dynamics and the geometry of spacetime, and therefore requires a treatment based on the FLRW metric.

Consider a source located at comoving radial coordinate χ , observed at redshift z . The angular part of the FLRW line element (Eq. 2.34) shows that two points separated by an angle $\delta\theta$ at that coordinate are separated by a comoving distance [2]

$$d_M = S_k(\chi), \quad (2.86)$$

known as the **transverse comoving distance**, where $S_k(\chi)$ is defined in Eq. 2.33. The corresponding physical transverse size at the time of emission is obtained by scaling with the scale factor at that epoch,

$$\ell = a(t_e) S_k(\chi) \delta\theta = \frac{S_k(\chi)}{1+z} \delta\theta, \quad (2.87)$$

Substituting this into the definition (Eq. 2.85) yields the angular diameter distance in an FLRW universe,

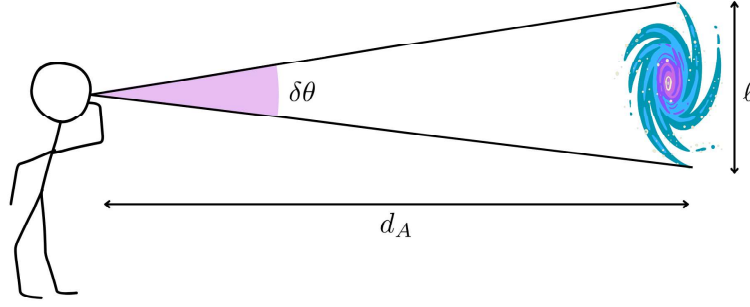


Figure 2.11: Illustration of the angular diameter distance d_A . An object of physical size ℓ subtends an angle $\delta\theta$ at the observer. The angular diameter distance relates the intrinsic size of a source to its observed angular extent and is one of the fundamental distance measures used in observational cosmology. Source: Prepared by the author (2026).

$$d_A = \frac{S_k(\chi)}{1+z} = \frac{d_M}{1+z}. \quad (2.88)$$

As we did with the luminosity distance, we can consider the relation between the scale factor and the redshift and rewrite d_A directly in terms of z and the expansion history $H(z)$,

$$d_A(z) = \frac{1}{1+z} S_k \left(c \int_0^z \frac{dz'}{H(z')} \right), \quad (2.89)$$

which, for the spatially flat universe adopted here, reduces to

$$d_A(z) = \frac{c}{1+z} \int_0^z \frac{dz'}{H(z')}. \quad (2.90)$$

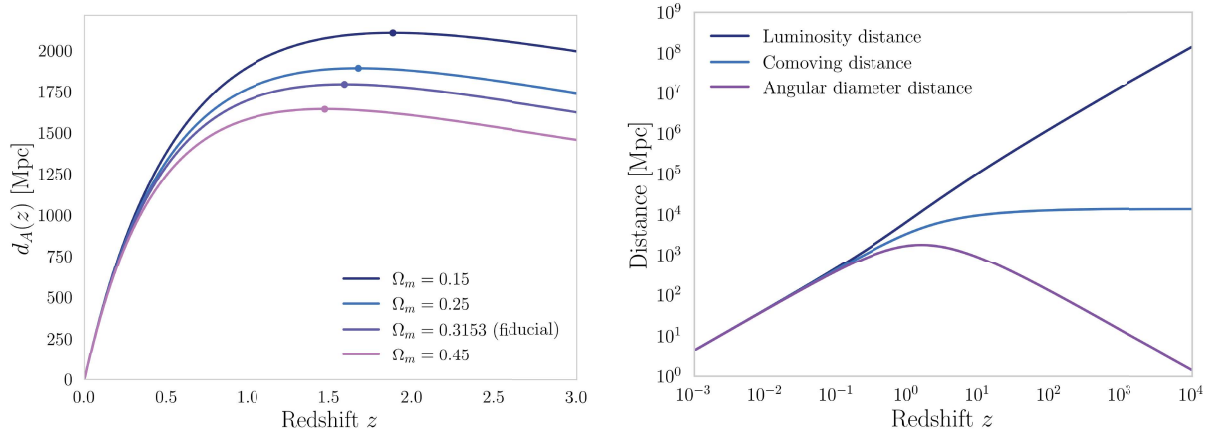
The angular diameter distance therefore depends both on the expansion history, through $H(z)$, and on the spatial geometry of the Universe, through S_k . A similar analysis to that of the distance-luminosity dependence on the matter density parameter can be performed for d_A , a relationship illustrated in the Figure 2.12a.

The luminosity distance and the angular diameter distance are related quantities in observational cosmology. Although defined from different observables, both depend on the same underlying spacetime geometry. Combining Eqs. 2.88 and 2.82 yields

$$d_L = (1+z)^2 d_A, \quad (2.91)$$

known as the **distance-duality** relation, or Etherington relation [96–98], which holds in any metric theory of gravity in which photons travel on null geodesics and the photon number is conserved. This relation plays an important role in observational cosmology, connecting independent cosmological probes within the same geometric framework.

Also, a distinctive feature of angular diameter distance, unlike luminosity distance, is that it does not increase monotonically with redshift. At low redshifts, the increase in comoving distance is the dominant effect, and $d_A(z)$ grows as objects recede. At higher redshifts, however, the factor $(1+z)^{-1}$ becomes dominant, and the angular diameter distance begins to decrease. As a result, $d_A(z)$ reaches a maximum around $z \simeq 1.7$ [99] for the fiducial cosmology adopted in this work. Beyond this point, objects of fixed physical size subtend increasingly larger angles in the sky, even when located at greater distances. This behavior is shown in Figure 2.12b.



(a) Angular diameter distance as a function of redshift.

(b) Relationship between cosmological distance measurements.

Figure 2.12: Left: angular diameter distance as a function of redshift for different values of the cosmological density parameters. The markers indicate the redshift at which the angular diameter distance reaches its maximum. The fiducial model corresponds to the matter density parameter inferred from Planck [15] (TT,TE,EE+lowE+lensing). Right: illustration of the relationship between the comoving distance χ , angular diameter distance d_A , and luminosity distance d_L . Source: Prepared by the author (2026).

2.5 Linear Perturbation Theory

In previous sections, the large-scale evolution of the Universe was described by the homogeneous and isotropic FLRW metric, whose expansion is governed by the scale factor $a(t)$ and the Friedmann equations. At smaller scales, however, the Universe is not homogeneous: matter is arranged into galaxies, clusters, filaments, and voids, forming the complex structure known as the cosmic web. These structures are understood to have grown from small primordial perturbations under the action of gravity [4], and while the fluctuations remain small, their evolution can be followed using linear perturbation theory [100], in which each quantity is split into a homogeneous background plus a small perturbation and only linear terms are retained.

This framework describes the early, linear growth of perturbations and the acoustic oscillations that arise in the photon-baryon plasma, which we develop in the remainder of

this section: we reformulate the fluid equations in comoving coordinates, linearize them around the homogeneous background, and follow the growth of perturbations through the matter era, before turning to the photon-baryon plasma, whose radiation pressure drives the acoustic oscillations and imprints the sound horizon at recombination.

2.5.1 Fluid Equations in an Expanding Universe

As seen in Section 2.1, the matter content of the Universe can be approximated as a continuous fluid. In the Newtonian regime, valid on scales well below the Hubble radius (Eq. 2.45) and for non-relativistic matter, the dynamics of a perfect fluid in a static spacetime are governed by the continuity, Euler, and Poisson equations. The **continuity equation** expresses conservation of mass [2, 101],

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0 \quad (2.92)$$

where $\rho = \rho(t, \vec{r})$ is the matter density and $\vec{v} = \vec{v}(t, \vec{r})$ is the velocity field, with $\vec{r} = (r, \theta, \phi)$ denoting the physical spatial coordinates. The conservation of momentum is governed by the **Euler equation** [2],

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v} \right) = -\vec{\nabla} p - \rho \vec{\nabla} \Phi, \quad (2.93)$$

where $p = p(t, \vec{r})$ is the pressure and $\Phi = \Phi(t, \vec{r})$ is the gravitational potential. The potential is defined through the **Poisson equation** [2, 101],

$$\nabla^2 \Phi = 4\pi G \rho. \quad (2.94)$$

In cosmology, however, the background geometry evolves with the expansion, making it convenient to reformulate these equations in comoving coordinates. The first step is to relate physical and comoving velocities: differentiating the comoving-physical distance relation given by Eq. 2.29 with respect to cosmic time gives the total velocity as a Hubble-flow term plus a peculiar velocity (Eq. 2.67). The subtlety is that, while ∂_t at fixed $\vec{r}$ and at fixed $\vec{x}$ coincide in a static space, they differ in an expanding one, since a point at fixed comoving coordinate is physically moving [2]. The spatial and temporal derivatives must therefore be transformed accordingly.

Using the relation $\vec{\chi} = \vec{r}/a$ between comoving and physical coordinates, the spatial gradient transforms as [2, 102, 103]

$$\vec{\nabla}_{\vec{r}} = \frac{1}{a} \vec{\nabla}, \quad (2.95)$$

where from now on $\vec{\nabla} \equiv \vec{\nabla}_{\vec{\chi}}$ denotes the gradient with respect to comoving coordinates. Similarly, the time derivative at fixed physical position differs from that at fixed comoving

position, since a comoving point moves with the expansion,

$$\left(\frac{\partial f}{\partial t}\right)_{\vec{r}} = \left(\frac{\partial f}{\partial t}\right)_{\vec{\chi}} - H \vec{\chi} \cdot \vec{\nabla} f, \quad (2.96)$$

where f is any function of space and time and the second term accounts for the motion of a point at fixed comoving coordinates. Applying these rules, together with the velocity decomposition (Eq. 2.67), to the physical fluid Eqs. 2.92 and 2.94, yields their comoving counterparts²,

$$\left(\frac{\partial \rho}{\partial t}\right)_{\vec{\chi}} + 3H\rho + \frac{1}{a} \vec{\nabla} \cdot (\rho \vec{v}_{\text{pec}}) = 0, \quad (2.97)$$

$$\left[\left(\frac{\partial}{\partial t}\right)_{\vec{\chi}} + \frac{\vec{v}_{\text{pec}}}{a} \cdot \vec{\nabla}\right] \vec{v} = -\frac{1}{a} \frac{\vec{\nabla} p}{\rho} - \frac{1}{a} \vec{\nabla} \Phi, \quad (2.98)$$

$$\nabla^2 \Phi = 4\pi G a^2 \rho. \quad (2.99)$$

With the fluid equations now written in comoving form, we are ready to examine the growth of small perturbations around the homogeneous background.

2.5.2 Evolution of Perturbations

The background solution is recovered by setting $\vec{v}_{\text{pec}} = 0$ in the comoving continuity and Euler equations. The structures we observe in the Universe, however, arise from small deviations from this scenario. To track their evolution, each quantity is decomposed into its background value and a small perturbation. [2], that is:

$$\rho = \bar{\rho}(1 + \delta), \quad p = \bar{p} + \delta p, \quad \Phi = \bar{\Phi} + \delta\Phi, \quad (2.100)$$

where $\bar{\rho} = \bar{\rho}(t)$, $\bar{p} = \bar{p}(t)$, and $\bar{\Phi} = \bar{\Phi}(t)$ are the homogeneous background values, and $\delta = \delta(t, \vec{\chi})$, $\delta p = \delta p(t, \vec{\chi})$, and $\delta\Phi = \delta\Phi(t, \vec{\chi})$ are the perturbations. The **density contrast** [2, 103]

$$\delta \equiv \frac{\delta\rho}{\bar{\rho}} = \frac{\rho - \bar{\rho}}{\bar{\rho}}, \quad (2.101)$$

quantifies the fractional deviation of the local density from its mean value. By construction, $\delta = 0$ corresponds to the mean density, $\delta > 0$ to an overdense region, and $\delta < 0$ to an underdense region.

Although the fluid equations are nonlinear, the primordial fluctuations were initially very small, which means that, while $|\delta| \ll 1$, any term quadratic in the perturbations

²The derivation of the Equations 2.97, 2.98 and 2.99, as well as the transformations in the spatial and temporal derivatives, are developed in Appendix A.2.

(such as $\delta \vec{v}_{\text{pec}}$ or $(\vec{v}_{\text{pec}} \cdot \vec{\nabla})\vec{v}_{\text{pec}}$) is negligible beside the linear ones. When substituting the decomposition from Eq. 2.100 into the comoving fluid equations (Eqs. 2.97, 2.98 and 2.99), the zeroth-order terms reproduce the background and cancel, while discarding the quadratic terms leaves the linearized continuity, Euler, and Poisson equations [2, 103],

$$\dot{\delta} = -\frac{1}{a}\vec{\nabla} \cdot \vec{v}_{\text{pec}}, \quad (2.102)$$

$$\dot{\vec{v}}_{\text{pec}} + H\vec{v}_{\text{pec}} = -\frac{1}{a\bar{\rho}}\vec{\nabla} \delta p - \frac{1}{a}\vec{\nabla} \delta \Phi, \quad (2.103)$$

$$\nabla^2 \delta \Phi = 4\pi G a^2 \bar{\rho} \delta. \quad (2.104)$$

which can be combined into a single second-order equation for δ . Differentiating the continuity equation (Eq. 2.102) in time and taking the divergence of the Euler equation (Eq. 2.103), then using the barotropic relation for the pressure perturbation, which defines the **sound speed** c_s [2, 102],

$$\delta p = \frac{\partial p}{\partial \rho} \delta \rho \equiv c_s^2 \delta \rho, \quad (2.105)$$

with $\delta \rho = \bar{\rho} \delta$, together with the linearized Poisson equation (Eq. 2.104), yields the equation governing the evolution of the perturbations [2, 102, 103],

$$\ddot{\delta} + 2H\dot{\delta} - \left(\frac{c_s^2}{a^2} \nabla^2 + 4\pi G \bar{\rho} \right) \delta = 0. \quad (2.106)$$

The first term in parentheses, $(c_s^2/a^2)\nabla^2\delta$, is the **pressure term**, which tends to smooth out the perturbation and drive oscillatory behaviour, while the gravity term $4\pi G \bar{\rho} \delta$ drives growth through gravitational instability. The second term on the left, $2H\dot{\delta}$, is the **Hubble drag**, a friction through which the expansion suppresses growth [102].

Since Eq. 2.106 is linear with spatially constant coefficients, distinct wavelengths evolve independently. Expanding $\delta(\vec{\chi}, t)$ in plane waves,

$$\delta(\vec{\chi}, t) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{\chi}} \delta(\vec{k}, t). \quad (2.107)$$

Spatial gradients acting on each plane-wave mode become $\vec{\nabla} \rightarrow i\vec{k}$, so $\nabla^2 \rightarrow -k^2$ with $k = |\vec{k}|$ [101], while time derivatives act only on $\delta(\vec{k}, t)$. Each mode then obeys [2, 102]

$$\ddot{\delta}(\vec{k}, t) + 2H\dot{\delta}(\vec{k}, t) + \left(\frac{c_s^2 k^2}{a^2} - 4\pi G \bar{\rho} \right) \delta(\vec{k}, t) = 0. \quad (2.108)$$

It is convenient to rewrite Eq. 2.108 as a damped-oscillator equation [101, 102],

$$\ddot{\delta}_k + 2H\dot{\delta}_k = -\omega^2 \delta_k, \quad (2.109)$$

where $\delta_k \equiv \delta(\vec{k}, t)$, and the gravity term has been absorbed, together with the pressure term, into a frequency

$$\omega^2 \equiv \frac{c_s^2 k^2}{a^2} - 4\pi G \bar{\rho} = \frac{c_s^2}{a^2} (k^2 - k_J^2), \quad (2.110)$$

written here in factored form by introducing the comoving **Jeans wavenumber** [2, 32],

$$k_J = \frac{\sqrt{4\pi a^2 G \bar{\rho}}}{c_s}, \quad (2.111)$$

the scale at which pressure support exactly balances gravity. The corresponding proper **Jeans length** is [101, 103]

$$\lambda_J \equiv \frac{2\pi a}{k_J} = c_s \sqrt{\frac{\pi}{G \bar{\rho}}}, \quad (2.112)$$

which is the distance a sound wave travels in a gravitational free-fall time $t_{\text{ff}} \sim (G \bar{\rho})^{-1/2}$ [102]. Neglecting the Universe expansion, Eq. 2.109 reduces to $\ddot{\delta}_k = -\omega^2 \delta_k$, whose behavior depends on whether pressure or gravity dominates³.

With the expansion restored, this picture is preserved qualitatively, but the Hubble drag attenuates both the pressure- and gravity-domination regimes [2, 102]: acoustic waves with wavelength below λ_J are slowly damped, while unstable modes above λ_J still grow, though as a power law rather than exponentially. Equivalently, collapse occurs only for regions exceeding the **Jeans mass** [101],

$$M_J \equiv \frac{4\pi}{3} \left(\frac{\lambda_J}{2} \right)^3 \bar{\rho} \sim c_s^3 \bar{\rho}^{-1/2}. \quad (2.113)$$

Alongside the Jeans length, the growth of perturbations is governed by a second characteristic scale: the Hubble horizon, the surface beyond which points recede faster than light, whose physical radius is the Hubble radius R_H (Eq. 2.45). A perturbation of comoving wavenumber k has physical wavelength $\lambda = 2\pi a/k$. A mode is called **sub-horizon** when its wavelength fits within the horizon, $\lambda \lesssim R_H$, equivalently $k \gtrsim aH/c$, and **super-horizon** when $\lambda \gtrsim R_H$, equivalently $k \lesssim aH/c$. In comoving coordinates the wavelength λ of a given mode is fixed, while the comoving Hubble radius $c/(aH)$ grows throughout the radiation- and matter-dominated eras. Any fluctuation that lies inside the Hubble radius today must therefore have been outside it at sufficiently early times, crossing the horizon at the moment when the growing Hubble radius first matched its comoving wavelength. The Newtonian fluid treatment developed above applies only to sub-horizon, non-relativistic perturbations, whereas super-horizon scales require the general-relativistic description [2].

³For $\lambda < \lambda_J$ ($\omega^2 > 0$), pressure dominates and $\delta_k \propto e^{\pm i\omega t}$ oscillates as a propagating acoustic wave. For $\lambda > \lambda_J$ ($\omega^2 < 0$), self-gravity dominates and $\delta_k \propto e^{\pm |\omega| t}$ becomes a non-propagating mode with growing and decaying branches [101, 102]; the growing branch is the **Jeans instability**, which seeds gravitational collapse.

2.5.3 Growth of Structures

For cold dark matter the sound speed is negligible, $c_s = 0$, so the pressure term in Eq. 2.108 drops out and the growth of perturbations is governed by [2, 103]

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\bar{\rho}\delta = 0. \quad (2.114)$$

The behaviour of its solutions depends on which component dominates the background expansion, so we can solve it in each era in turn.

Radiation era. The expansion is radiation-driven, $a \propto t^{1/2}$ and $H = 1/2t$. Radiation does not cluster on sub-horizon scales and matter is subdominant, so the gravitational source is negligible against the Hubble drag, $4\pi G\bar{\rho} \ll H^2$, and Eq. 2.114 reduces to

$$\ddot{\delta} + 2H\dot{\delta} = \ddot{\delta} + \frac{1}{t}\dot{\delta} \approx 0. \quad (2.115)$$

The power-law $\delta \propto t^p$ gives the indicial equation $p^2 = 0$, a double root, whose two solutions are the constant and logarithmic modes [103, 104],

$$\delta(t) = A_1 + A_2 \ln t. \quad (2.116)$$

Cold dark matter perturbations therefore grow at most logarithmically, essentially frozen. This stagnation is the Mészáros effect [105], which suppresses small-scale power.

Matter era. The expansion is matter-dominated, $a \propto t^{2/3}$ and $H = 2/3t$, so the source term becomes $4\pi G\bar{\rho} = \frac{3}{2}H^2$ and Eq. 2.114 takes the equidimensional (Euler) form [2]

$$\ddot{\delta} + \frac{4}{3t}\dot{\delta} - \frac{2}{3t^2}\delta = 0, \quad (2.117)$$

whose solution is in the form [104]

$$\delta(t) = B_1 t^{2/3} + B_2 t^{-1}, \quad (2.118)$$

where B_1 and B_2 are constants. Since the decaying mode quickly becomes negligible, the late-time evolution is set by the growing mode: in the matter era the density contrast grows with the scale factor, $\delta \propto a \propto t^{2/3}$ [103].

Dark-energy era. Once dark energy dominates, the expansion is nearly exponential, $a \propto e^{H_\Lambda t}$ with $H = H_\Lambda$ constant. Matter is again negligible beside the vacuum energy, so the source term drops and [104]

$$\ddot{\delta} + 2H_\Lambda\dot{\delta} = 0, \quad (2.119)$$

with solution [3, 104]

$$\delta(t) = C_1 + C_2 e^{-2H_\Lambda t}. \quad (2.120)$$

The decaying exponential dies away and δ tends to a constant: structure growth freezes once dark energy dominates.

In general, any solution of Eq. 2.114 can be written as

$$\delta(\vec{k}, t) = \delta_+(\vec{k})D_+(t) + \delta_-(\vec{k})D_-(t), \quad (2.121)$$

where $\delta_\pm(\vec{k})$ are the Fourier modes of the initial density field and the k -independent functions $D_+(t)$ and $D_-(t)$ are the growing and decaying modes, describing the linear evolution [2]. Specifically, the growth factor is called the **linear growth function**, and it is written as

$$D_+(a) \equiv \frac{\delta(a)}{\delta(a_0)}, \quad D_+(a_0) = 1. \quad (2.122)$$

For the case where dark energy is a cosmological constant ($w = -1$) and assuming spatial flatness ($\Omega_k = 0$, so that $\Omega_m + \Omega_\Lambda = 1$), one can show that the growing solution is

$$D_+(a) \propto \frac{H(a)}{H_0} \int_0^a \frac{da'}{[\Omega_m/a' + \Omega_\Lambda a'^2 + (1 - \Omega_m - \Omega_\Lambda)]^{3/2}}. \quad (2.123)$$

The growth described above applies to a pressureless fluid. The physics underlying the BAO feature comes instead from the baryons, which before recombination form a tightly coupled plasma with the photons.

2.5.4 The Photon-Baryon Plasma and Acoustic Oscillations

Before recombination ($z \gtrsim 1100$) radiation and matter behaved as a single fluid, tightly coupled by Thomson scattering between photons and free electrons and Coulomb scattering between protons and electrons. Its adiabatic sound speed follows from the definition presented in Eq. 2.105, evaluated on the combined system. The pressure is supplied by the radiation, $p = \frac{1}{3}\rho_\gamma c^2$ [32], baryons being pressureless, while the total density is $\rho = \rho_\gamma + \rho_b$. Since both are functions of the scale factor, with $\rho_\gamma \propto a^{-4}$ and $\rho_b \propto a^{-3}$, so that $d\rho_\gamma/da = -4\rho_\gamma/a$ and $d\rho_b/da = -3\rho_b/a$,

$$c_s^2 = \frac{\partial p}{\partial \rho} = \frac{dp/da}{d\rho/da} = \frac{c^2}{3} \frac{4\rho_\gamma}{4\rho_\gamma + 3\rho_b} = \frac{c^2}{3} \frac{1}{1 + \frac{3\rho_b}{4\rho_\gamma}}, \quad (2.124)$$

which gives

$$c_s^2 = \frac{c^2}{3(1+R)}, \quad R \equiv \frac{3\rho_b}{4\rho_\gamma}, \quad (2.125)$$

with R the baryon-to-photon momentum ratio. When radiation is dominating ($R \ll 1$), the sound speed is relativistic,

$$c_s \approx \frac{c}{\sqrt{3}} \approx 0.58 c, \quad (2.126)$$

so the Jeans length (Eq. 2.112) is of order the Hubble radius, and essentially all sub-horizon baryon perturbations oscillate rather than grow [2]. In this regime the gravitational term in Eq. 2.108 becomes negligible, reducing the evolution equation to

$$\ddot{\delta}(\vec{k}, t) + 2H\dot{\delta}(\vec{k}, t) + \frac{c_s^2 k^2}{a^2} \delta(\vec{k}, t) \simeq 0. \quad (2.127)$$

Consistently, the gravity term in the frequency (Eq. 2.110) drops out and ω reduces to the ordinary sound-wave relation, $\omega \simeq c_s k/a$. Since the Hubble drag in Eq. 2.127 varies slowly over an oscillation period, the equation admits approximate oscillatory solutions, $\delta(\vec{k}, t) \propto \exp[\pm i \phi(t)]$ [2], whose phase is the time integral of this frequency

$$\phi(t) = \int_0^t \omega(t') dt' \simeq k \int_0^t \frac{c_s(t')}{a(t')} dt' \equiv k r_s(t), \quad (2.128)$$

where the comoving wavenumber $k = |\vec{k}|$ has been factored out of the integral. The remaining integral defines the **comoving sound horizon**,

$$r_s(t) \equiv \int_0^t \frac{c_s(t')}{a(t')} dt' \Rightarrow r_s(z) = \int_z^\infty \frac{c_s(z')}{H(z')} dz', \quad (2.129)$$

the maximum comoving distance an acoustic wave can travel by time t . Each Fourier mode therefore oscillates as a function of the single dimensionless combination $k r_s$. During these oscillations, baryons and photons alternately undergo compression (when the overdensity is maximal) and rarefaction (when the density is minimal) [6], corresponding to the peaks and troughs of the acoustic wave. After decoupling, the phase reached by each mode is effectively frozen. This freezing imprints a spherical shell of enhanced baryon density at a radius r_s around every initial overdensity. These concentric shells, reflecting the last phase of compression or rarefaction, constitute the physical origin of the BAO feature in the matter distribution.

Because r_s is fixed by pre-recombination physics and is insensitive to the late-time expansion, it acts as a **standard ruler** whose comoving length, calibrated by the CMB, is known a priori [18, 106]. The scale relevant for galaxy surveys is the sound horizon at the drag epoch z_d , $r_d \equiv r_s(z_d)$, when the baryons are released from the photon drag. Being a comoving length, r_d relates directly to the transverse comoving distance $d_M(z)$, introduced in Eqs. 2.86 and 2.88: oriented transverse to the line of sight the ruler subtends an angle

$\delta\theta = r_d/d_M(z)$, while along the line of sight it spans a redshift interval $\delta z = r_d H(z)/c$. The transverse and radial BAO signals therefore constrain $d_M(z)$ and $H(z)$, respectively [106].

In practice the acoustic scale is measured relative to a template computed in a fiducial cosmology, and the resulting rescaling of the peak position is quantified by the dilation parameters [20, 107]

$$\alpha_{\perp} = \frac{d_M(z)/r_d}{[d_M(z)/r_d]_{\text{fid}}}, \quad \alpha_{\parallel} = \frac{[H(z) r_d]_{\text{fid}}}{H(z) r_d}, \quad \alpha = \frac{d_V(z)/r_d}{[d_V(z)/r_d]_{\text{fid}}}, \quad (2.130)$$

with $d_V(z) = [d_M^2(z) c z / H(z)]^{1/3}$ the volume-averaged distance used in the isotropic case [18]. Since r_d is set by early-Universe physics, these ratios isolate the geometric information, making the transverse comoving distance $d_M(z)$ the central observable of the BAO analysis developed in the following chapters.

2.5.5 Matter Power Spectrum

The previous sections tracked the evolution of individual Fourier modes: the growing mode $D_+(t)$ for the pressureless dark matter, and the acoustic oscillations of the baryon–photon fluid, each obeying Eq. 2.108 independently. What we compare with observations, however, is not a single mode but the statistical distribution of the density field. By construction, the density contrast (Eq. 2.101) has zero mean [3],

$$\langle \delta(\vec{x}, t) \rangle = 0, \quad (2.131)$$

so the first non-trivial statistic is its two-point function. Working with the Fourier modes of Eq. 2.107, homogeneity and isotropy of the background imply [2]

$$\langle \delta(\vec{k}, t) \delta^*(\vec{k}', t) \rangle = (2\pi)^3 \delta_D(\vec{k} - \vec{k}') P(k, t), \quad (2.132)$$

which defines the **power spectrum** $P(k, t)$, the Fourier transform of the real-space two-point correlation function $\xi(r)$ [2, 101] (which we will discuss in detail in Chapter 3).

A density field whose Fourier components have uncorrelated phases is called a **Gaussian field** [3]. When such a field is in addition homogeneous and isotropic, its entire statistical content is captured by the power spectrum alone. This case is of particular interest in cosmology since most inflationary scenarios predict that the density fluctuations generated during inflation form precisely such a homogeneous and isotropic Gaussian field [3, 102]. Inflation further predicts that the corresponding primordial power spectrum is well described by a power law [2, 102],

$$P(k, t_i) = A_s k^{n_s}, \quad (2.133)$$

with amplitude A_s and spectral index n_s . The scale-invariant Harrison-Zel’dovich value

$n_s = 1$ [108, 109], one of the six free parameters for the Λ CDM model described in Section 2.3.2, gives equal power per logarithmic interval in k . Recent CMB observations have measured this value as $n_s = 0.9649 \pm 0.0042$ [15].

As primordial fluctuations evolve through radiation- and matter-dominated eras, the spectrum given in Eq. 2.133 is reshaped into the characteristic form of the matter power spectrum in later times. The origin of this distortion is the scale-dependent growth discussed in Section 2.5.3: modes that entered the horizon during radiation domination had their growth interrupted by the Mészáros effect, while those that entered during matter domination grew unimpeded. Consequently, the spectrum retains its primordial slope on large scales, while on small scales the suppression accentuates it to $P(k) \propto k^{n_s-4}$ [2]. The two regimes meet in a transition defined by the comoving horizon scale in the matter-radiation equality, $k_{\text{eq}} \approx 0.015 h \text{ Mpc}^{-1}$ [2].

Within linear theory, this form is fixed, so the only effect of cosmic evolution is to rescale the overall amplitude of the spectrum, which grows monotonically as the structure develops. The power spectrum of matter is therefore conventionally specified at the current epoch, $z = 0$, its value at any other redshift differing only by this amplitude. The acoustic physics of Section 2.5.4 is similarly imprinted on $P(k, z)$ as a fixed feature: the moving sound horizon r_s defines the location of baryonic acoustic oscillations, a scale established by pre-recombination physics that remains constant in comoving coordinates and is therefore preserved under subsequent linear growth.

Figure 2.13 illustrates the evolution of the linear matter power spectrum with redshift, showing the decrease in amplitude as structure growth progresses. The comparison between the full and no-wiggle spectra highlights the baryon acoustic oscillation features, which are more clearly visible in the ratio shown in Figure 2.14. These oscillations correspond to the imprint of primordial sound waves and provide a key cosmological standard ruler.

Galaxy bias: Galaxies do not exactly replicate the distribution of matter in the universe. They form within halos of dark matter, which are located at the peaks of the density field and are therefore more tightly packed than matter as a whole. On large scales, this relationship is captured by a single bias parameter $b(z)$,

$$\delta_g = b \delta, \quad P_g(k, z) = b^2(z) P(k, z), \quad (2.134)$$

so the observed galaxy power spectrum is a rescaled version of the underlying matter spectrum. The bias depends on the tracer population and redshift, and becomes scale dependent once nonlinear effects set in [2, 101].

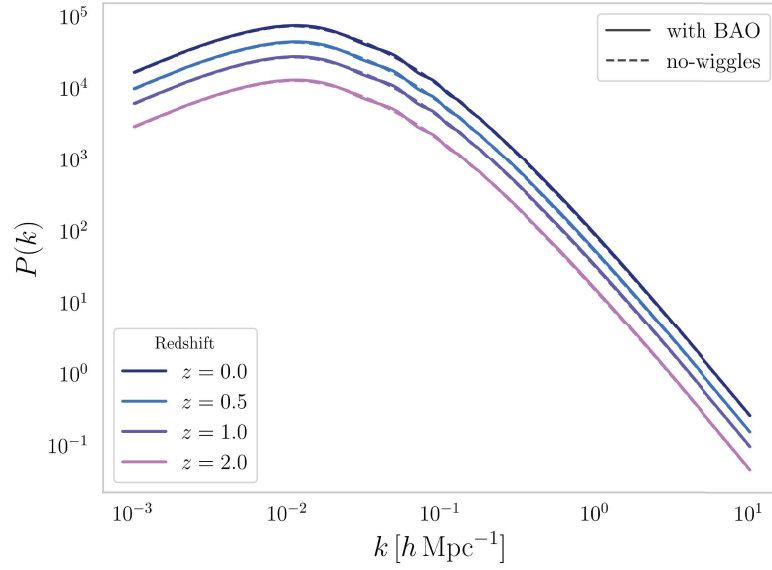


Figure 2.13: Linear matter power spectrum, $P(k, z)$, for a fiducial Λ CDM cosmology at different redshifts. Solid lines show the full spectrum, including baryon acoustic oscillations, while dashed lines correspond to the Eisenstein-Hu no-wiggle approximation. The amplitude decreases with increasing redshift due to linear structure growth, whereas the BAO scale remains fixed in co-moving coordinates. Source: Prepared by the author (2026).

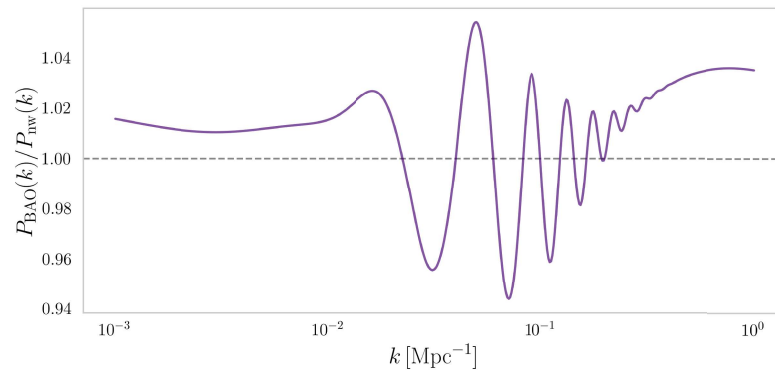


Figure 2.14: Ratio between the linear matter power spectrum including baryon acoustic oscillations and the corresponding Eisenstein-Hu no-wiggle spectrum, $P_{\text{BAO}}(k)/P_{\text{nw}}(k)$, at $z = 0$. The oscillatory pattern highlights the imprint of acoustic waves propagating in the tightly coupled photon-baryon plasma before recombination. Source: Prepared by the author (2026).

Chapter 3

Statistics Of The Galaxy Distribution

The linear theory developed in Section 2.5 describes the evolution of individual Fourier modes of the density field. What is actually compared with observations, however, is not a single mode but the matter distribution traced by any type of galaxies. This chapter establishes the statistical framework within which that comparison is carried out. To this end, the matter distribution is treated as a random field and its discrete tracers as a point source, with the homogeneous Poisson process adopted to describe complete spatial randomness. The two-point correlation function $\xi(r)$ is then defined as a measure of the excess clustering of the tracers, and its projection onto the sky yields the angular correlation function $w(\theta)$. We then turn to the Fourier-space description, introducing the power spectrum $P(k)$ and its angular counterpart, the angular power spectrum C_ℓ . Finally, we describe the pseudo- C_ℓ estimator, which recovers C_ℓ from the partial-sky observations characteristic of a real survey, such as the one analyzed in this work.

3.1 Random Fields and Point Processes

The large-scale distribution of matter in the Universe exhibits a complex structure, composed by galaxies, clusters, filaments, and voids. Instead of individually characterizing each of these components, modern cosmology adopts a statistical description, in which the distribution of matter is treated as the realization of a random process [110, 111]. The interest, therefore, does not lie in the exact positions of individual galaxies, but in the statistical properties of the underlying field of which they are a part.

To make this statistical description precise, the distribution of matter is considered a realization of a random field, that is, a realization that assigns a random variable to each point $\mathbf{x} \in \mathbb{R}^3$ [110]. For the matter density field, this variable is the local density, and the observed Universe is just one realization of this field. The relevant statistical quantities are therefore ensemble averages, denoted by $\langle \cdot \rangle$, taken over the hypothetical set of all possible realizations of the process [17].

Two simplifying assumptions underlie the construction of random fields in cosmology.

The first is statistical homogeneity and isotropy: the statistical properties of the field are invariant under translations and rotations, depending only on relative separations and not on absolute position or direction [110], since these fields are defined for a specific background geometry and share its symmetry properties. The second is ergodicity: since we observe a single Universe, we do not have access to an ensemble of independent realizations over which to average [110, 112, 113]. It is therefore assumed that ensemble averages can be replaced by spatial averages calculated over a sufficiently large volumes [110].

The statistical properties of a random field are encoded in the hierarchy of its moments. For the density contrast $\delta(\mathbf{x})$ defined in Eq. 2.101, the first moment is its mean value, $\langle\delta(\mathbf{x})\rangle$, while the second moment describes the correlation between the field at two different positions, $\langle\delta(\mathbf{x}_1)\delta(\mathbf{x}_2)\rangle$. More generally, the N -point moment is defined as

$$\langle\delta(\mathbf{x}_1)\delta(\mathbf{x}_2)\cdots\delta(\mathbf{x}_N)\rangle.$$

The knowledge of the complete hierarchy constitutes, in principle, a full statistical description of the field [110]. For the density contrast, the first moment vanishes by construction (Eq. 2.131), so the lowest-order non-trivial statistic is the second moment, which defines the **two-point correlation function** (2PCF) [107],

$$\xi(\mathbf{x}_1, \mathbf{x}_2) = \langle\delta(\mathbf{x}_1)\delta(\mathbf{x}_2)\rangle. \quad (3.1)$$

Under the homogeneity and isotropy established above, this quantity depends only on the scalar separation $r = |\mathbf{x}_1 - \mathbf{x}_2|$, so that [107, 110]

$$\xi(r) = \langle\delta(\mathbf{x})\delta(\mathbf{x} + \mathbf{r})\rangle. \quad (3.2)$$

For a Gaussian random field, as predicted by inflation (Section 2.5), all connected moments of order higher than two vanish identically, so the complete statistical information reduces to $\xi(r)$ alone [111].

The density contrast is, however, a continuous field that is not observed directly. Galaxy surveys provide only a discrete set of positions of galaxies, clusters, or quasars that trace the underlying luminous matter [110]. Such observations are more naturally described as a **spatial point process**, that is, a random distribution of discrete points in space [110, 113]. The simplest example is the **homogeneous Poisson process**, in which objects are distributed independently and uniformly throughout space with a constant mean number density $\bar{n}$, the average number of objects per unit volume, $\bar{n} = \langle N \rangle / V$. The probability of finding an object within a volume element δV is then

$$\delta P = \bar{n} \delta V, \quad (3.3)$$

and, for a random distribution, the presence of objects in disjoint volumes is statistically

independent. The joint probability of finding one object in δV_1 and another in δV_2 , separated by r , is therefore the product of the individual probabilities,

$$\delta P_{12}^{\text{Poisson}} = \bar{n}^2 \delta V_1 \delta V_2. \quad (3.4)$$

In the Universe, however, gravitational instability causes galaxies to cluster, making it more likely to find pairs at close separations in some regions (and less clustered in other) than this Poisson result predicts. The shift from that baseline is what the two-point correlation function quantifies.

3.2 Two-Point Correlation Function

The 2PCF function defined in Eq. 3.2 has an equivalent operational interpretation in terms of pair probabilities. Generalizing the Poisson result in Eq. 3.4 to include correlations, the joint probability of finding one object in δV_1 and another in δV_2 , separated by r , is [101, 111]

$$\delta P_{12} = \bar{n}^2 [1 + \xi(r)] \delta V_1 \delta V_2, \quad (3.5)$$

so that $\xi(r) = 0$ recovers the uncorrelated Poisson case (Eq. 3.4). Equivalently, given that an object is found in δV_1 , the conditional probability of finding a second in δV_2 is

$$\delta P(2|1) = \bar{n} [1 + \xi(r)] \delta V_2. \quad (3.6)$$

The sign of $\xi(r)$ distinguishes three physical regimes,

$$\xi(r) \begin{cases} > 0, & \text{clustering (excess of close pairs),} \\ = 0, & \text{Poisson distribution (no correlation),} \\ < 0, & \text{anticorrelation (deficit of pairs).} \end{cases} \quad (3.7)$$

On intermediate scales, the observed galaxy correlation function is well approximated by a power law [68, 114],

$$\xi(r) \simeq \left(\frac{r}{r_0} \right)^{-\gamma}, \quad (3.8)$$

where r_0 is the correlation length, which depends on the galaxy sample in analysis, and $\gamma \approx 1.8$ [110, 114] is a typical value for the slope of the correlation function.

In practice, $\xi(r)$ is estimated by comparing pair counts in the observed catalogue with those from a synthetic **random catalogue**, a realization of the homogeneous Poisson process with the same survey geometry and mean number density $\bar{n}$ as the data. One defines $DD(r)$, $DR(r)$, and $RR(r)$ as the numbers of data-data, data-random, and

random-random pairs, respectively, falling in a bin centered at r , each normalized by the total number of pairs of its kind. Several estimators combining these counts have been proposed in the literature [112, 115–118]. The most widely adopted is the Landy-Szalay estimator [118],

$$\xi(r) = \frac{DD(r) - 2DR(r) + RR(r)}{RR(r)}, \quad (3.9)$$

which was constructed to minimize the variance of the measurement. In a systematic comparison against cosmological simulations, Kerscher et al. [119] found the Landy-Szalay estimator to have the smallest deviation from the true correlation function, with a bias negligible compared to its variance. These properties make it particularly well suited to surveys with irregular footprints and masked regions.

When radial distances are unavailable or imprecise, or when one studies the projected distribution on the celestial sphere, the angular analogue of $\xi(r)$ is the **two-point angular correlation function** (2PACF) $w(\theta)$. In this case, the joint probability of finding two objects in solid-angle elements $\delta\Omega_1$ and $\delta\Omega_2$ separated by an angle θ is [111]

$$\delta P_{12} = \bar{n}^2 [1 + w(\theta)] \delta\Omega_1 \delta\Omega_2. \quad (3.10)$$

In practice, $w(\theta)$ is estimated from angular pair counts using the same estimators introduced above, with separations measured as angles rather than comoving distances. The 2PACF has been employed in astronomy and cosmology to investigate the large-scale distribution of matter, probe the statistical isotropy and homogeneity of the Universe, detect the BAO feature, and constrain cosmological models (see, e.g., [47, 120–124, 124–134]).

One of the most important features that can be detected in the 2PCF $\xi(r)$ is the BAO signal, which appears as a localized excess of galaxy pairs at a characteristic comoving separation of approximately $100 h^{-1} \text{ Mpc}$ [15, 18] (Figure 3.1). This scale corresponds to the acoustic waves signature imprinted in the primordial baryon-photon plasma prior to recombination, as discussed in Section 2.5. Because this scale is set by well-understood early-Universe physics, the BAO feature acts as a **standard ruler**: measuring its apparent size as a function of redshift constrains the expansion history and, with it, the properties of dark energy [18].

Finally, the two-point function is not the only statistic available for the analysis of density fields. Higher-order statistics, such as the three-point correlation function $\zeta(r_{12}, r_{23}, r_{31})$ [120] and its Fourier counterpart, the bispectrum [111], carry additional information about structure growth and primordial non-Gaussianity. These are, however, beyond the scope of the present analysis, which is based throughout on two-point statistics.

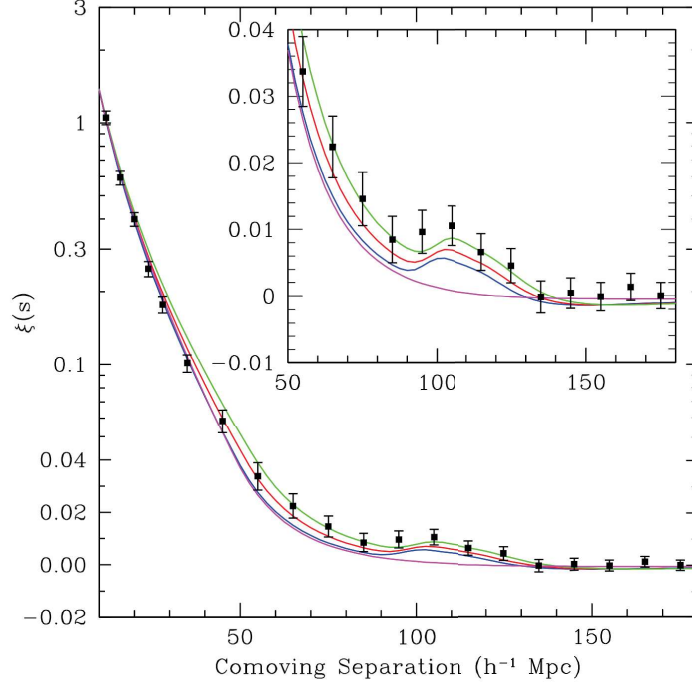


Figure 3.1: Two-point correlation function for a sample of luminous red galaxies from Sloan Digital Sky Survey (SDSS) as a function of comoving separation r , measured in units of h^{-1} Mpc. Credits: [18].

3.3 Power Spectrum

The 2PCF, from Eq. 3.2, characterizes the clustering of matter in configuration space. A complementary description is provided in Fourier space by the **power spectrum** $P(k)$ (defined in Eq. 2.132), which is the most natural object for theoretical predictions, since linear perturbation theory treats each Fourier mode independently. The two quantities form a Fourier-transform pair [17, 112],

$$\xi(r) = \frac{1}{(2\pi)^3} \int d^3k P(k) e^{i\mathbf{k}\cdot\mathbf{r}}, \quad (3.11)$$

$$P(k) = \int d^3r \xi(r) e^{-i\mathbf{k}\cdot\mathbf{r}}, \quad (3.12)$$

so they carry complementary statistical information, differing only in representation. Under statistical isotropy, these equations reduce to one-dimensional integrals involving the spherical Bessel function $j_0(kr) \equiv \sin(kr)/(kr)$ [111],

$$\xi(r) = \frac{1}{2\pi^2} \int_0^\infty dk k^2 P(k) \frac{\sin(kr)}{kr}, \quad (3.13)$$

$$P(k) = 4\pi \int_0^\infty dr r^2 \xi(r) \frac{\sin(kr)}{kr}. \quad (3.14)$$

Formally, the power spectrum is defined through the two-point function of the Fourier

modes of the density contrast. Statistical homogeneity implies that modes of different wavevectors are uncorrelated, while isotropy restricts the variance to depend only on the magnitude $k = |\mathbf{k}|$ [2, 17], as stated in Eq. 2.132. The power spectrum therefore quantifies the variance of density fluctuations contributed by modes of wavenumber k : the larger $P(k)$, the stronger the clustering at the corresponding scale k^{-1} .

As well as discussed for the 2PCF, the formalism above presupposes knowledge of the complete three-dimensional position of each tracer, which we do not necessarily have. In these cases, the relevant observable is instead the projected density contrast on the sphere. For a direction (θ, ϕ) on the sky, Eq. 3.16 is defined as

$$\delta(\theta, \phi) \equiv \frac{n(\theta, \phi) - \bar{n}}{\bar{n}}, \quad (3.15)$$

where $n(\theta, \phi)$ is the observed angular number density of objects. Because the full sphere admits a complete orthogonal basis of spherical harmonics $Y_{\ell m}(\theta, \phi)$, the projected field is expanded as [111]

$$\delta(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi), \quad (3.16)$$

with expansion coefficients

$$a_{\ell m} = \int d\Omega \delta(\theta, \phi) Y_{\ell m}^*(\theta, \phi). \quad (3.17)$$

The multipole ℓ plays on the sphere the role of the wavenumber k in a three-dimensional space: low ℓ corresponds to large angular scales and high ℓ to small angular scales, with the geometric correspondence $\theta = 180^\circ/\ell$.

Assuming statistical isotropy of the data observed on the sphere implies that harmonic coefficients at different multipoles are uncorrelated and that their variance is independent of m [17, 135],

$$\langle a_{\ell m} a_{\ell' m'}^* \rangle = C_\ell \delta_{\ell\ell'} \delta_{mm'}, \quad (3.18)$$

which defines the **angular power spectrum** C_ℓ , the spherical-harmonic analogue of $P(k)$. For each multipole ℓ there are $2\ell + 1$ independent modes m , and since these are the only realizations of that angular scale available within the single observable Universe, the fractional uncertainty of any estimator of C_ℓ is bounded below by [17]

$$\frac{\Delta C_\ell}{C_\ell} \geq \sqrt{\frac{2}{2\ell + 1}}, \quad (3.19)$$

an irreducible uncertainty known as **cosmic variance**, which is most significant at low ℓ .

Theoretically, C_ℓ is related to the three-dimensional power spectrum $P(k)$ through a

line-of-sight projection. For a galaxy sample with normalized redshift distribution $\phi(z)$ and linear bias $b(z)$, C_ℓ is given by [136–138]

$$C_\ell = \frac{2}{\pi} \int dk k^2 P(k) \Psi_\ell^2(k), \quad (3.20)$$

where the projection kernel,

$$\Psi_\ell(k) \equiv \int dz b(z) D(z) \phi(z) j_\ell[k\chi(z)], \quad (3.21)$$

encodes how three-dimensional fluctuations of wavenumber k are mapped onto the angular scale ℓ , with j_ℓ the spherical Bessel function of order ℓ , $\chi(z)$ the comoving distance to redshift z , and $D(z)$ the linear growth factor. The full computation is numerically demanding due to the rapidly oscillatory nature of j_ℓ [139, 140], which motivates the **Limber approximation** [141]. It approximates the spherical Bessel function by a Dirac delta centred at $x = \ell + 1/2$, that is $j_\ell(kr) \rightarrow \sqrt{\pi/(2\ell + 1)} \delta_D(\ell + 1/2 - kr)$ [140], establishing the correspondence $k \simeq (\ell + 1/2)/\chi(z)$,

$$C_\ell \approx \int_0^{z_{\max}} dz \frac{H(z)}{c \chi^2(z)} [\phi(z) b(z) D(z)]^2 P\left(\frac{\ell + 1/2}{\chi(z)}\right), \quad (3.22)$$

where the prefactor $H(z)/c$ arises from converting the integration variable via Eq. 2.76. The approximation is accurate on small angular scales (large ℓ) and for broad radial selection functions, but it degrades on large angular scales and for narrow redshift bins, where the full computation involving the Bessel integral (Eq. 3.20) is required [140].

The single preferred scale imprinted by BAO manifests itself differently in configuration and Fourier space. A localized bump in the two-point correlation function at the acoustic scale (r_d) corresponds to a series of oscillatory features in the power spectrum, superimposed on its smooth broadband shape [106]. The wavelength of these oscillations is inversely proportional to the sound horizon and the apparent size of the BAO feature depends on the geometry of the Universe.

As discussed in Section 2.5.5, the observed acoustic scale is measured relative to a template computed in a fiducial cosmology and is commonly parameterized through the shift parameters $(\alpha_\perp)$, $(\alpha_\parallel)$, and (α) defined in Eq. 2.130. These quantities measure the shift of the BAO feature with respect to its fiducial position and are directly related to cosmological distance measurements. Consequently, measuring the position of the acoustic peak in the 2PCF, or equivalently the phase of the oscillatory pattern in the power spectrum (at several redshift bins), provides a direct probe of the expansion history of the Universe.

Figure 3.2 illustrates this correspondence between the acoustic peak in the angular two-point correlation function and the oscillatory feature in the angular power spectrum. Both quantities are normalized by their corresponding smooth, non-oscillatory reference

spectra in order to isolate the BAO signal from the broadband clustering shape. In the analyses developed throughout this dissertation, the cosmological information is extracted from the shift in the position of these features, which are quantified through the parameter α .

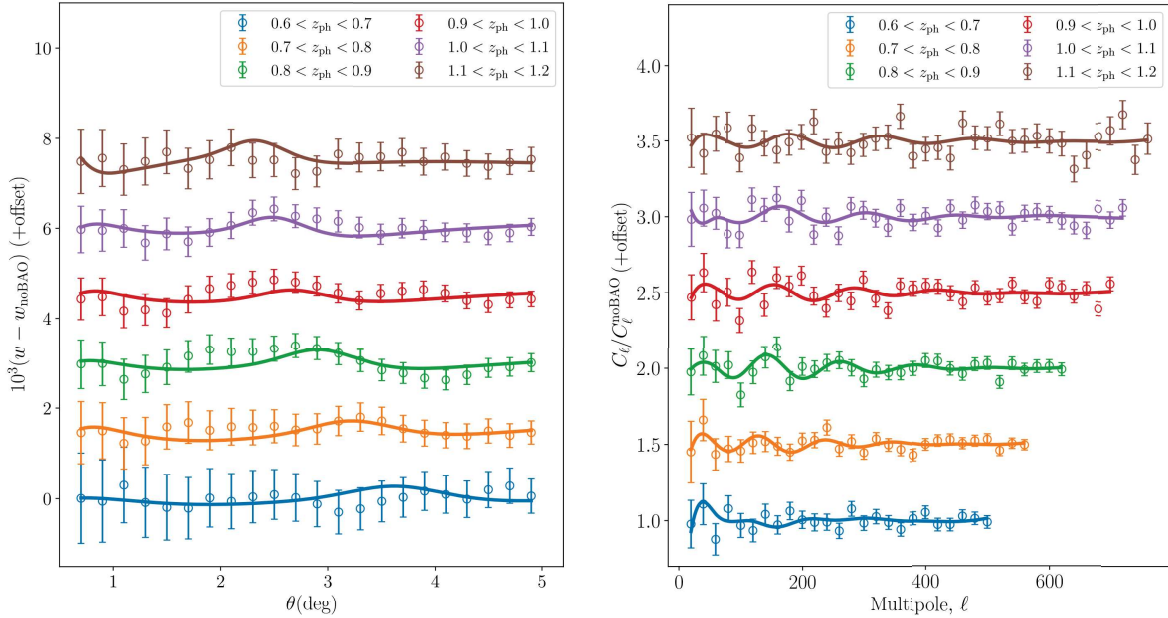


Figure 3.2: Isolated BAO feature measured in configuration space through the angular correlation function $[w(\theta)]$ (left) and in harmonic space through the angular power spectrum (C_ℓ) (right), for different tomographic redshift bins from the Dark Energy Survey. Measurements are shown with error bars, while the solid curves represent the best-fit BAO model. As the redshift increases, the BAO feature shifts toward smaller angular separations in $[w(\theta)]$ and correspondingly toward larger multipoles in (C_ℓ) , reflecting the approximately constant comoving size of the sound horizon projected onto progressively larger cosmological distances. Source: [23].

It is worth emphasizing, however, that the spherical harmonics form a complete orthogonal basis only over the full sphere. In realistic surveys, data is not available on the full sky, and regions within the footprint are further removed due to bright stars, observational problems, or Galactic contamination. The overdensity field is therefore observed only through a mask, and once the projection is no longer onto a full sphere the orthogonality of the spherical harmonics is broken: different multipoles become coupled, and a direct harmonic transform of the observed field yields a spectrum in which power from neighbouring scales is mixed [136]. Recovering an unbiased estimate of C_ℓ from partial-sky data therefore requires a dedicated estimator, the **pseudo- C_ℓ** method, whose formalism we describe next.

3.3.1 Pseudo- C_ℓ estimation

The pseudo- C_ℓ method estimates the true C_ℓ from a partial-sky field by modelling the masking effect analytically and correcting for it [135]. Considering two overdensity fields

associated with samples in the redshift bins i and j , the true, full-sky angular power spectrum is defined through the covariance of their harmonic coefficients [142],

$$\langle a_{\ell m}^i a_{\ell' m'}^{j*} \rangle \equiv C_{\ell}^{ij} \delta_{\ell\ell'} \delta_{mm'}, \quad (3.23)$$

which for a single sample ($i = j$) reduces to the auto-power spectrum C_{ℓ} of Eq. 3.18. This is the quantity we wish to recover. The incomplete coverage is described by a mask, or weight map, $W(\theta, \phi)$, which vanishes (or down-weights) wherever the field is not observed, so that what we actually access is the masked overdensity field,

$$\delta^W(\theta, \phi) \equiv W(\theta, \phi) \delta(\theta, \phi). \quad (3.24)$$

On the full sky, statistical isotropy implies that C_{ℓ}^{ij} is the same for all $2\ell + 1$ modes at fixed ℓ , so the spectrum can be estimated by averaging over the available m [135, 142],

$$\hat{C}_{\ell}^{ij} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} a_{\ell m}^i a_{\ell m}^{j*}. \quad (3.25)$$

In general, the two samples do not need to share the same coverage area, therefore each field has its own weight map. In practice, we only have the masked coefficients, so natural generalization replaces the full-sky coefficients with the masked ones, defining the pseudo- C_{ℓ} estimator [135, 142],

$$\tilde{C}_{\ell}^{ij} \equiv \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} a_{\ell m}^{i,V} a_{\ell m}^{j,W*}, \quad (3.26)$$

where $a_{\ell m}^{i,V}$ and $a_{\ell m}^{j,W}$ are the harmonic coefficients of fields i and j masked by V and W , respectively. The tilde indicates that this is not the quantity we want: the mask couples different multipoles, making $\tilde{C}_{\ell}^{ij}$ a biased, distorted version of C_{ℓ}^{ij} .

Taking the expectation value of Eq. 3.26 and using statistical isotropy, the average pseudo-spectrum is related to the true spectrum by a single linear operation,

$$\langle \tilde{C}_{\ell}^{ij} \rangle = \sum_{\ell'} M_{\ell\ell'} C_{\ell'}^{ij}, \quad (3.27)$$

where $\langle \cdot \rangle$ denotes the ensemble expectation value and $M_{\ell\ell'}$ is the **mode-coupling matrix**, whose off-diagonal terms quantify the leakage of power between scales caused by the missing sky [135]. Since it depends only on the mask geometry, it can be computed in advance, without knowledge of the data in study. For a scalar field, such as the galaxy overdensity, it is given in terms of the mask cross-power spectrum and the Wigner 3- j symbols [135, 136],

$$M_{\ell\ell'} = \frac{2\ell' + 1}{4\pi} \sum_{\ell''} (2\ell'' + 1) \tilde{C}_{\ell''}^{VW} \begin{pmatrix} \ell & \ell' & \ell'' \\ 0 & 0 & 0 \end{pmatrix}^2, \quad (3.28)$$

where $\tilde{C}_{\ell''}^{VW}$ is the cross-power spectrum of the masks. In the full-sky limit the mask becomes trivial, $M_{\ell\ell'}$ reduces to the identity, and the pseudo-spectrum coincides with the true one.

Because masking destroys information, $M_{\ell\ell'}$ generally cannot be inverted directly. The standard solution is to bin the coupled pseudo- C_ℓ , denoted $\tilde{C}_\ell^{ij}$, into **bandpowers** $\tilde{B}_q^{ij}$, each defined by a set of multipoles and normalized weights w_q^ℓ [135],

$$\tilde{B}_q^{ij} \equiv \sum_{\ell \in q} w_q^\ell \tilde{C}_\ell^{ij}. \quad (3.29)$$

where q indexes the bandpower. Assuming the true spectrum is approximately constant within each bandpower, one constructs the binned mode-coupling matrix $\mathcal{M}_{qq'}$, which is now invertible, yielding an unbiased estimate of the decoupled bandpowers [135],

$$B_q^{ij} = \sum_{q'} (\mathcal{M})_{qq'}^{-1} \tilde{B}_{q'}^{ij}. \quad (3.30)$$

Since the true spectrum is not exactly constant, any theoretical prediction must pass through the same chain of operations before being compared with the data [135].

Finally, for an auto-spectrum of a single sample ($i = j$, $v = w$) the expectation value receives a contribution from both signal and noise [135, 142],

$$\langle \tilde{C}_\ell \rangle = \sum_{\ell'} M_{\ell\ell'} (C_{\ell'} + N_{\ell'}), \quad (3.31)$$

where N_ℓ is the shot-noise contribution. For a galaxy survey the noise is dominated by Poisson sampling of discrete objects [111], so the shot-noise spectrum is white,

$$N_\ell = \frac{1}{\bar{n}}, \quad (3.32)$$

with $\bar{n}$ the mean angular number density of the sample. This bias is subtracted from the calculated $\langle \tilde{C}_\ell \rangle$ to obtain an unbiased estimate [135]. For cross-spectra between independent samples, however, the noise is uncorrelated and thus averages to zero, so this bias does not arise.

Taken together, the developments of this chapter provide the theoretical framework on which our work rests: the two-point statistics of the galaxy distribution, their representation in configuration and harmonic space, the imprint of the BAO as a standard ruler, and the procedures required to estimate the angular power spectrum from realistic, partial-sky observations. With these tools in place, we now present the data set used in our analyses, before describing in Chapter 5 how the estimator developed here is applied

in practice to extract the cosmological signal.

Chapter 4

Observational data

Modern observational cosmology relies on large astronomical surveys capable of mapping the distribution of objects across significant portions of the sky, providing the statistical power necessary for precise studies of the large-scale structure of the Universe. Stage IV surveys, such as the Legacy Survey of Space and Time (LSST) [143], the Dark Energy Spectroscopic Instrument (DESI) [144, 145], Euclid [146], and the Roman Space Telescope [147], represent the current generation of large cosmological surveys designed to achieve extremely high-precision measurements of cosmic expansion, large-scale structure and weak gravitational lensing. These are expected to increase both the area of the observed sky and the number of cosmic objects detected, producing datasets with unprecedented statistical precision. In this context, earlier surveys, such as the Sloan Digital Sky Survey (SDSS), played a fundamental role in the development and validation of the analytical techniques currently employed for studying the large-scale structure of the universe.

Within this context, the general features of SDSS and the data release used in this work are presented in Section 4.1. In Section 4.2, we describe the selection criteria adopted for the sample of blue galaxies used for the analysis.

4.1 The Sloan Digital Sky Survey

The Sloan Digital Sky Survey (SDSS) is one of the largest astronomical surveys ever conducted, combining wide-field imaging and spectroscopy to map the distribution of astronomical objects over a significant fraction of the sky. The survey is carried out with a dedicated 2.5m modified Ritchey–Chrétien wide-field telescope at Apache Point Observatory in New Mexico [93, 148]. The telescope was designed with a focal ratio of $f/5$, producing a flat 3° field of view with a plate scale of 16.51 arcsec/mm [148]. Imaging observations are obtained using a large mosaic CCD camera operating in drift-scan (time-delay-and-integrate) mode, in which the camera continuously scans the sky along great circles rather than taking discrete exposures. As a result, each point on the sky is observed

sequentially through five photometric filters [93].

The SDSS photometric system, referred to as *ugriz*, was specifically designed for the survey and covers the wavelength range from the near-ultraviolet to the near-infrared through five essentially non-overlapping passbands. The *u*, *g*, *r*, *i*, and *z* filters peak at approximately 3500 Å, 4800 Å, 6250 Å, 7700 Å, and 9100 Å, respectively [148]. In addition to the main telescope, SDSS also employs auxiliary instruments for photometric calibration and atmospheric monitoring, including the 0.5 m Photometric Telescope (PT), a seeing monitor, and a 10 μm cloud scanner [93].

Spectroscopic observations are carried out using a pair of multi-fibre spectrographs fed by fibres manually plugged into aluminium plates drilled for each observed field. The spectrographs cover the optical wavelength range with a spectral resolution of $\lambda/\Delta\lambda \sim 1800$ [148]. These measurements provide precise redshifts for galaxies, quasars, and stars, enabling three-dimensional mapping of the large-scale structure of the Universe.

Survey operations officially began in 2000 after the telescope received first light in 1998. Over two decades, the SDSS evolved through several observational phases. SDSS-I, operating until 2005, focused primarily on the Legacy Survey, consisting of wide-field imaging and spectroscopy of galaxies and quasars [93]. SDSS-II, conducted between 2005 and 2008, completed the Legacy Survey and expanded the scientific program to include repeated imaging observations for supernova searches and spectroscopic studies of the Milky Way stellar population [149]. SDSS-III, operating from 2008 to 2014, introduced several new surveys, including the Baryon Oscillation Spectroscopic Survey (BOSS), which significantly increased the spectroscopic galaxy sample used for cosmological analyses [149, 150].

Throughout its observational phases, SDSS released its data publicly through a sequence of Data Releases (DRs). The twelfth release, DR12, which will be used for the present work, contains the complete dataset from SDSS-III and includes imaging and spectroscopy covering more than 14,000 square degrees of the sky, primarily in the Northern Galactic Cap together with several disconnected regions in the south (see Figure 4.1). The DR12 catalog contains approximately 4.7×10^8 objects, including about 1.37×10^6 spectroscopically confirmed galaxies [149].

The combination of large sky coverage, homogeneous photometry, and extensive spectroscopic observations made SDSS one of the most influential surveys in modern observational cosmology. Its galaxy samples enabled high-precision studies of the large-scale structure of the Universe, including measurements of galaxy clustering, BAO and cosmological parameters analyses. In particular, SDSS data played a central role in the first robust detections of the BAO feature in the galaxy distribution [18].

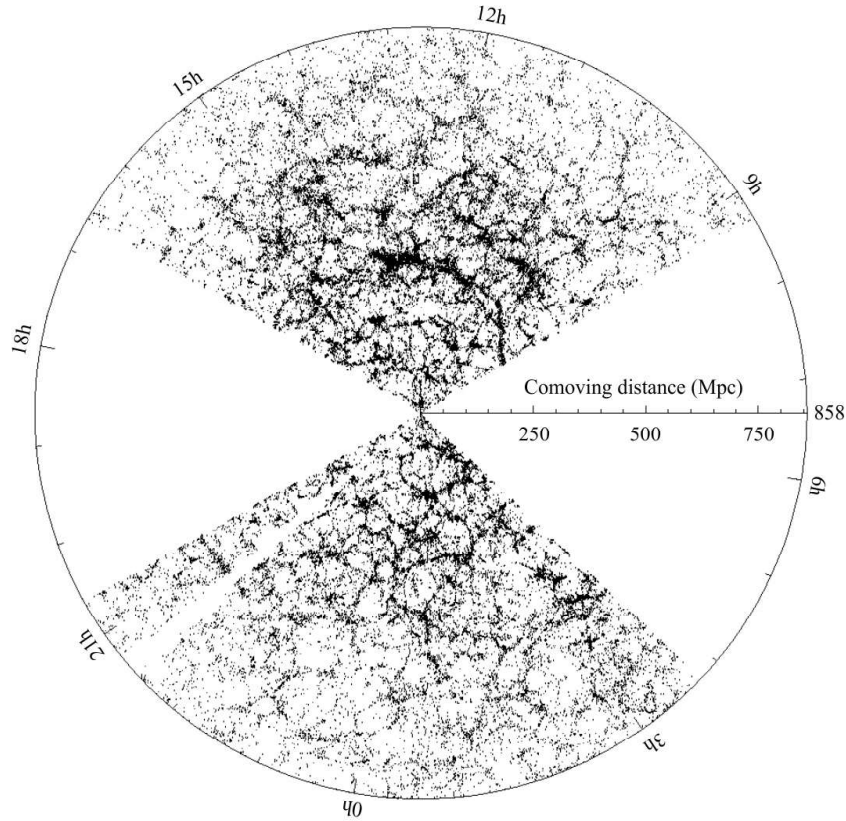


Figure 4.1: Distribution of SDSS objects in shown as a function of right ascension (in hours) and comoving distance (out to ≈ 858 Mpc). The filaments, clusters, and voids trace the large-scale structure of the cosmic web. Source: Adapted from [?].

4.2 Blue Galaxies Sample

The galaxy population exhibits a well-defined bimodality in color, revealed when the color distribution, quantified by indices such as $(u-r)$ and $(g-r)$, is represented as a histogram or in a color-magnitude diagram: two well-defined peaks appear, separated by a sparsely populated transition region known as the Green Valley, and this pattern is consistently observed in large photometric surveys, including the SDSS [151?]. Figure 4.2 shows the bimodal color distribution of the SDSS DR12 galaxy sample from which the data used for our analyses were selected.

The two peaks correspond to two physically distinct populations. The peak at lower color values defines the blue cloud: galaxies with relatively young stellar populations, high star formation rates, and predominantly late-type morphologies such as spirals and irregulars [152?]. At higher color values lies the red sequence, populated by galaxies with older stars, low star formation activity, and predominantly early-type morphologies, such as elliptical and lenticular galaxies [102, 153? , 154]. The Green Valley between them is interpreted as a transition region occupied by galaxies evolving from active to quiescent star formation regimes [155, 156]. Although this connection between color and morphology provides a useful framework, it is not absolute. Dust extinction can redden

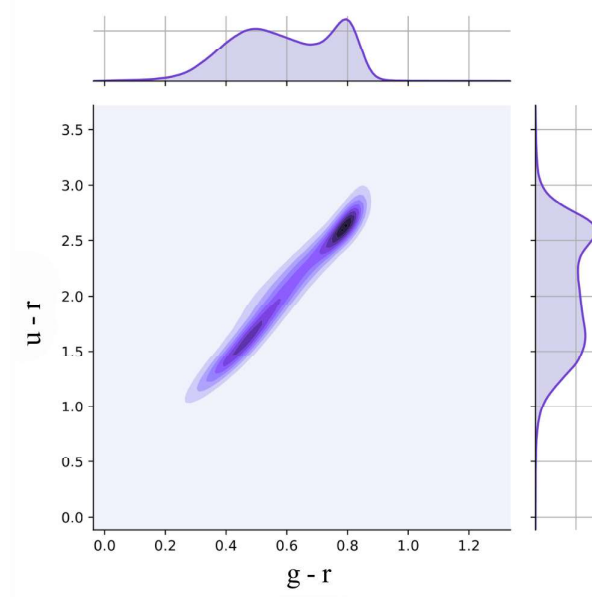


Figure 4.2: Distribution of the SDSS DR12 galaxies in the $(u - g)$ versus $(r - i)$ color-color plane after the application of K-corrections, as well as Galactic and intrinsic extinction corrections. The figure shows the full galaxy sample prior to the color-based classification into blue and red populations. The bimodal distribution reflects the well-known separation between star-forming blue galaxies and more passive red galaxies, providing the basis for the color selection adopted in this work. Source: Adapted from [44].

an intrinsically blue late-type galaxy, and a recent stellar explosion can temporarily bluen an otherwise quiescent early-type galaxy [102]. Furthermore, clustering analyses confirm that the two populations are not merely distinct in colors, but occupy statistically different large-scale environments, with blue galaxies residing preferentially in lower-density regions and exhibiting weaker spatial correlations than their red counterparts [148, 157, 158].

Among the available color indices, the $u - r$ color is particularly well suited for separating these populations, being sensitive to the contrast between the ultraviolet emission from young stars and the redder continuum of evolved populations [151].

In the present work the separation of the blue galaxies is refined through the joint color selection

$$0.0 < g - r < 0.6, \quad 0.0 < u - r < 2.0, \quad (4.1)$$

applied to rest-frame colors that account for Galactic extinction, the k -correction, and intrinsic dust reddening. This color-color calibration was carried out in Avila et al. 2019 [45] and is summarized here for completeness.

First, the magnitudes were converted to the rest frame through the K-correction, which removes the effect of the redshifting of the galaxy spectral energy distribution (SED) and enables a consistent comparison of galaxy luminosities and colors across different redshifts [159]. The K-correction was computed using the Python implementation of the calculator developed by Chilingarian et al. [160, 161].

The photometry was also corrected for Galactic extinction, which arises from the absorption and scattering of light by interstellar dust in the Milky Way. This foreground attenuation was estimated from the reddening maps of Schlegel et al. [162], which provide the dust column along each line of sight.

In addition to foreground extinction, blue star-forming galaxies are also affected by internal dust attenuation. Since ongoing star formation occurs within dusty molecular clouds, a significant fraction of the emitted radiation is absorbed and scattered before escaping the galaxy [?]. As a consequence, galaxies appear systematically redder and fainter than they would in the absence of dust. To recover the intrinsic colors of the galaxies in the sample, the SDSS u , g , and r magnitudes were corrected for internal extinction using the observed H_α and H_β emission-line fluxes [44, 163].

The color excess $E(B-V)$ was estimated from the Balmer decrement following Calzetti et al. [164],

$$E(B-V) = 0.935 \ln \left(\frac{H_\alpha/H_\beta}{2.88} \right) \times 0.44. \quad (4.2)$$

The resulting colour excess was then converted into extinction in each SDSS photometric band according to

$$A_\lambda = k(\lambda) E(B-V), \quad (4.3)$$

where $k(\lambda)$ is the attenuation curve proposed by Calzetti et al. [165],

$$k(\lambda) = \begin{cases} 1.17 \left(-2.156 + \frac{1.509}{\lambda} - \frac{0.198}{\lambda^2} + \frac{0.011}{\lambda^3} \right) + 1.78, & 0.12 \leq \lambda \leq 0.63, \mu\text{m}, \\ 1.17 \left(-1.857 + \frac{1.040}{\lambda} \right) + 1.78, & 0.63 < \lambda \leq 2.20, \mu\text{m}. \end{cases} \quad (4.4)$$

After applying the K-correction and extinction correction, one obtains the intrinsic magnitudes and colors for all galaxies in the sample.

In addition to the photometric selection, a set of geometrical cuts was applied to remove regions with low completeness and to define a homogeneous survey footprint suitable for clustering analyses, considering only the Northern Galactic Hemisphere (NGH). The excluded regions are defined by

$$\text{RA} > 250^\circ \quad \text{and} \quad \text{Dec} > 50^\circ, \quad (4.5)$$

$$100^\circ < \text{RA} < 300^\circ, \quad \text{Dec} > -3^\circ, \quad (4.6)$$

$$\text{RA} > 260^\circ \quad \text{and} \quad \text{Dec} < 20^\circ, \quad (4.7)$$

where RA and Dec denote the right ascension and declination, respectively. The resulting footprint and redshift distribution of the sample, before and after the application of the geometrical selection criteria, are shown in Figures 4.3 and 4.4. The final catalog contains 255 456 galaxies distributed over an area of approximately $7\,383.3\,\text{deg}^2$, corresponding to a sky fraction of $f_{\text{sky}} = 0.179$. Because the sample spans the redshift interval $0.0 \leq z \leq 0.3$, the comoving volume is, approximately, $V \simeq 4.35 \times 10^8 h^{-3} \text{Mpc}^3$.

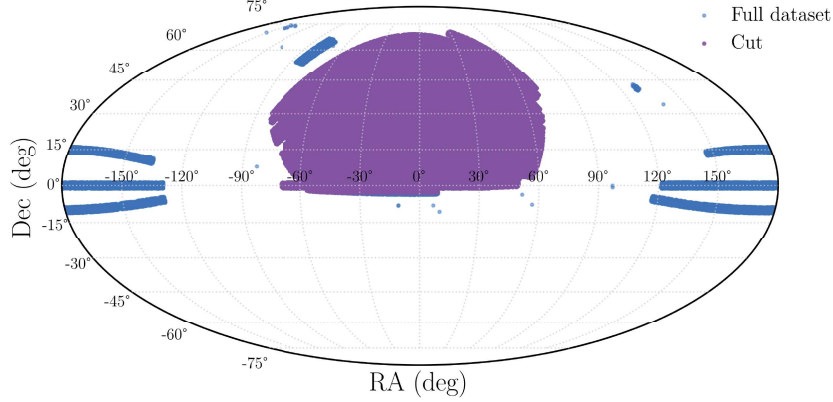


Figure 4.3: Mollweide projection of the blue-galaxy sample in right ascension (RA) and declination (DEC). The blue points show the full catalog, whereas the purple points indicate the galaxies retained after the geometrical selection. The adopted cuts remove the SGH, isolated structures and regions with irregular angular coverage, producing a more homogeneous survey mask for the estimation of the angular power spectrum and subsequent BAO analysis.

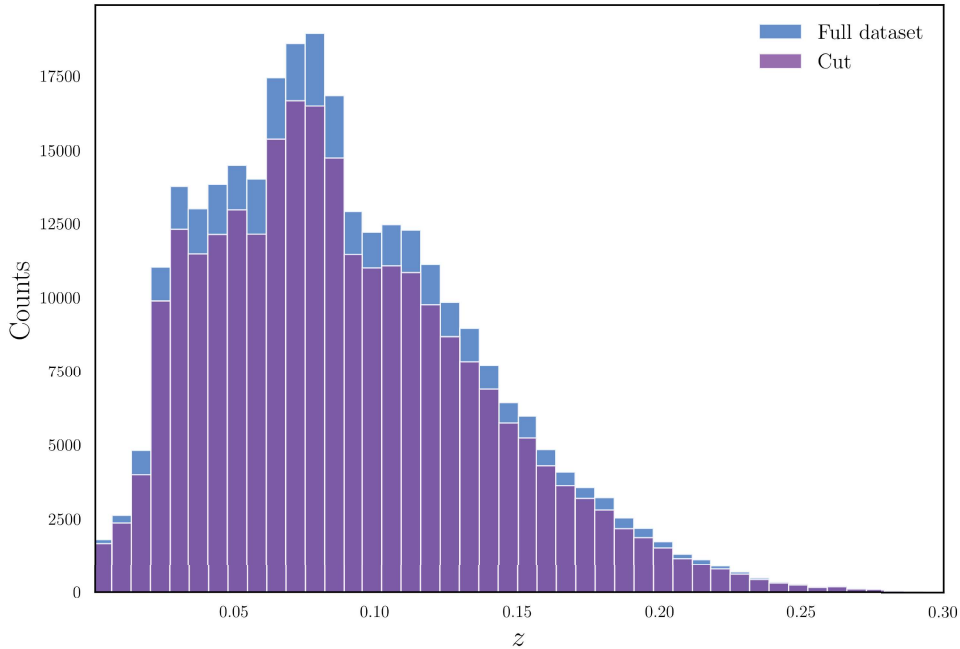


Figure 4.4: Histogram of galaxy redshifts for the full blue-galaxy catalog (blue) and for the final sample after the application of the geometrical cuts (purple).

The choice of a sample of blue, star-forming galaxies in the local Universe is intentional and aims to fill a specific gap in BAO studies. Most BAO measurements target luminous

red galaxies for two reasons: they are highly biased tracers of the underlying matter field, which increases the clustering amplitude and therefore the signal-to-noise ratio of the acoustic feature, and they are typically observed at intermediate to high redshifts where large moving volumes are available [18, 166–168]. In contrast, the BAO signal in the very local Universe remains poorly explored, both because the accessible volume is smaller and because the lower bias of typical local tracers weakens the signal [169–172].

In this context, blue galaxies are a less common choice because their weaker clustering makes the acoustic feature more difficult to detect. Measuring BAO with such a cosmic tracer is therefore valuable as an independent and complementary probe, since it tests the validity of the BAO phenomenon as a standard ruler analyzing a sample with a different bias and environment tracer. Furthermore, existing measurements with blue galaxies have been performed almost exclusively in configuration space, using the correlation function [171, 173?]. The present work, instead, analyzes the blue galaxies in the Fourier space, through the angular power spectrum described in Section 3.3.

The use of angular statistics constitutes the second methodological motivation. The three-dimensional correlation function and the power spectrum require the conversion of measured redshifts into comoving distances, a step that presupposes a fiducial cosmological model and can impose model-dependent biases on the inferred clustering. The angular power spectrum C_ℓ (and its counterpart in the configuration space $w(\theta)$), instead, requires only angular positions and redshifts, without any assumed distance-redshift relationship [23, 136, 138, 174]. The recovered acoustic scale, therefore, depends only weakly on the assumed cosmology, which makes the angular approach a robust and weakly model-dependent route to the BAO scale. Applying it to a local sample of blue galaxies combines a relatively unexplored tracer and redshift regime with a method that minimizes cosmological assumptions.

A third, complementary motivation comes from the existing body of work within our own group on this same sample of blue galaxies. Previous studies had already characterized the angular correlation function $\omega(\theta)$ for this tracer [171, 172], establishing its basic clustering properties in configuration space. Rather than repeating that measurement, the present work extends the characterization of the sample to harmonic space, through the angular power spectrum, providing a complementary estimator built on the same underlying dataset.

Chapter 5

Methodology

As discussed in previous chapters, the angular power spectrum C_ℓ is one of the most useful statistics used to characterize the large-scale distribution of matter and to extract cosmological information from galaxy surveys. In the context of this work, the goal is to use the angular clustering of the selected blue galaxy sample to identify the BAO feature and measure its characteristic scale. Achieving this, however, is a non-trivial task: the survey covers only a fraction of the sky, the galaxy distribution is a discrete realization of the underlying density field, and the statistical uncertainties must be accurately quantified. The observed power spectrum must therefore be complemented with realistic covariance estimates and theoretical modeling in order to constrain the acoustic scale and its associated uncertainty. This chapter presents the complete analysis pipeline adopted to accomplish this task.

Section 5.1 characterizes the sample, defining the redshift bins, the angular mask, and the redshift distributions used throughout. Section 5.2 presents the estimation of the angular power spectrum in the presence of incomplete sky coverage. Section 5.3 describes the generation of the mock catalogs employed for uncertainty estimation and pipeline validation, and Section 5.4 the construction of the covariance matrix. Finally, Section 5.5 describes the template fit to the measured spectrum, the extraction of the acoustic scale, and the resulting cosmological inference.

5.1 Sample Characterization

Following the photometric and geometric selection described in Section 4.2, the resulting catalog is divided into two redshift bins of equal width $\Delta z = 0.1$,

$$z_1 \in [0.05, 0.15], \quad z_2 \in [0.15, 0.25], \quad (5.1)$$

which are analyzed independently throughout this work. The higher-redshift bin ($0.15 \leq z \leq 0.25$) constitutes the main analysis sample, as it provides the largest effective volume

and therefore the highest statistical sensitivity to the BAO feature. The lower-redshift bin ($0.05 \leq z \leq 0.15$) is treated as an internal consistency and robustness check of the pipeline, allowing us to test the stability of the methodology in a regime with reduced comoving volume and weaker signal-to-noise.

The choice of bin width reflects a compromise between two competing requirements. A robust detection of the BAO feature requires a sufficiently high number density of galaxies per shell, which favors wider bins. At the same time, the transverse BAO signal is extracted from angular clustering, and the imprint of the acoustic scale is progressively washed out as the width of the shell increases; to preserve sensitivity, thin bins are therefore generally preferred [174]. The adopted width $\Delta z = 0.1$ was found to provide a balanced compromise between galaxy and comoving volume, optimizing the signal-to-noise ratio of the angular power spectrum in each bin.

For each bin we computed the comoving volume subtended by the corresponding spherical shell, restricted to the surveyed sky fraction, and the mean comoving number density of galaxies $\bar{n}$. These quantities are summarized in Table 5.1. The lower-redshift bin contains more galaxies but probes a smaller comoving volume, whereas the higher-redshift bin spans a larger volume with fewer objects, so the mean number density drops by more than an order of magnitude between the two bins. The number density also sets the shot-noise level (Eq. 3.32), whose contribution is not subtracted from the measured spectra but is instead absorbed at the fitting stage, as described in Section 5.2.

Redshift bin	N_{gal}	Volume [Mpc ³]	$\bar{n}$ [Mpc ⁻³]
[0.05, 0.15]	167 822	1.89×10^8	8.86×10^{-4}
[0.15, 0.25]	29 124	6.48×10^8	4.49×10^{-5}

Table 5.1: Summary of the two redshift bins used in this analysis, including the number of galaxies, comoving volume, and mean number density.

The angular selection function of the sample is encoded in a binary HEALPIX mask, in which every pixel containing at least one galaxy is assigned the value unity and all remaining pixels are set to zero. The footprint is initially defined at a resolution of $N_{\text{side}} = 64$. At finer resolutions, many pixels are left empty within the surveyed region because no galaxies fall on them, producing a mask with holes that would erroneously remove observed area. A pixel scale of $N_{\text{side}} = 64$ is sufficiently coarse that essentially every pixel inside the footprint is occupied, so that the mask faithfully traces the survey coverage rather than the discrete sampling of the tracers.

For the generation of the mock catalogs and the subsequent covariance estimation and statistical analysis, the mask is upgraded to $N_{\text{side}} = 512$ by means of the `ud_grade` routine of `healpy`. Although power spectrum analysis is limited to low multipoles, values for which, formally, a coarser resolution would suffice, finer pixelization is retained to precisely define the survey boundary and small-scale masked regions when generating

simulated catalogs, avoiding spurious blocky edges that a coarser grid would introduce. This operation preserves the surveyed area and sky fraction exactly, subdividing each coarse pixel into finer ones without modifying the underlying geometry. The properties of both masks are summarized in Table 5.2 and the resulting footprint is shown in Figure 5.1.

N_{side}	f_{sky}	Valid pixels	Comment
64	0.179	8 797	Base mask (geometry definition)
512	0.179	563 008	Upgraded-resolution mask

Table 5.2: Summary of the HEALPIX angular masks used in this analysis. The high-resolution mask is obtained by upgrading the low-resolution map with `ud_grade`, preserving the survey footprint while increasing the angular resolution.

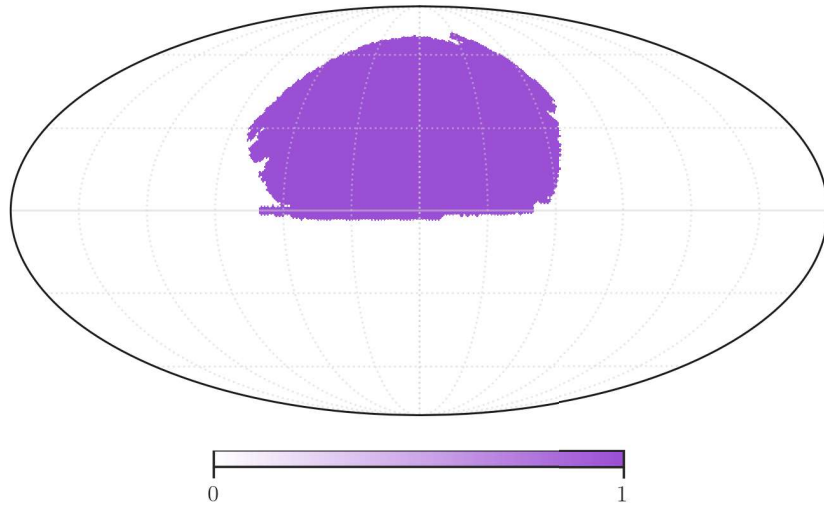


Figure 5.1: Binary HEALPIX survey mask in Mollweide projection. The colored region ($w = 1$) marks the surveyed footprint after the geometric selection; the white region denotes unobserved sky ($w = 0$).

For the present sample at low effective redshift ($z_{\text{eff}} \sim 0.1$ and $z_{\text{eff}} = 0.2$), the BAO feature is expected to appear at low multipoles, corresponding to large angular scales, since the angular diameter distance is small. Therefore, the base resolution $N_{\text{side}} = 64$ is sufficient to resolve C_ℓ over the multipole range of interest, while the upgraded mask provides the finer sampling required to generate the mock maps without reintroducing the holes described above.

Finally, the redshift distribution of the galaxy sample enters the analysis through two related, but distinct, quantities. The first is the redshift distribution within each bin, $n(z)$, which describes the normalized number of galaxies as a function of redshift in that bin. In this work, $n(z)$ is constructed separately for each redshift interval using histogram distributions of the galaxy redshifts. For a given bin $[z_{\text{min}}, z_{\text{max}}]$, the distribution is obtained by dividing the redshift range into fine intervals and computing the corresponding galaxy counts per unit redshift.

Another quantity required for the analysis is the angular number density dN/dz , expressed in galaxies per arcmin^{-2} per redshift interval. In contrast to $n(z)$, which is constructed separately for each redshift bin, dN/dz is computed from the full sample over the entire range $0.0 \leq z \leq 0.3$, since the mock catalogs are generated across this whole interval before any redshift cut is applied. The effective survey area is obtained directly from the binary mask: with N_{obs} occupied pixels at resolution N_{side} , each subtending a solid angle $\Omega_{\text{pix}} = 41253 \text{ deg}^2/N_{\text{pix}}$, the surveyed area is $\Omega = N_{\text{obs}} \Omega_{\text{pix}}$, corresponding to a sky fraction $f_{\text{sky}} = N_{\text{obs}}/N_{\text{pix}}$ and to $\Omega = 4\pi f_{\text{sky}}$ in steradians. Counting the n_i galaxies in each redshift interval of width Δz , the angular number density is

$$\frac{dN_i}{dz_i} = \frac{n_i}{\Delta z \Omega}, \quad (5.2)$$

with Ω expressed in arcmin^2 . This quantity sets the expected number of galaxies per shell and is used directly to populate the mock catalogs (Section 5.3). The normalized redshift distributions $n(z)$ for the two bins, together with the angular number density dN/dz of the full sample, are shown in Figures 5.2 and 5.3, respectively.

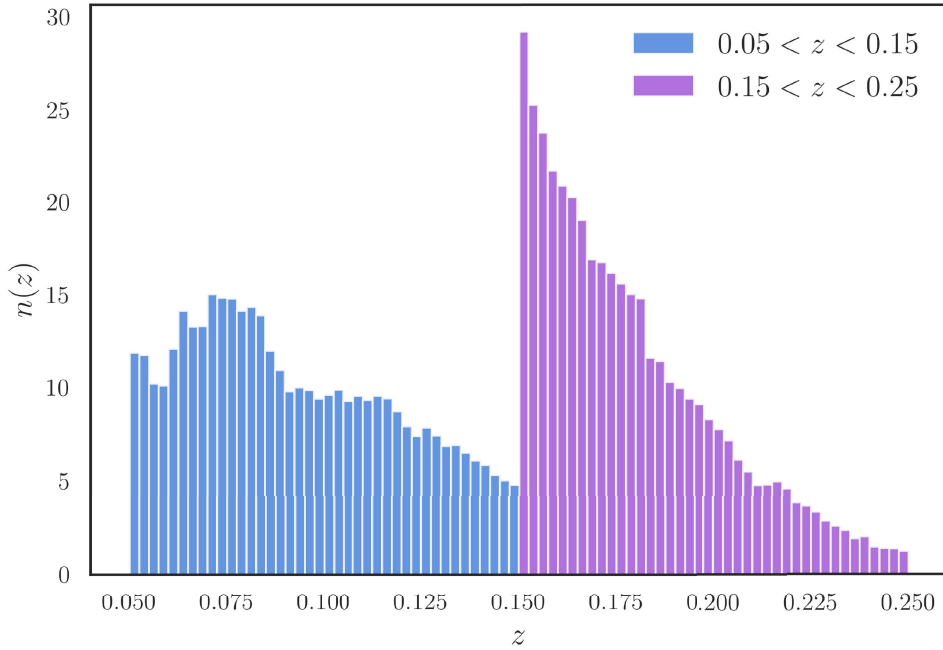


Figure 5.2: Normalized redshift distribution $n(z)$ for the two redshift bins, $0.05 < z < 0.15$ and $0.15 < z < 0.25$, used in the angular clustering analysis. Each distribution is a histogram of the galaxy redshifts normalized to unit integral.

5.2 Measuring the Angular Power Spectrum

Combining the previous results, the complete procedure to estimate the angular power spectrum of the galaxy overdensity map from partial-sky data is as follows:

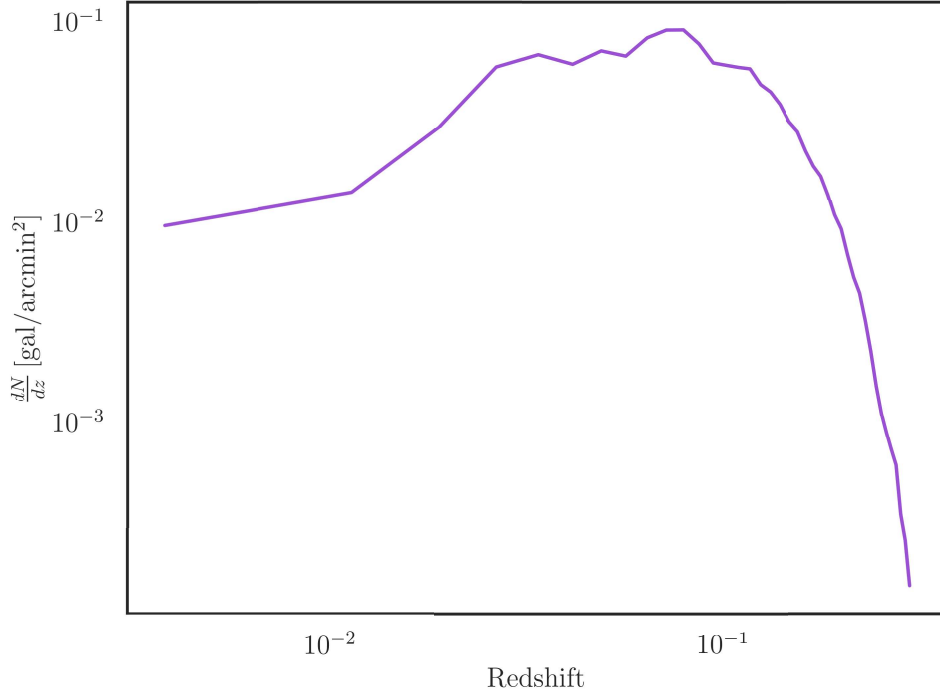


Figure 5.3: Angular number density dN/dz of the full sample over $0.0 < z < 0.3$, in gal arcmin^{-2} per redshift interval, used to populate the mock catalogs.

1. **Construct the field.** Build the masked overdensity map from the galaxy counts (Eq. 3.24) and compute its harmonic coefficients $a_{\ell m}^w$ (Eq. 3.17).
2. **Compute the coupled pseudo-spectrum.** Estimate the pseudo- C_ℓ from the masked harmonic coefficients (Eq. 3.26).
3. **Build the mode-coupling matrix.** From the mask power spectrum, compute $M_{\ell\ell'}$ (Eq. 3.28), which describes how the mask mixes the multipoles.
4. **Bin into bandpowers.** Average the coupled spectrum into bandpowers using the chosen weights (Eq. 3.29).
5. **Decouple.** Invert the binned mode-coupling matrix $\mathcal{M}_{qq'}$ and apply it to the binned spectrum to obtain the final unbiased bandpowers (Eq. 3.30).

All these operations are implemented in the public software package NAMASTER¹ [135], which provides a unified framework for estimating angular power spectra from partial-sky data. The code is written in C with a Python interface (`pymaster`) and is organized around two main objects: a *field* object, which stores the mask and the observed overdensity map, and a *workspace* object, which computes and stores the mode-coupling matrix. We use NAMASTER to estimate all the angular power spectra presented in Chapter 6.

¹<https://github.com/LSSTDESC/NaMaster>

Starting from the HEALPIX count map, we build the fluctuation field of Eq. 3.15 within the surveyed footprint, which is the quantity for which the pseudo- C_ℓ formalism of Section 3.3.1 is defined. The overdensity maps of the two samples, shown in Figure 5.4, are pixelized at $N_{\text{side}} = 512$ to match the upgraded survey mask.

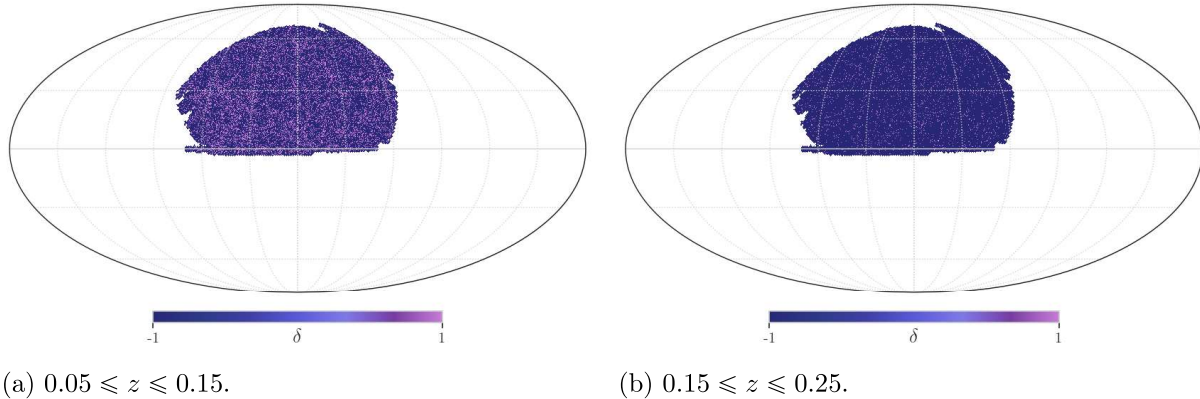


Figure 5.4: Galaxy overdensity field $\delta(\hat{\mathbf{n}})$ in Mollweide projection, constructed at $N_{\text{side}} = 512$ within the survey footprint, for the two redshift bins.

The effect of the mask on the recovered spectrum is encoded in the mode-coupling matrix $M_{\ell\ell'}$, computed from the power spectrum of the mask alone. Figure 5.5 shows its structure over the low-multipole range relevant to this work. The matrix is strongly diagonal-dominant, so most of the power at a given multipole is preserved at that multipole, while the power of the off-diagonal terms, which represent the leakage of power between neighboring multipoles induced by the incomplete sky coverage, fall off rapidly away from the diagonal. This locality of the coupling justifies the binning into bandpowers: since the mask mixes only nearby multipoles, the true spectrum can be taken as approximately constant within each band, making the binned mode-coupling matrix $\mathcal{M}_{qq'}$ invertible and the decoupled bandpowers unbiased.

For the first bin, with $z_{\text{eff}} = 0.1$, the coupled pseudo-spectrum is binned into bandpowers of width $\Delta\ell = 4$. The relatively narrow binning is motivated by the low redshift of this sample: since the acoustic feature is expected at correspondingly low multipoles, a finer binning avoids averaging the signal over a range of ℓ too broad, thereby diluting the signal. For the second bin, with $z_{\text{eff}} = 0.2$, a width of $\Delta\ell = 8$ is adopted, since it gave the most stable and best-constrained acoustic scale among the binning widths tested. The sensitivity of the results to the binning choice for both bins was assessed by repeating the full pipeline for bandpower widths $\Delta\ell = 4, 6, 8, 10, 12, 16$ and 20, and the recovered acoustic scale for the main bin is compared in Appendix B.

Finally, the shot-noise contribution arising from the discrete sampling of galaxies is not subtracted from the measured spectrum. Instead, it is modeled at the fitting stage as an additive term in the template used to extract the acoustic scale, and is therefore marginalized over together with the broadband shape of the spectrum.

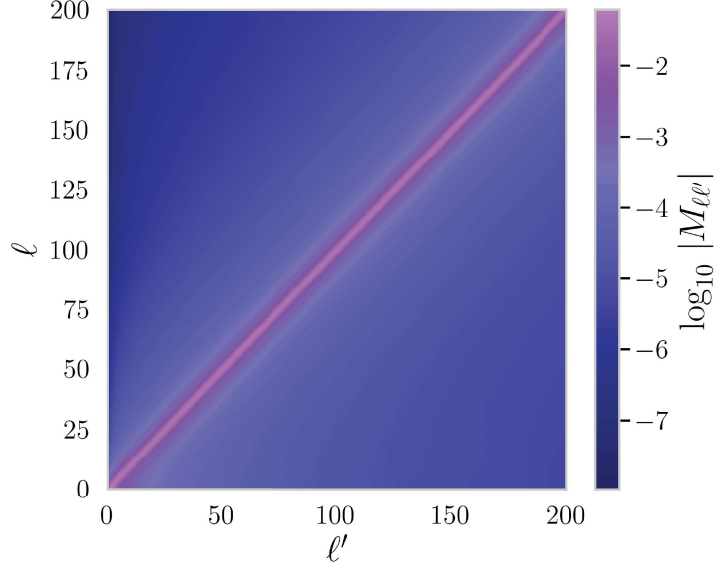


Figure 5.5: Mode-coupling matrix $M_{\ell\ell'}$ induced by the survey mask, over $0 \leq \ell \leq 200$. The strong diagonal indicates that power is mostly preserved at each multipole, while the off-diagonal terms quantify the leakage between neighboring multipoles caused by the partial-sky coverage.

5.3 Simulated Galaxy Catalogues: GLASS

Mock galaxy catalogs play a fundamental role in modern cosmological analyses. They are widely used to validate data analysis pipelines, estimate covariance matrices, quantify statistical uncertainties, and test the robustness of parameter-inference methods. Since observational data provide only a single realization of the Universe, simulations are essential for characterizing the expected statistical fluctuations of cosmological observables.

Cosmological simulations can generally be divided into two broad categories: physical and statistical simulations. Physical simulations, such as N -body and hydrodynamic simulations, model the evolution of matter by directly solving the gravitational and fluid equations governing the formation of cosmic structures [175–177]. These simulations provide highly realistic descriptions of the matter distribution and its non-linear evolution, but they are computationally expensive, particularly for large cosmological volumes.

An alternative is provided by statistical simulations, in which realizations of cosmological observables are generated directly from their statistical properties rather than from the full dynamical evolution of matter. These methods are far more computationally efficient and are therefore widely used to generate large ensembles of mock catalogs for covariance estimation and simulation-based inference [178–180].

In this work we employ the public code GLASS² (Generator for Large Scale Structure), designed to produce realizations of the large-scale matter distribution that reproduce a prescribed angular power spectrum while remaining computationally inexpensive [180].

The simulated sky is constructed by discretizing the observer’s past light cone into a

²<https://glass.readthedocs.io/stable/>

series of concentric spherical shells, each defined by a redshift interval. The matter density contrast within each shell is projected onto the celestial sphere through an appropriate radial window function, and the resulting field is discretized on a HEALPIX grid of chosen resolution N_{side} (Figure 5.6). The statistical properties of the projected field are determined by the theoretical angular power spectra, computed from a fiducial cosmological model and used as input for the simulation.

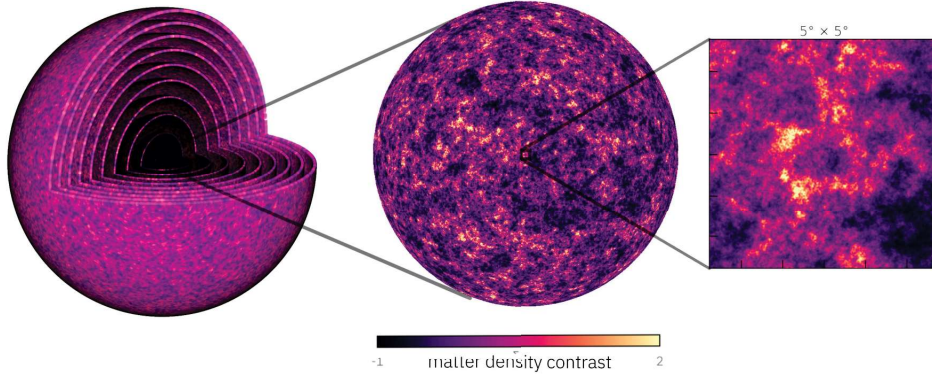


Figure 5.6: Matter density field generated with GLASS. The distribution is discretized into ten redshift shells spanning $0 \leq z \leq 1.0$, each of width $\Delta z = 0.1$. The left panel illustrates the nested shell structure, the center panel shows a hemisphere of the innermost shell at full resolution ($N_{\text{side}} = 8192$), and the right panel a $5^\circ \times 5^\circ$ zoom highlighting the small-scale features. Source: [180].

To generate the density fields, GLASS first produces correlated Gaussian random fields whose angular power spectra match the theoretical prediction. These are subsequently transformed into lognormal fields, introducing a realistic level of non-Gaussianity while preserving the desired two-point statistics [180]. Building on this framework, we generate mock catalogs designed to reproduce both the cosmological signal and the observational characteristics of our sample. The simulation is specified by two ingredients, the fiducial cosmological model and the observational configuration of the survey, both summarized in Table 5.3. The fiducial model is a flat Λ CDM cosmology whose matter power spectrum is computed with CAMB [181].

Table 5.3: Survey configuration and cosmological parameters used to generate the mock catalogues. A linear window function is used, with $h = 0.6736$. All cosmological parameters are taken from the *Planck* 2018 results [15], using the TT,TE,EE+lowE+lensing likelihood combination.

Survey configuration	Cosmological parameters
$z_{\text{grid}} = [0.0-0.3]$	$\Omega_c h^2 = 0.1200$
$n_c = 3$	$\Omega_b h^2 = 0.02237$
$b = 1.0$	$n_s = 0.9649$
$N_{\text{side}} = 512$	$\ln(10^{10} A_s) = 3.044$
$\ell_{\text{max}} = 1024$	$\Sigma m_\nu = 0.06 \text{ eV}$

Following the procedure described above, the light cone between $z = 0$ and $z = 0.3$ is

sliced into concentric spherical shells, each described by a top-hat radial window function, with the shell spacing set to $\Delta x = 100$ Mpc in comoving distance. For every pair of shells the theoretical angular power spectrum is evaluated from the matter power spectrum and discretized onto the HEALPIX grid at $N_{\text{side}} = 512$, retaining cross-correlations only between the $n_c = 3$ nearest shells. Each shell is then realized as a lognormal field and populated with galaxies according to a constant linear galaxy bias $b = 1$, applied through the relation $\delta_g = b \delta_m$ (Eq. 2.134). The monopole of the overdensity field is removed before galaxy positions are drawn (Eq. 2.131), ensuring that the mean density within the surveyed footprint is consistent with the input dN/dz .

To assess the sensitivity of the results to the treatment of non-linear evolution, two independent ensembles of $N_m = 1000$ catalogues were generated, differing only in the input matter power spectrum:

- **Linear ensemble:** the angular power spectra are computed from the linear matter power spectrum (`NonLinear=NonLinear_none` in CAMB). This prescription preserves the full amplitude of the BAO and is the most directly connected to the linear perturbation theory of Section 2.5. It is well suited to the present sample, since blue galaxies are weakly biased tracers ($b \simeq 1$) that preferentially inhabit low-density environments where non-linear clustering is suppressed [172, 182? –184].
- **Non-linear ensemble:** the angular power spectra are computed with `NonLinear=NonLinear_both`, which applies the HALOFIT prescription [185] to both the matter transfer function and the lensing potential. This introduces realistic non-linear mode coupling and BAO damping into the input spectra, producing mock catalogs whose two-point statistics more closely resemble those of the observed universe at the multipoles relevant to this work.

All remaining simulation parameters, such as the shell spacing, cosmological parameters, bias model, survey mask, angular number density dN/dz , HEALPIX resolution, and random-seed structure, are identical between the two ensembles, ensuring that any difference in the recovered angular power spectra or in the covariance matrix can be attributed solely to the non-linear corrections in the input power spectrum. To make the mocks reproduce the observational sample as closely as possible, the simulations also incorporate the survey mask and the measured angular number density dN/dz (Section 5.1). The mask is supplied so that galaxies are placed only within the observed footprint, while dN/dz fixes the expected number of galaxies in each shell. Figure 5.7 shows the distribution of the total number of galaxies across the non-linear ensemble. The mean is consistent with the count of the observed sample, confirming that the partitioning of dN/dz over the shells and the masked sampling reproduce the expected sample size. An equivalent result is obtained for the linear mock ensemble, whose galaxy-count distribution is statistically indistinguishable from that of the non-linear mocks.

The non-linear mock ensemble is adopted for the final analysis, as it provides a more realistic description of the statistical properties of the galaxy field relevant for covariance estimation. While the BAO feature is modeled using a linear template (discussed in details in Section 5.5), the covariance matrix must capture the full sample variance of the observed density field, which is sensitive to non-linear evolution even at the relatively low redshifts and large angular scales considered in this work. In this regime, non-linear effects primarily modify the broadband amplitude and mode coupling of the angular power spectrum, while leaving the position of the acoustic feature largely unchanged. The inclusion of HALOFIT corrections in the mock generation therefore ensures a more consistent description of the variance of the measured bandpowers and of the damping of power on quasi-linear scales, without affecting the modeling of the BAO shift parameter itself.

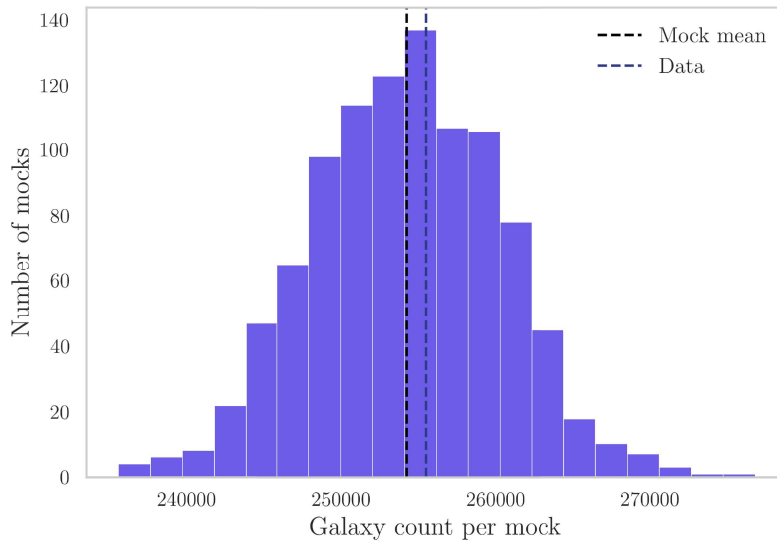


Figure 5.7: Distribution of the total number of galaxies per mock, generated using a non-linear power spectrum, across the $N_m = 1000$ realizations. The ensemble mean is consistent with the observed sample, validating the redshift partitioning and masked sampling of the mocks.

5.4 Covariance Matrix and Uncertainty Estimation

The angular power spectrum is measured from a finite number of galaxies observed over a limited region of the sky, so the recovered bandpowers are subject to statistical fluctuations and are correlated across multipoles (as seen in Section 3.3.1). Characterizing both the uncertainty of each bandpower and the correlations between them requires the covariance matrix of the estimator, which enters the analysis through the likelihood used to fit the acoustic scale.

The covariance is estimated from the ensemble of the $N_m = 1000$ mock catalogs

described in Section 5.3. The full measurement pipeline is applied to each mock, and the covariance is taken as their sample covariance,

$$\text{Cov}[\text{APS}]_{\ell\ell'}^{ij} \equiv \frac{1}{N_m - 1} \sum_{k=1}^{N_m} \left(C_{\ell}^{i(k)} - \bar{C}_{\ell}^i \right) \left(C_{\ell'}^{j(k)} - \bar{C}_{\ell'}^j \right), \quad (5.3)$$

where i and j label the redshift bins, ℓ and ℓ' the bandpowers, $k = 1, \dots, N_m$ indexes the mocks, and $\bar{C}_{\ell}^i$ is the mean spectrum over the ensemble. Since every mock shares the same survey mask, the off-diagonal terms of Eq. 5.3 also carry the correlations between multipoles induced by the partial sky coverage. The diagonal elements give the variance of each bandpower, and their square root $\sigma_{\ell} = \sqrt{\text{Cov}_{\ell\ell}}$ defines the 1σ uncertainty.

Because the covariance is estimated from a finite number of mocks, its inverse is a biased estimator of the true precision matrix. This bias is corrected by the Hartlap factor [186–188],

$$\hat{C}^{-1} = \frac{N_m - n_b - 2}{N_m - 1} C_{\text{mock}}^{-1}, \quad (5.4)$$

where n_b is the number of bandpowers in the fit window. For the first redshift bin, $0.05 < z < 0.15$, using $\Delta\ell = 4$ and $n_b = 48$ bins, the Hartlap correction (multiplicative factor) was 0.951. For the second bin, $0.15 < z < 0.25$, using $\Delta\ell = 8$ and $n_b = 24$ bins, it was 0.975. In both cases the correction is small given the large number of mock realizations used, but it is nonetheless applied for statistical consistency.

The amplitude of the covariance and its correlation structure can be separated through the correlation matrix,

$$\text{Corr}_{\ell\ell'}^{ij} = \frac{\text{Cov}_{\ell\ell'}^{ij}}{\sqrt{\text{Cov}_{\ell\ell}^{ii} \text{Cov}_{\ell'\ell'}^{jj}}}, \quad (5.5)$$

which is normalized to unity on the diagonal and measures the degree of correlation between pairs of bandpowers.

As a validation, the mock-based uncertainties can also be compared with the analytic Gaussian prediction for the covariance of the angular power spectrum [136],

$$\text{Cov}[\text{APS}]_{\ell\ell'}^{ij} = \frac{2}{f_{\text{sky}} \Delta\ell (2\ell + 1)} \left(C_{\ell}^{ij} + \frac{\delta_{ij}}{\bar{n}_i} \right)^2 \delta_{\ell\ell'}, \quad (5.6)$$

where δ_{ij} , $\delta_{\ell\ell'}$ are Kronecker deltas. This expression is diagonal in ℓ and accounts for both cosmic variance and shot noise, while the factor $1/f_{\text{sky}}$ encodes the loss of independent modes due to the incomplete sky coverage. Eq. 5.6 was also used for the best-fit parameters (discussed in details in Section 5.5) for each of the mock realizations (Appendix B).

The mock-based covariance is computed from simulated catalogs whose input cosmology and galaxy bias are fixed to fiducial values. If the true amplitude of the angular

power spectrum differs from the fiducial one, the covariance will be inaccurate: an underestimated amplitude leads to error bars that are too small, giving excessive statistical weight to the data in the likelihood, while an overestimated amplitude inflates the uncertainties and dilutes the constraining power. Since the effective galaxy bias b_{eff} enters the angular power spectrum as $C_\ell \propto b_{\text{eff}}^2$, even a modest mismatch between the assumed and true bias produces a significant rescaling of the variance. In particular, the mock catalogs are generated with $b = 1$, while the best-fit effective bias recovered from the data may differ from this value, making the raw mock covariance inconsistent with the amplitude of the observed spectrum. To ensure self-consistency between the error model and the best-fit parameters, we adopt an iterative calibration procedure inspired by the approach of Van Uitert et al. [189], which is described in detail in the following subsection.

5.4.1 Iterative calibration of the covariance matrix

As discussed in Chan et al. [190], when the sample properties differ from those assumed in the mock catalogs, the Gaussian covariance scales as C_ℓ^2 (Eq. 5.6), and the mock covariance can be rescaled accordingly. We implement this correction through the following iterative procedure:

1. **Initial fit:** the BAO template of Eq. 5.9 is fitted to the data using the mock-based covariance (corrected by the Hartlap factor of Eq. 5.4), yielding initial best-fit values for the shift parameter α and the nuisance parameters (B_0, a, A_0) . The effective galaxy bias is inferred as $b_{\text{eff}} = \sqrt{|B_0^2|}$.
2. **Covariance rescaling:** the best-fit model C_ℓ^{model} is compared with the mean of the mock spectra $\overline{C}_\ell^{\text{mock}}$, and a per-multipole rescaling factor is computed as

$$r_\ell = \left(\frac{C_\ell^{\text{model}}}{\overline{C}_\ell^{\text{mock}}} \right)^2, \quad (5.7)$$

which rescales the covariance amplitude, based on the ratio between the best-fit model angular power spectrum and the mean of the mock ensemble. The covariance matrix is then updated by rescaling its elements as

$$\text{Cov}_{\ell\ell'}^{(\text{new})} = \sqrt{r_\ell r_{\ell'}} \text{Cov}_{\ell\ell'}^{(\text{mock})}, \quad (5.8)$$

where $\text{Cov}^{(\text{mock})}$ is always the original, unmodified mock covariance. This rescaling adjusts the diagonal elements (the variances) to reflect the amplitude of the best-fit model rather than that of the fiducial mocks, while preserving the correlation structure encoded in the off-diagonal terms. The factor enters as $\sqrt{r_\ell r_{\ell'}}$ because the covariance scales as the square of the power spectrum amplitude.

3. **Re-fit:** the template fit is repeated with the updated covariance matrix, yielding new best-fit parameters and a new estimate of b_{eff} .
4. **Convergence:** steps 2 and 3 are iterated until the recovered parameters stabilise. In practice, three iterations are performed: after the second iteration, the shift parameter α and the effective bias b_{eff} change by less than their statistical uncertainties, indicating convergence.

If the parameter values adopted in the covariance model are too far from the best-fit region, the inferred model becomes inconsistent with the error model used in the likelihood function. Iterative rescaling corrects this discrepancy without requiring the regeneration of the entire set of simulation catalogs at each step, which would be computationally expensive. The converged covariance matrix is adopted for all subsequent analyses, including the profile-likelihood estimation of α , the MCMC sampling, and the robustness tests presented in Appendix B.

5.5 Template Fitting and Parameter Inference

With the galaxy sample fully characterized and validated through mock catalogs, and with the covariance matrix estimated, we are now in a position to perform the statistical inference of the BAO feature. Cosmological information is contained in the position of the acoustic feature in the harmonic space, and the shift parameter α (introduced in Section 2.5) quantifies its displacement relative to the fiducial cosmology. The complete inference proceeds by fitting a parametrized model to the data to extract the BAO shift parameter, which is then converted into the distance ratio D_M/r_d and the angular scale θ_{BAO} at the effective redshift of each bin. Finally, a compilation of $\theta_{\text{BAO}}(z)$ measurements, including our own, is used to constrain the cosmological parameters ($\Omega_{m,0}, h r_d$).

The starting point is a theoretical angular power spectrum, denoted C^{temp} , computed from the matter power spectrum of the fiducial cosmology adopted (Table 5.4) and evaluated with the Core Cosmology Library [191], using the redshift distribution $n(z)$ of the bin as the radial selection function. The parametrization used to fit the data is based on Seo et al. [192],

$$C_{\ell}^{\text{model}} = \left(B_0^2 + \frac{a}{\ell} \right) C^{\text{temp}}(\ell/\alpha) + A_0, \quad (5.9)$$

which contains four free parameters. The parameter α rescales the multipole axis of the template and is the quantity of interest, with $\alpha = 1$ corresponding to perfect agreement between the observation and the fiducial model. The remaining parameters absorb the broadband shape of the spectrum, to which the acoustic information is not tied: B_0 sets the overall amplitude (through a galaxy-bias-like factor B_0^2), the term a/ℓ allows for a

smooth scale-dependent tilt, and the constant A_0 absorbs the white shot-noise contribution.

Table 5.4: Fiducial flat- Λ CDM parameters adopted in the template construction and in the conversion of α into the distance ratio D_M/r_d , taken from the *Planck* 2018 [15] results using the TT,TE,EE+lowE+lensing likelihood combination.

Parameter	Value
$\Omega_{m,0}$	0.3153 ± 0.0073
H_0 [km s ⁻¹ Mpc ⁻¹]	67.36 ± 0.54
r_d [Mpc]	147.09 ± 0.26
w_0	-1
w_a	0

Note that the amplitude parameter B_0 also plays a role in the construction of the covariance matrix. As described in Section 5.4.1, the mock-based covariance is rescaled using the best-fit model to correct for the amplitude mismatch between the simulated and observed spectra.

To evaluate the sensitivity of the recovered α to the modeling, alternative constructions of C^{temp} are considered, differing in two independent choices: (i) the input matter power spectrum $P(k)$ used to compute the template, and (ii) the method used to project this power spectrum onto the sphere to obtain C^{temp} . For the input spectrum, besides the linear $P_{\text{lin}}(k)$, a damped spectrum is also employed,

$$P(k) = [P_{\text{lin}}(k) - P_{\text{nw}}(k)] e^{-k^2 \Sigma_{\text{nl}}^2} + P_{\text{nw}}(k), \quad (5.10)$$

where $\Sigma_{\text{nl}} = 5.2 h^{-1} \text{Mpc}$ is the non-linear damping scale [190], and $P_{\text{nw}}(k)$ is the no-wiggle spectrum, with the same broadband shape as P_{lin} but with the baryonic oscillations removed, computed with the Eisenstein-Hu fitting formula [9]. For the projection, in addition to the Limber approximation (Eq. 3.22), the full angular spectrum of Eqs. 3.20 and 3.21 is also evaluated.

As with the fiducial template, the damped spectrum and the full projection are evaluated on a grid of multipoles and interpolated with a cubic spline. The combination of two choices for the input spectrum ($P_{\text{lin}}(k)$ or $P(k)$) and two projection schemes (Limber or full) results in four distinct template configurations, which are compared in Appendix B. The recovered α parameter is consistent across all of them. The simplest one, the linear power spectrum with the Limber projection, is therefore adopted as the fiducial template throughout this work, being also the least computationally expensive to evaluate.

The parameters of Eq. 5.9 are inferred within a Bayesian framework. The fit minimizes the χ^2 statistic,

$$\chi^2(\boldsymbol{\theta}) = \sum_{\ell, \ell'} [C_\ell - C_\ell^{\text{model}}(\boldsymbol{\theta})] \text{Cov}_{\ell\ell'}^{-1} [C_{\ell'} - C_{\ell'}^{\text{model}}(\boldsymbol{\theta})], \quad (5.11)$$

where $\boldsymbol{\theta} = (B_0, A_0, a, \alpha)$, C_ℓ are the measured bandpowers (binned values), $C_\ell^{\text{model}}(\boldsymbol{\theta})$ the template of Eq. 5.9, and Cov^{-1} the inverse of the covariance matrix. Following Verde [193], the bandpowers are assumed to follow a multivariate Gaussian distribution, so that the likelihood of the data given the parameters is

$$\mathcal{L}(\boldsymbol{\theta}) \propto \exp\left[-\frac{1}{2} \chi^2(\boldsymbol{\theta})\right], \quad (5.12)$$

and minimizing χ^2 is therefore equivalent to maximize the likelihood. By the Bayes theorem, the posterior probability of the parameters given the data is [193]

$$\mathcal{P}(\boldsymbol{\theta} \mid \mathbf{C}) = \frac{\mathcal{L}(\mathbf{C} \mid \boldsymbol{\theta}) \pi(\boldsymbol{\theta})}{\int \mathcal{L}(\mathbf{C} \mid \boldsymbol{\theta}) \pi(\boldsymbol{\theta}) d\boldsymbol{\theta}}, \quad (5.13)$$

where $\mathcal{L}(\mathbf{C} \mid \boldsymbol{\theta})$ is the likelihood of Eq. 5.12 and $\pi(\boldsymbol{\theta})$ the prior. We adopt the flat priors of Table 5.5, and the posterior is sampled with the ensemble sampler EMCEE [194].

Table 5.5: Flat prior distributions adopted for the four parameters of the fitting template.

Parameter	Prior range
B_0	$[0.2, 10]$
A_0	$[-0.01, 0.01]$
a	$[-100, 100]$
α	$[0.5, 1.5]$

The fit is performed over the multipole range $10 \leq \ell \leq 200$, which excludes the lowest multipoles, where the mode-coupling and cosmic variance are largest, while covering the range in which the acoustic feature is expected for the sample in study. The parameter constraints are reported as the median and the 16th-84th percentiles of the marginalized posterior.

To assess whether the data truly exhibit the acoustic characteristic, the best-fit template is compared with a featureless (no-wiggle) version of the same model, built from the no-wiggle spectrum $P_{\text{nw}}(k)$ introduced above. The significance is evaluated by profiling over α : at each value on a grid, the remaining parameters (B_0, A_0, a) that minimize the χ^2 are determined by least squares, yielding a one-dimensional profile $\chi^2(\alpha)$ whose minimum gives the best-fit shift. The detection significance is then

$$\Delta\chi^2 = \chi_{\text{no-wiggle}}^2 - \chi_{\text{wiggle}}^2, \quad S = \sqrt{\Delta\chi^2}, \quad (5.14)$$

where $\Delta\chi^2$ is the improvement in fit quality from including the oscillations and S is the corresponding detection significance, in units of standard deviations (σ). This significance is also tested against the mocks, by comparing the data value of $\Delta\chi^2$ with the distribution obtained from the mock ensemble (Appendix B).

The shift parameter is then converted into a physical distance using the fact that the

angular acoustic scale measures the transverse comoving distance. Recalling its definition (Eq. 2.130),

$$\alpha = \frac{D_M(z_{\text{eff}})/r_d}{(D_M(z_{\text{eff}})/r_d)_{\text{fid}}} \Rightarrow \frac{D_M(z_{\text{eff}})}{r_d} = \alpha \left(\frac{D_M(z_{\text{eff}})}{r_d} \right)_{\text{fid}}.$$

The fiducial values used for the conversion are shown in the Table 5.4. The associated uncertainty is obtained by Monte-Carlo propagation, sampling α together with the fiducial $\Omega_{m,0}$ and $h r_d$ within their uncertainties. The angular BAO scale then follows as

$$\theta_{\text{BAO}}(z_{\text{eff}}) = \frac{r_d}{D_M(z_{\text{eff}})} = \left(\frac{D_M(z_{\text{eff}})}{r_d} \right)^{-1}, \quad (5.15)$$

which gives θ_{BAO} in radians and is converted to degrees for the comparisons that follow.

To place the measurement in a cosmological context, the same relation is applied to a compilation of $\theta_{\text{BAO}}(z)$ measurements, with the distance ratio modeled for a spatially flat cosmology as

$$\frac{D_M(z)}{r_d} = \frac{c}{100 (h r_d)} \int_0^z \frac{dz'}{E(z')}, \quad E(z) = \frac{H(z)}{H_0}, \quad (5.16)$$

and the dimensionless Hubble function written for a generalized dark-energy equation of state, reducing to Λ CDM for $w_0 = -1$ and $w_a = 0$,

$$E(z) \equiv \sqrt{\Omega_{m,0} a^{-3} + (1 - \Omega_{m,0}) e^{-3w_a(1-a)} a^{-3(1+w_0+w_a)}}. \quad (5.17)$$

The posterior of $(\Omega_{m,0}, h r_d)$ is sampled with EMCEE under flat priors, the combination $h r_d$ being the quantity directly constrained by the angular BAO data rather than H_0 and r_d separately. Two fits are performed: one combining the present measurement with independent $\theta_{\text{BAO}}(z)$ determinations from the literature, and a second that additionally includes the BAO measurements from DESI. Comparing both fits isolates the effect of the DESI data on the recovered constraints. These analyses and the resulting parameter constraints are presented in Chapter 6.

Chapter 6

Results

This chapter presents the main results of this work: the measurement of the angular power spectrum of the blue-galaxy sample described in Chapter 4, the extraction of the BAO feature using the methodology developed in Chapter 5, and the cosmological constraints that follow. The analysis is carried out independently in two redshift bins. The bin $0.15 \leq z \leq 0.25$ ($z_{\text{eff}} = 0.2$) yields the stronger acoustic signal and constitutes the headline measurement. The bin $0.05 \leq z \leq 0.15$ ($z_{\text{eff}} = 0.1$) is presented as a consistency check.

6.1 Measured Angular Power Spectra and Template Fit

6.1.1 Main analysis: $0.15 \leq z \leq 0.25$

We begin with the higher-redshift bin. The angular power spectrum is estimated with the pseudo- C_ℓ pipeline described in Section 5.2, using the galaxy sample and mask defined in Section 5.1. The same pipeline is applied consistently to both mock catalogs and data, ensuring a homogeneous treatment of survey geometry, binning, and mode coupling.

The covariance matrix is obtained from an ensemble of $N_m = 1000$ mocks described in Section 5.3. Mock catalogs generated with both linear and non-linear matter power spectra are considered, and the resulting covariance matrices are found to be fully consistent within statistical uncertainties, indicating that the inferred bandpower correlations are insensitive to the specific clustering model adopted in the simulations. The covariance matrix used in the main analysis is therefore derived from the non-linear mock ensemble, which provides the most realistic description of the small-scale clustering regime relevant to this work.

For the iterative covariance estimated from the linear mocks, the fitted amplitude corresponds to $B_0 \simeq 1.31$, while in the non-linear case the iterative calibration converges to $B_0 \simeq 1.36$. In both cases, the measured clustering amplitude remains systematically above the mock expectation prior to rescaling, indicating that the mock covariance

underestimates the variance of the data and motivating the recalibration discussed in Section 5.4.1. The recalibration has a negligible impact on the best-fit acoustic scale, with α remaining stable across iterations (Appendix B), but it increases the parameter uncertainties and yields a more conservative estimate of the BAO detection significance, as expected from the enlarged error bars. After this rescaling, the resulting correlation matrix, shown in Figure 6.1, is strongly diagonal-dominant, confirming that correlations between neighbouring bandpowers are weak and that the binned mode-coupling matrix effectively decouples the multipoles over the fit range.

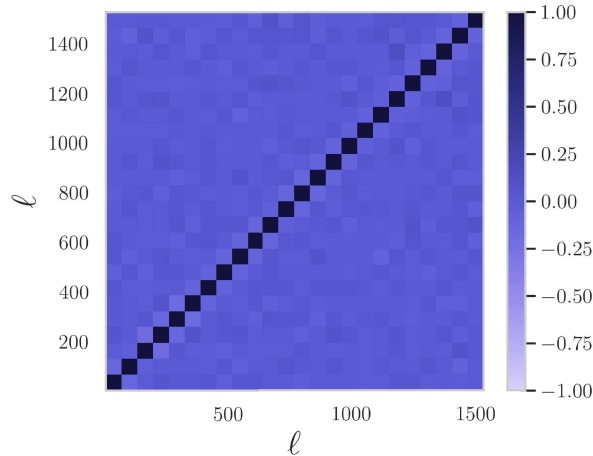


Figure 6.1: Correlation matrix of the mock-based covariance for the $0.15 \leq z \leq 0.25$ bin. The dominant diagonal confirms that correlations between neighbouring bandpowers are weak.

As a further robustness check, we compare four template configurations obtained by combining two choices of the input matter power spectrum (linear and linear with BAO damping) with two projection schemes (Limber approximation and full spherical Bessel integration). The recovered acoustic scale is stable across all four combinations, with best-fit values ranging from $\alpha = 1.033$ to $\alpha = 1.036$, a maximum spread of 0.003, well within the statistical uncertainty of the measurement ($\sigma_\alpha \simeq 0.09\text{--}0.16$). The BAO detection significance is similarly insensitive to the template choice, varying between 1.17σ and 1.18σ . Given this stability, the simplest configuration, the linear matter power spectrum projected with the Limber approximation, is adopted as the fiducial template throughout the remainder of this work.

It is worth emphasizing the weakly model-dependent character of this methodology. The dilation parameter α measures the ratio of the observed acoustic scale to that predicted by the fiducial cosmology, and is therefore insensitive to the overall normalization and broadband shape of the angular power spectrum, which are absorbed by the nuisance parameters (B_0 , a , A_0). The constraint on α is driven by the *position* of the BAO feature in multipole space, not by the amplitude or tilt of the spectrum. Furthermore, as confirmed by the robustness tests of Appendix B, the recovered acoustic scale is stable across different template configurations, binning schemes, and fitting ranges, indicating that the

measurement is not sensitive to the specific modelling choices made in the analysis.

The marginalized parameter constraints for this redshift bin are summarized in Table 6.1, and the corresponding posterior distribution is shown in Figure 6.2. Both the profile likelihood and MCMC analyses yield values of α consistent with unity, although the statistical uncertainties remain large ($\bar{\sigma}_\alpha \simeq 0.22$), reflecting the moderate sky coverage, the relatively low number density of the sample, and the limited comoving volume accessible at this redshift.

The parameter B_0 rescales the amplitude of the fiducial template to match the observed clustering signal. Since the template is constructed from the matter power spectrum and the angular power spectrum of galaxies scales as the square of the linear bias, B_0 can be interpreted as an effective bias parameter. The recovered value indicates that the observed sample clusters somewhat more strongly than the fiducial mocks, which assumed $b = 1$. This is consistent with expectations for blue, star-forming galaxies at these redshifts, for which previous studies report effective biases in the range $b \simeq 1.0\text{--}1.4$ [172, 184?].

The parameter a/ℓ is a broadband correction that absorbs residual scale-dependent differences between the measured spectrum and the theoretical template. It carries no direct physical interpretation and serves to prevent imperfections in the broadband modeling from biasing the acoustic scale. As shown in Appendix B, removing this term shifts the recovered α appreciably, confirming that it plays an important role in isolating the BAO feature from the broadband shape of the spectrum.

The constant term A_0 can be compared with the theoretical shot-noise contribution of Eq. 3.32. For the present sample, the expected Poisson level is 7.72×10^{-5} , so the MCMC best-fit value for A_0 is slightly above but consistent with the shot-noise prediction. The small excess indicates that A_0 absorbs not only the Poisson contribution but also residual scale-independent offsets from the broadband model. It is therefore best interpreted as an effective constant offset rather than a pure shot-noise measurement.

The posterior distribution also reveals correlations between α and the broadband parameters, particularly the tilt term a . When the acoustic feature is only marginally detected, small shifts in the position of the template can be partially compensated by adjustments to the broadband shape. The marginalization over (B_0, a, A_0) accounts for these degeneracies and ensures that the reported uncertainty on α reflects the full parameter covariance.

Table 6.1: Marginalized constraints (median and 16th–84th percentiles) for the template parameters in the $0.15 \leq z \leq 0.25$ bin.

Parameter	Value
B_0	$1.368^{+0.387}_{-0.210}$
A_0	$(8.381^{+0.457}_{-0.460}) \times 10^{-5}$
a	$-10.25^{+8.314}_{-15.56}$
α	$0.974^{+0.194}_{-0.248}$

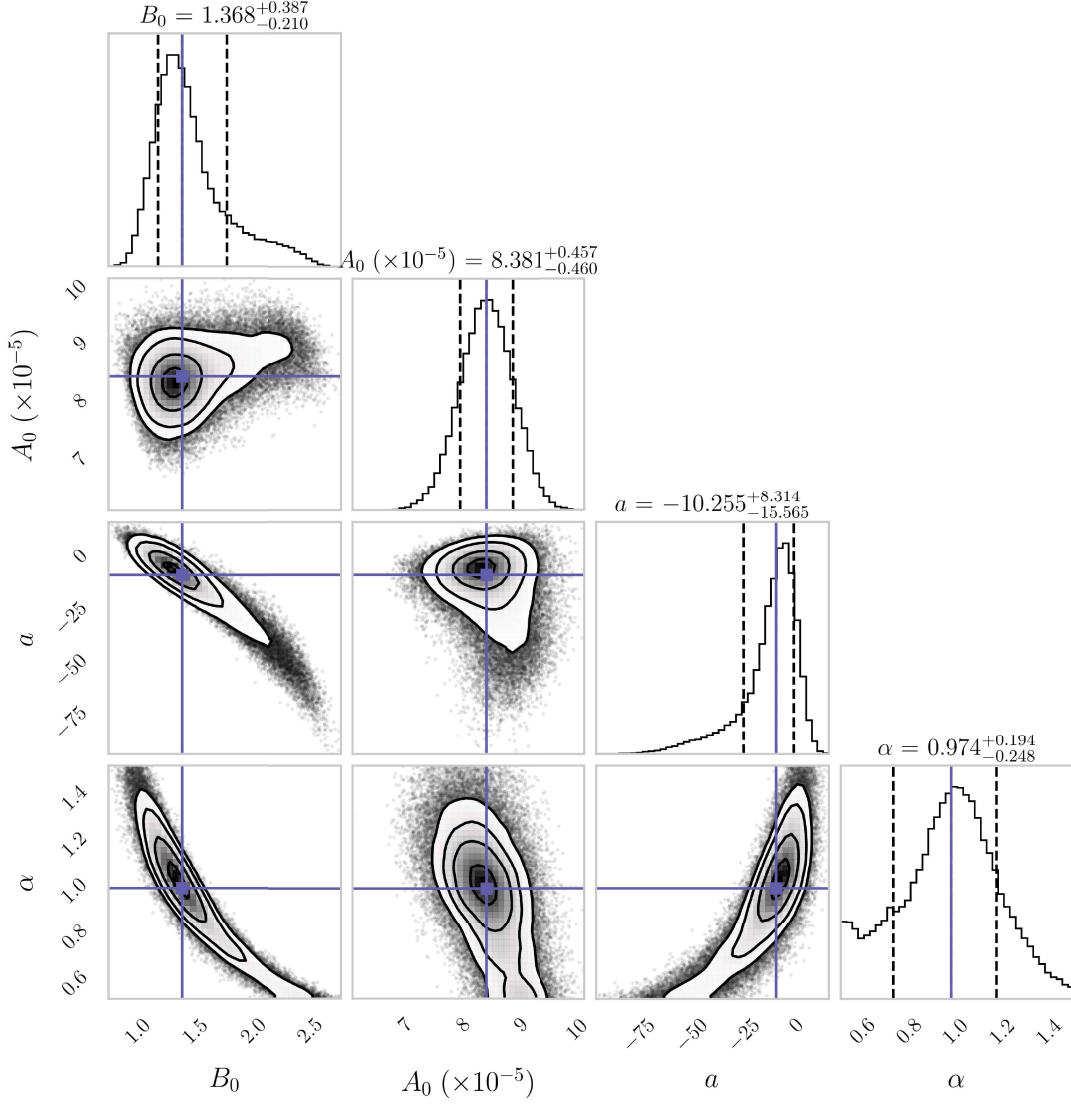


Figure 6.2: Marginalized posterior distributions and two-dimensional contours (68% and 95% credible regions) for the four template parameters of Eq. 5.9, obtained from the MCMC sampling for the $0.15 \leq z \leq 0.25$ bin. The purple cross indicates the median values, and the dashed lines on the diagonal panels mark the 16th and 84th percentiles. The α - a and α - B_0 panels show the expected degeneracies between the acoustic scale and the broadband nuisance parameters.

Figure 6.3 shows the measured angular power spectrum together with the best-fitting template and the mean of the mock ensemble. The observed spectrum has a larger amplitude than the mock mean, as expected given that the mocks assume a unit galaxy bias while the data prefer $B_0 > 1$. To facilitate the visual comparison, the mock mean and its dispersion have been rescaled by the best-fitting B_0^2 . After this rescaling, the data fall well within the mock scatter, confirming that the amplitude mismatch is the dominant difference between the observed and simulated spectra and that the broadband shape and acoustic structure are faithfully reproduced by the mocks.

The robustness of the measurement was also assessed by varying the bandpower width, the fitting range, the template parametrization, and the prior widths; the full set of tests

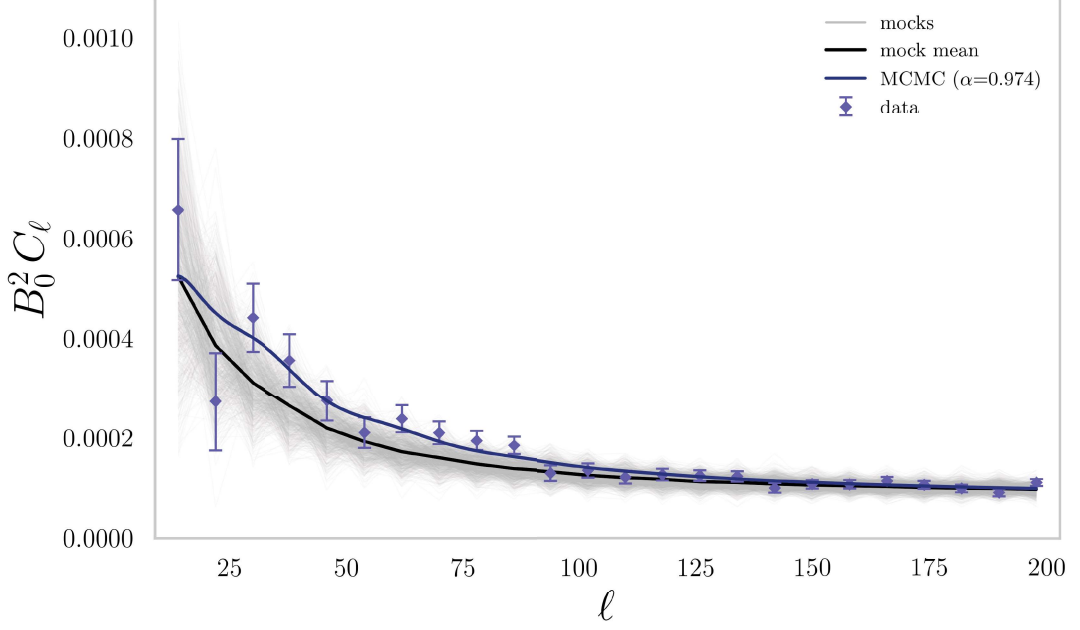


Figure 6.3: Measured angular power spectrum (diamonds with error bars) for the $0.15 \leq z \leq 0.25$ bin, compared with the best-fitting BAO template from the MCMC analysis ($\alpha = 0.974$, solid blue curve). The grey lines show individual mock realizations rescaled by the best-fitting amplitude B_0^2 , and the black curve is the corresponding rescaled mock mean.

is presented in Appendix B and summarized here.

For bandpower widths $\Delta\ell = 4, 6, 8$, and 9 , the recovered α is consistent with unity within the statistical uncertainties. For coarser binning ($\Delta\ell \gtrsim 10$), the recovered α becomes increasingly biased and the uncertainties grow significantly, indicating that the wider bins smooth out the oscillatory BAO signal and the fit loses sensitivity to the acoustic feature. This test therefore motivates the fiducial choice $\Delta\ell = 8$, as it yields a well-constrained acoustic scale while preserving the BAO information, and its result is consistent with the narrower binning schemes.

The sensitivity to the template parametrization was tested by modifying the broadband terms in Eq. 5.9. The recovered acoustic scale is insensitive to the treatment of the amplitude: replacing the fiducial $(B_0^2 + a/\ell)$ by $(B_0 + a/\ell)^2$ yields nearly identical results. In contrast, the tilt term a/ℓ proves to be important. Both its complete removal and its replacement by a term proportional to ℓ shift α substantially and eliminate the statistical preference for the BAO feature. This indicates that the a/ℓ term absorbs residual large-scale shape differences between the data and the template. Without it, these mismatches are compensated by the acoustic-scale parameter itself, biasing the measurement. The term is therefore retained in the fiducial model throughout.

To quantify the impact of the fitting range, $\ell_{\min}$ and $\ell_{\max}$ were varied independently while holding the complementary boundary fixed at its fiducial value (Appendix B). The acoustic scale is stable against variations in $\ell_{\max}$ across the full range tested. Varying $\ell_{\min}$

shows a stronger sensitivity: setting $\ell_{\min} \gtrsim 18$, thereby excluding the lowest multipoles from the fit, is sufficient to degrade or eliminate the BAO detection in several configurations. This is consistent with the expectation that the acoustic feature for this low-redshift sample is concentrated at very low multipoles, so that cutting even a narrow range at the low- ℓ end removes a substantial fraction of the signal. For $\ell_{\min} \gtrsim 50$, the best-fit α drifts upward and the uncertainties grow as the relevant modes are progressively lost. The fiducial choice $\ell_{\min} = 10$ retains the BAO signal while excluding the lowest multipoles, where the mode-coupling correction and cosmic variance are largest.

To assess the presence of the acoustic feature, the fiducial template was compared with the corresponding no-wiggle model following the procedure of Section 5.5. The best-fitting wiggle template yields $\chi^2 = 20.71$ for 20 degrees of freedom ($\chi^2/\text{dof} = 1.035$, $p = 0.414$), indicating that the model provides an adequate description of the measured spectrum. The improvement over the featureless no-wiggle template is $\Delta\chi^2 = 1.38$, corresponding to a 1.17σ preference for the oscillatory model. This marginal significance is consistent with the distribution of detection strengths recovered from the mock ensemble (Appendix B): a substantial fraction of realizations at this redshift and sky coverage yield no significant BAO detection, confirming that a $\sim 1\sigma$ result is within the expected statistical regime for this survey configuration.

Figure 6.4 shows the $\chi^2(\alpha)$ profiles for both templates: the wiggle profile exhibits a shallow minimum near $\alpha \approx 1.03$, while the no-wiggle profile is nearly flat, confirming that the acoustic feature is only marginally favoured by the data. Although this does not constitute a statistically significant detection of BAO, the data are better described by the oscillatory template than by a smooth spectrum, and the recovered acoustic scale is consistent with the fiducial Λ CDM prediction.

The profile likelihood analysis yields $\alpha = 1.034^{+0.155}_{-0.162}$, while the MCMC sampling gives $\alpha = 0.974^{+0.247}_{-0.194}$. The two estimates are fully consistent within their uncertainties and both compatible with the fiducial value $\alpha = 1$, indicating that the position of the acoustic feature is in agreement with the prediction of the reference Λ CDM cosmology. The uncertainty on α reflects the limited statistical power of the present sample: at the low redshifts probed here, the accessible comoving volume is small, the sky fraction is modest ($f_{\text{sky}} \approx 0.18$), and the angular power spectrum at the relevant multipoles is dominated by cosmic variance. While the data are consistent with the acoustic feature predicted by the standard model, they do not provide sufficient precision to establish a high-significance BAO detection from this bin alone. This is illustrated in Figure 6.5, which shows the measured spectrum divided by the best-fitting smooth (no-wiggle) model. The expected oscillatory pattern from the wiggle template is visible as a subtle modulation at low multipoles, but the data points scatter broadly around unity, confirming that the BAO signal is present at a level comparable to the statistical noise.

To summarize the performance of the BAO fit, Table 6.2 reports the best-fit values of

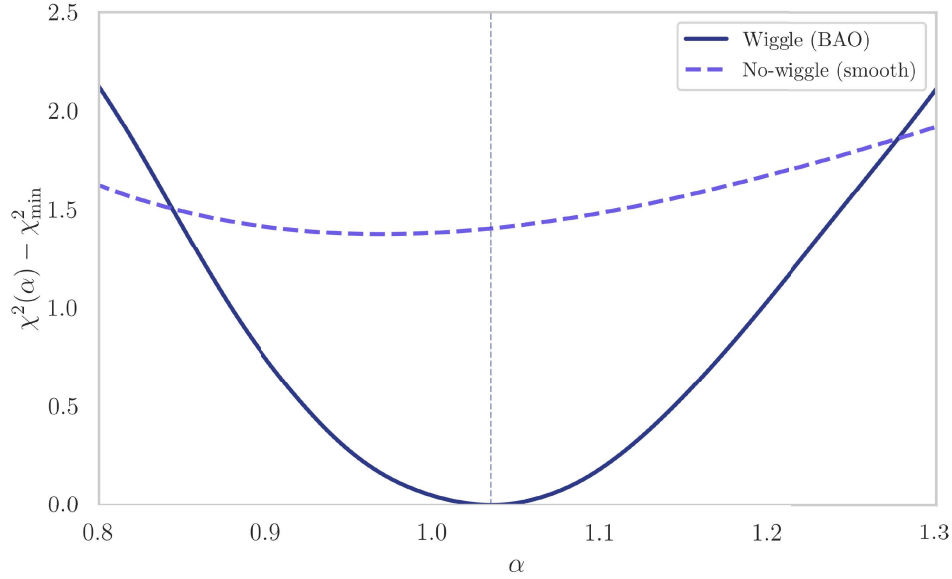


Figure 6.4: Profile $\chi^2(\alpha) - \chi_{\min}^2$ for the wiggle (solid) and no-wiggle (dashed) templates in the $0.15 \leq z \leq 0.25$ bin. The wiggle template exhibits a shallow minimum near $\alpha \approx 1.03$, while the no-wiggle profile is nearly flat. The difference between the two minima, $\Delta\chi^2 = 1.38$ (1.17σ), indicates a marginal preference for the oscillatory model. The vertical dotted line marks the best-fit α .

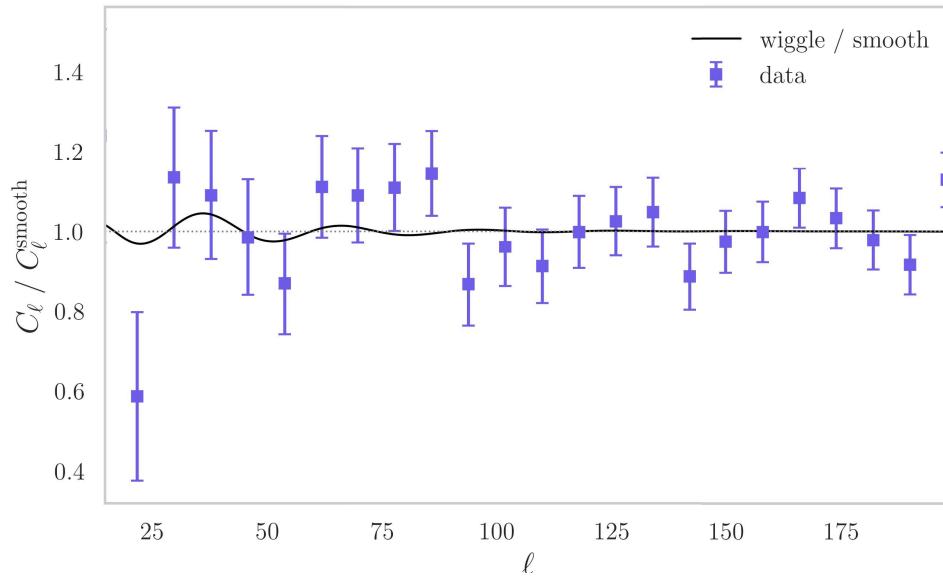


Figure 6.5: Ratio of the measured angular power spectrum to the best-fitting no-wiggle (smooth) model for the $0.15 \leq z \leq 0.25$ bin. The solid curve shows the corresponding ratio for the best-fitting wiggle template, highlighting the expected acoustic oscillation. The data are consistent with the oscillatory pattern but the uncertainties are too large to establish a significant detection.

the acoustic scale parameter α , together with the corresponding goodness-of-fit statistics and the significance of the BAO detection as quantified by the difference between the wiggle and no-wiggle models.

Table 6.2: Summary statistics of the BAO fit in the fiducial configuration. The two inference methods yield consistent constraints on the acoustic scale.

Quantity	Value
MCMC α	$0.974^{+0.247}_{-0.194}$
Profile-likelihood α	$1.034^{+0.155}_{-0.162}$
$\chi^2_{\min}/\text{dof}$	1.035
$\Delta\chi^2$ (wiggle vs. no-wiggle)	1.38
BAO significance	1.17σ

6.1.2 Consistency check: $0.05 \leq z \leq 0.15$

The same analysis pipeline was applied to the lower-redshift bin. No statistically significant preference for the BAO template was found ($\Delta\chi^2 < 0.1$, less than 0.3σ), and the recovered acoustic scale showed strong sensitivity to the fitting configuration, behaviour characteristic of an unconstrained fit. This outcome is expected: although this bin contains the larger number of galaxies (Table 5.1), it probes a substantially smaller comoving volume, increasing the cosmic variance at the low multipoles where the acoustic feature is concentrated. In addition, the BAO scale in $\Delta z = 0.1$ subtends a broader range of angular scales, further reducing the contrast of the oscillatory signal in the angular power spectrum. The lower-redshift bin is therefore not used for cosmological inference but is retained as a validation of the analysis pipeline.

Figure 6.6 illustrates the measured angular power spectrum together with the best-fitting broadband model, which provides a good description of the data, while the right panel shows the $\chi^2(\alpha)$ profiles for the wiggle and no-wiggle templates, which are nearly indistinguishable, confirming the absence of a detectable acoustic feature in this bin.

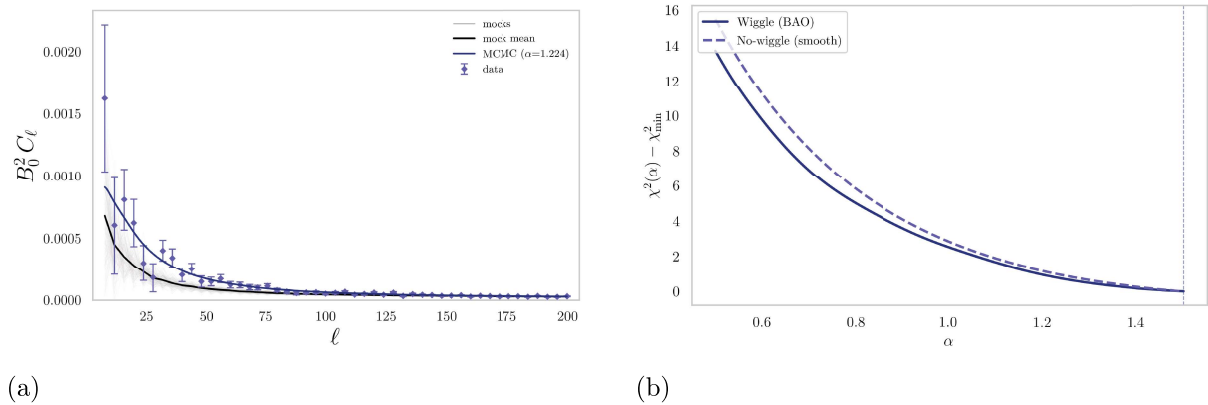


Figure 6.6: Left: measured angular power spectrum for the $0.05 \leq z \leq 0.15$ bin (diamonds), compared with the MCMC best-fit template ($\alpha = 1.5$, solid curve) and the rescaled mock mean (black curve). Right: profile $\chi^2(\alpha) - \chi^2_{\min}$ for the wiggle (solid) and no-wiggle (dashed) templates. The two profiles are nearly indistinguishable, with $\Delta\chi^2 < 0.1$ ($< 0.3\sigma$), confirming the absence of a statistically significant BAO detection in this bin. The broadband model provides an adequate fit to the data, validating the analysis pipeline even in the regime where the acoustic feature cannot be resolved.

6.2 Cosmological Analysis

The acoustic-scale measurement from the $0.15 \leq z \leq 0.25$ bin, which provides the most robust constraint, is now translated into cosmological distance information and combined with external BAO data to constrain the parameters of the flat Λ CDM model.

6.2.1 Constraints on the Transverse BAO Scale

The shift parameter α measured in Section 6.1 encodes the ratio of the observed transverse comoving distance to the sound horizon, relative to the same ratio in the fiducial cosmology. Applying Eq. 2.130 with the MCMC best-fit $\alpha = 0.974$ and the fiducial distance $D_M^{\text{fid}}(z_{\text{eff}})/r_d^{\text{fid}}$, and propagating the uncertainties on both α and the fiducial parameters via a Monte Carlo procedure, we obtain, at the effective redshift $z_{\text{eff}} = 0.2$,

$$\frac{D_M(z_{\text{eff}})}{r_d} = 5.606 \pm 1.276. \quad (6.1)$$

Although the uncertainty is large, this measurement provides an independent determination of the transverse BAO distance scale at low redshift. The corresponding angular size of the acoustic horizon, given by Eq. 5.15, is

$$\theta_{\text{BAO}} = (10.21 \pm 2.326)^\circ. \quad (6.2)$$

This large angular scale is characteristic of the BAO feature at low redshift, where the sound horizon subtends a substantial fraction of the sky. The value is consistent with previous angular BAO determinations [121, 167].

Before combining the measurement with external data, it is useful to assess its consistency with current cosmological models. Following the tension metric introduced by Avila et al. [195], we compare the measured D_M/r_d with the predictions of two reference cosmologies: the Planck 2018 flat Λ CDM model [15] and the DESI flat $w_0 w_a$ CDM model [196]. The resulting tension parameters are

$$T_{\text{Planck}} = 0.120, \quad T_{\text{DESI}} = 0.063. \quad (6.3)$$

Both values are well below unity, indicating full consistency with the predictions of both models. The present measurement therefore does not have sufficient precision to discriminate between Λ CDM and dynamical dark energy, but provides an independent low-redshift anchor that validates the standard cosmological framework. Figure 6.7 shows the evolution of D_M/r_d as a function of redshift, together with recent BAO measurements and the predictions from both cosmological models.

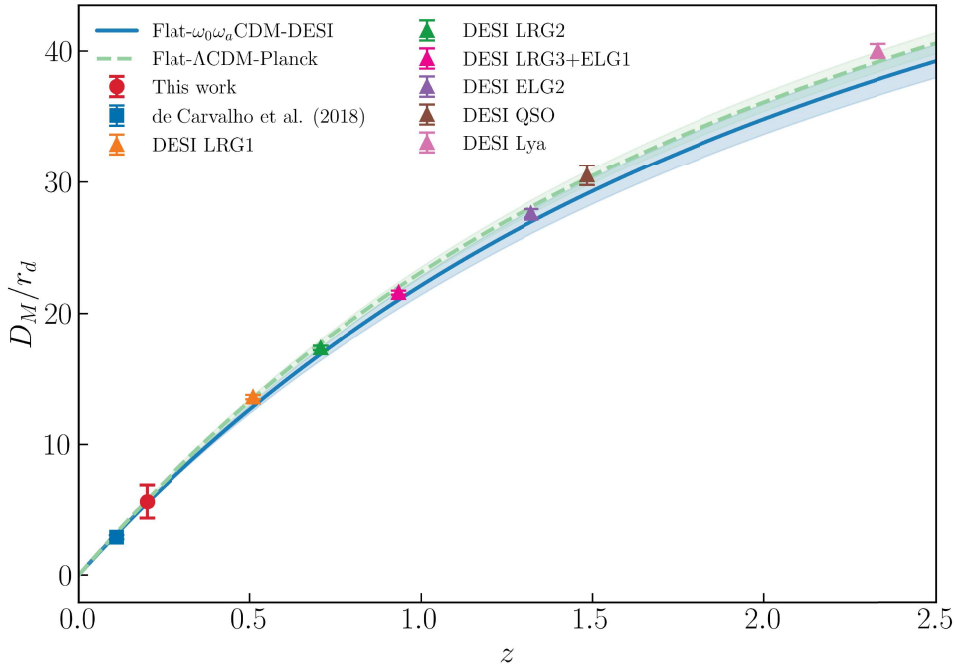


Figure 6.7: Comparison of measurements of the comoving angular diameter distance in units of the sound horizon, D_M/r_d , as a function of redshift. The red point corresponds to the result of this work, while other points show recent BAO measurements from galaxy and quasar surveys, including DESI LRG, ELG, QSO, and Ly α samples. The solid blue curve represents the prediction of a flat w_0w_a CDM model calibrated to DESI, while the dashed green curve shows the Planck-calibrated flat Λ CDM prediction. Shaded regions indicate the corresponding model uncertainties. Our measurement is consistent with both cosmological models within the statistical uncertainties and follows the expected redshift evolution of D_M/r_d .

6.2.2 Joint Constraints on $\Omega_{m,0}$ and $h r_d$

To explore the cosmological constraining power of angular BAO measurements [197], the present result is combined with a compilation of transverse observations spanning $0.05 \leq z \leq 2.33$. The compilation includes 17 angular BAO measurements from Sabogal et al. [198], six recent DESI determinations [196], and the measurement obtained in this work. The analysis assumes a spatially flat Λ CDM cosmology with two free parameters, $\Omega_{m,0}$ and the combination $h r_d$, which is the quantity directly constrained by angular BAO data rather than H_0 and r_d separately. The theoretical $\theta_{\text{BAO}}(z)$ is computed from Eqs. 5.16 and 5.17, and the posterior is sampled with EMCEE under the flat priors $0.1 \leq \Omega_{m,0} \leq 0.7$ and $80 \leq h r_d \leq 120$ Mpc.

The resulting constraints on the cosmological parameters excluding DESI data points are

$$\Omega_{m,0} = 0.422^{+0.098}_{-0.083}, \quad (6.4)$$

$$h r_d = 98.74^{+2.63}_{-2.78} \text{ Mpc}. \quad (6.5)$$

These results are consistent with those reported by [198], which are based on 2D BAO measurements alone. The relatively larger uncertainties reflect, as expected, the reduced constraining power of purely transverse information in comparison to full 3D BAO analyses. The preferred value of $\Omega_{m,0}$ is slightly higher than the Planck [15] best-fit value, reflecting the degeneracy between $\Omega_{m,0}$ and $h r_d$: at fixed effective distance, a higher matter density can be compensated by a smaller sound horizon, and vice versa. This degeneracy is clearly visible in the $\Omega_{m,0}$ – $h r_d$ plane of the posterior distribution (Figure 6.8), where elongated contours follow the expected direction of partial parameter compensation.

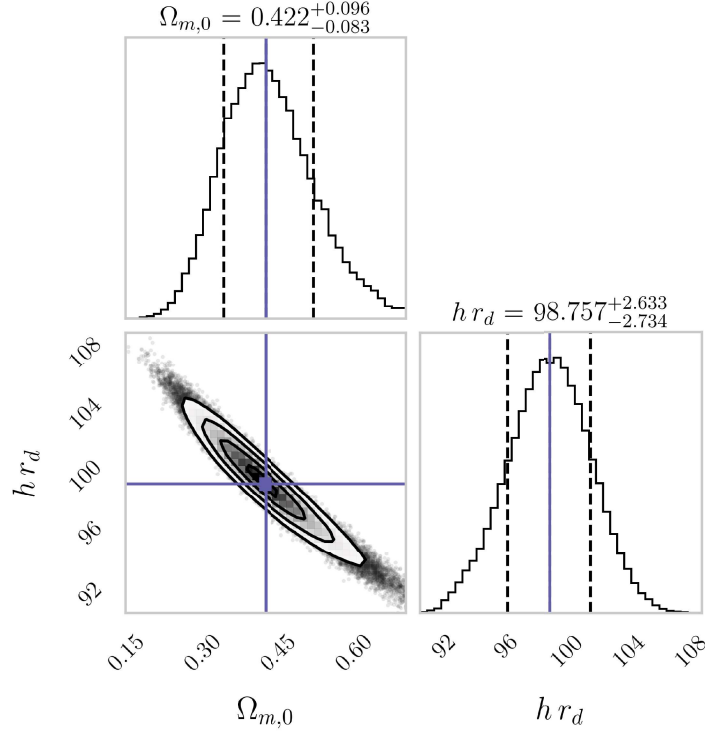


Figure 6.8: Marginalized posterior distributions and two-dimensional contours (68% and 95% credible regions) for $\Omega_{m,0}$ and $h r_d$, obtained without including DESI measurements. The purple cross indicates the median values, and the dashed lines on the diagonal panels mark the 16th and 84th percentiles.

Including the DESI measurements significantly tightens the constraints, yielding

$$\Omega_{m,0} = 0.269^{+0.018}_{-0.017}, \quad (6.6)$$

$$h r_d = 103.11^{+0.98}_{-0.99} \text{ Mpc}. \quad (6.7)$$

Compared to the BAO-only case, the inclusion of DESI data results in a much narrower posterior (Figure 6.9) and a reduced $\Omega_{m,0}$ – $h r_d$ correlation. This improvement is driven by the higher precision and broader redshift coverage of the DESI measurements, which provide multiple anchors for the evolution of $\theta_{\text{BAO}}(z)$ across redshift.

Finally, from the best-fit cosmological parameters inferred in both configurations, we

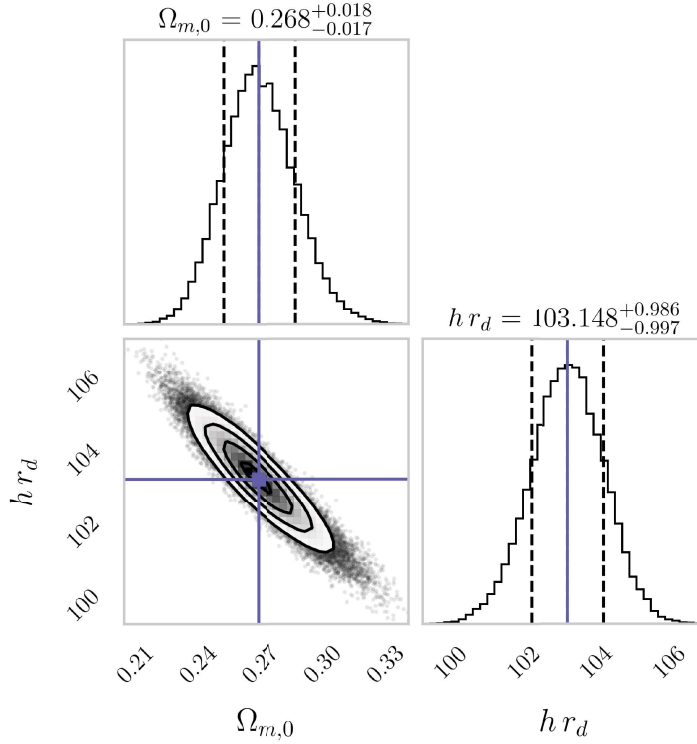


Figure 6.9: Marginalized posterior distributions and two-dimensional contours (68% and 95% credible regions) for $\Omega_{m,0}$ and $h r_d$, obtained including DESI measurements. The purple cross indicates the median values, and the dashed lines on the diagonal panels mark the 16th and 84th percentiles.

compute the corresponding theoretical angular BAO scale using Eqs. 5.15, 6.1, and 5.17. Figure 6.10 shows the resulting $\theta_{\text{BAO}}(z)$ curves compared with the full compilation of angular BAO measurements, including our data point. Both reconstructions exhibit the expected monotonic decrease of the angular acoustic scale with redshift, with the DESI-included model providing a slightly better visual agreement across the full redshift range, particularly at intermediate and high redshifts.

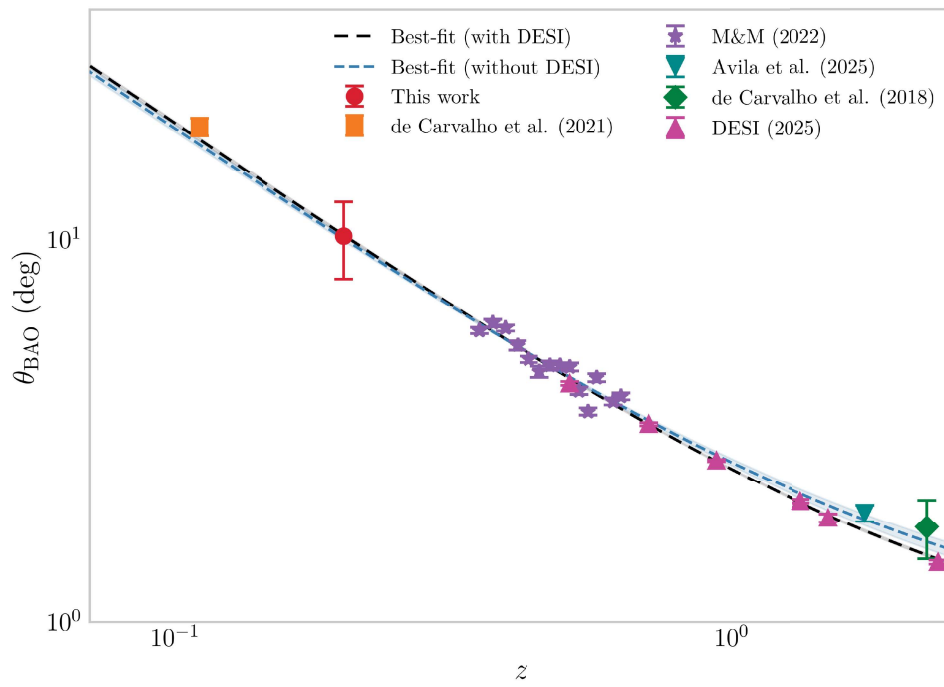


Figure 6.10: Compilation of angular BAO measurements $\theta_{\text{BAO}}(z)$ from the literature together with the result obtained in this work. The curves show the reconstructed theoretical predictions from the best-fit cosmological models with and without DESI measurements.

Chapter 7

Conclusion

The BAO feature, imprinted on the matter distribution by acoustic waves propagating through the primordial plasma, constitutes one of the most robust standard rulers available to observational cosmology. In this work, we have investigated its low-redshift signature through the angular clustering of a population of star-forming blue galaxies drawn from the twelfth data release of the Sloan Digital Sky Survey. This tracer population has been comparatively less explored for BAO studies relative to luminous red galaxies and quasars, and its viability as a cosmological probe at low redshift was one of the central questions motivating this work.

The analysis was carried out using the pseudo- C_ℓ formalism to estimate the angular power spectrum, with a covariance matrix constructed from 1000 lognormal mock catalogs generated with GLASS and iteratively recalibrated to match the observed clustering amplitude. The BAO shift parameter α was extracted through both profile likelihood and MCMC analyses, and its robustness was assessed against variations in bandpower width, fitting range, template parametrization, and broadband model. A key feature of the methodology is its weakly model-dependent character: the constraint on α is driven exclusively by the position of the BAO feature in multipole space, with the broadband shape of the angular power spectrum absorbed by nuisance parameters, minimizing sensitivity to the detailed modelling of galaxy clustering.

For the primary redshift bin $0.15 \leq z \leq 0.25$ ($z_{\text{eff}} = 0.2$), the analysis yields $\alpha = 1.034^{+0.155}_{-0.162}$ (profile likelihood) and $\alpha = 0.974^{+0.247}_{-0.194}$ (MCMC). Both estimates are consistent with the fiducial Λ CDM prediction $\alpha = 1$, though with large uncertainties that reflect the modest statistical power of the sample. The comparison between the wiggle and no-wiggle templates gives $\Delta\chi^2 = 1.38$, corresponding to a 1.17σ marginal preference for the oscillatory model, insufficient to claim a detection, but consistent with the expected signal strength for this survey configuration, as confirmed by the mock ensemble analysis presented in Appendix B. For the lower-redshift bin $0.05 \leq z \leq 0.15$ ($z_{\text{eff}} = 0.1$), no preference for the BAO template was found ($\Delta\chi^2 < 0.1$), as expected from the reduced comoving volume and increased cosmic variance at this redshift.

The measurement translates into a transverse distance ratio $D_M/r_d = 5.606 \pm 1.276$ at $z_{\text{eff}} = 0.20$, consistent with both the Planck 2018 Λ CDM and the DESI w_0w_a CDM predictions, with tension parameters $T_{\text{Planck}} = 0.120$ and $T_{\text{DESI}} = 0.063$. When combined with a compilation of angular BAO measurements from the literature and with DESI DR2 data, the joint analysis yields $\Omega_{m,0} = 0.269^{+0.018}_{-0.017}$ and $h r_d = 103.11^{+0.98}_{-0.99}$ Mpc, consistent with current cosmological constraints.

The central conclusion of this work is not a high-significance BAO detection, but rather the demonstration that the full pipeline, from mock generation and covariance estimation through pseudo- C_ℓ measurement, template fitting, and cosmological inference, performs as expected in a realistic low-redshift setting. The robustness tests show that the recovered acoustic scale is stable across a wide range of modelling choices, the covariance estimation is well-controlled, and the fitting machinery correctly identifies the absence of a detectable signal in the lower-redshift bin while returning a consistent, unbiased estimate in the primary bin. The mock ensemble distributions of Appendix B further confirm that a $\sim 1\sigma$ result is within the statistical regime expected for a survey of this volume and sky coverage, and that the estimator is unbiased on average across both linear and non-linear mock ensembles.

The main limitation of the analysis is the modest comoving volume accessible at $z \lesssim 0.25$, which drives large cosmic variance at the multipoles relevant to the BAO feature and limits the achievable signal-to-noise per realization. These are intrinsic constraints of the dataset rather than methodological shortcomings. Extending the analysis to larger sky coverage, higher galaxy number density, or broader redshift ranges would directly increase the statistical power of the measurement.

In summary, this work establishes the pseudo- C_ℓ pipeline as a viable and well-validated tool for angular BAO analysis with blue galaxy samples at low redshift. The methodology is robust, weakly model-dependent, and directly transferable to upcoming photometric surveys, where the increased statistical power will substantially improve sensitivity to the acoustic feature. Whether or not a significant detection is achievable with the present dataset, the infrastructure developed here is ready for application to larger and deeper samples, where the BAO signal will progressively emerge with greater clarity.

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Appendix A

Mathematical Derivations

This appendix gathers the mathematical details underlying some of the theoretical concepts introduced in Chapter 2. The sections that follow derive the Friedmann equations and the fluid equations in comoving coordinates, which provide the foundation for the description of the homogeneous expansion of the Universe and the growth of cosmic structure.

A.1 The Friedmann Equations

This appendix collects the explicit computation underlying the Friedmann and continuity equations obtained in Section 2.3. Starting from the FLRW metric, we compute the connection coefficients, the curvature tensors, and the components of the stress-energy tensor, and then combine them in Einstein's equations and in the conservation law.

Computed directly from Eq. (2.8), the nonzero Christoffel symbols of the FLRW metric are

$$\Gamma_{11}^0 = \frac{a\dot{a}}{c^2(1 - kr^2/R_0^2)}, \quad \Gamma_{22}^0 = \frac{a\dot{a}r^2}{c^2}, \quad \Gamma_{33}^0 = \frac{a\dot{a}r^2 \sin^2 \theta}{c^2},$$

$$\Gamma_{01}^1 = \Gamma_{10}^1 = \Gamma_{02}^2 = \Gamma_{20}^2 = \Gamma_{03}^3 = \Gamma_{30}^3 = \frac{\dot{a}}{a},$$

$$\Gamma_{11}^1 = \frac{kr/R_0^2}{1 - kr^2/R_0^2}, \quad \Gamma_{22}^1 = -r(1 - kr^2/R_0^2), \quad \Gamma_{33}^1 = -r(1 - kr^2/R_0^2) \sin^2 \theta,$$

$$\Gamma_{12}^2 = \Gamma_{21}^2 = \Gamma_{13}^3 = \Gamma_{31}^3 = \frac{1}{r},$$

$$\Gamma_{33}^2 = -\sin \theta \cos \theta, \quad \Gamma_{23}^3 = \Gamma_{32}^3 = \cot \theta.$$

From the Christoffel symbols, we construct the Ricci tensor using Eq. 2.7. For the FLRW spacetime, all off-diagonal components vanish as a consequence of homogeneity and isotropy, leaving only

$$\begin{aligned} R_{00} &= -3\frac{\ddot{a}}{a}, \\ R_{11} &= \frac{1}{c^2(1 - kr^2/R_0^2)} \left(a\ddot{a} + 2\dot{a}^2 + \frac{2kc^2}{R_0^2} \right), \\ R_{22} &= \frac{r^2}{c^2} \left(a\ddot{a} + 2\dot{a}^2 + \frac{2kc^2}{R_0^2} \right), \\ R_{33} &= \frac{r^2}{c^2} \sin^2 \theta \left(a\ddot{a} + 2\dot{a}^2 + \frac{2kc^2}{R_0^2} \right). \end{aligned}$$

The Ricci scalar follows from contracting with the inverse metric,

$$R = \frac{6}{c^2} \left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2 R_0^2} \right).$$

Combining these results, the Einstein tensor is constructed according to Eq. 2.5. The time-time component is:

$$G_{00} = 3 \left(\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2 R_0^2} \right),$$

$$G_{00} = 3 \left(\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2 R_0^2} \right),$$

while the spatial components give

$$\begin{aligned}
G_{11} &= -\frac{1}{1 - \frac{kr^2}{R_0^2}} \left[\frac{2a\ddot{a} + \dot{a}^2}{c^2} + \frac{k}{R_0^2} \right], \\
G_{22} &= -r^2 \left(\frac{2a\ddot{a} + \dot{a}^2}{c^2} + \frac{k}{R_0^2} \right), \\
G_{33} &= -r^2 \sin^2 \theta \left(\frac{2a\ddot{a} + \dot{a}^2}{c^2} + \frac{k}{R_0^2} \right).
\end{aligned}$$

On the matter side, the perfect-fluid stress-energy tensor (2.10), evaluated in comoving coordinates, has nonzero components

$$T_{00} = \rho c^4, \quad T_{11} = \frac{p a^2}{1 - kr^2/R_0^2}, \quad T_{22} = p a^2 r^2, \quad T_{33} = p a^2 r^2 \sin^2 \theta.$$

Substituting the Einstein tensor and the stress-energy tensor into Einstein's equations (2.4), the time-time component yields the first Friedmann equation (2.36), while the spatial components reduce, under isotropy, to a single independent relation that gives the second Friedmann equation (2.37).

The continuity equation follows from the conservation law (2.38). Expanding the divergence in terms of the Christoffel symbols gives

$$\nabla_\mu T^{\mu\nu} = \partial_\mu T^{\mu\nu} + \Gamma_{\mu\lambda}^\mu T^{\lambda\nu} + \Gamma_{\mu\lambda}^\nu T^{\mu\lambda} = 0.$$

Evaluating the contravariant tensor (2.11) in comoving coordinates, where the four-velocity is $u^\mu = (1, 0, 0, 0)$ with $u_\mu u^\mu = -c^2$, its non-vanishing components are

$$T^{00} = \rho, \quad T^{ij} = p g^{ij},$$

with the pressure terms cancelling in T^{00} since $g^{00} = -1/c^2$. Evaluating the time component ($\nu = 0$),

$$\partial_\mu T^{\mu 0} + \Gamma_{\mu\lambda}^\mu T^{\lambda 0} + \Gamma_{\mu\lambda}^0 T^{\mu\lambda} = 0,$$

each term can be computed directly from the FLRW connection coefficients. The first involves only $T^{00} = \rho$, yielding $\partial_0 T^{00} = \dot{\rho}$; the second uses $\Gamma_{\mu 0}^\mu = 3\dot{a}/a$, so that $\Gamma_{\mu 0}^\mu T^{\mu 0} = 3(\dot{a}/a)\rho$; and the third involves the spatial symbols Γ_{ij}^0 , namely

$$\Gamma_{11}^0 T^{11} = \frac{a\dot{a}}{c^2 (1 - kr^2/R_0^2)} \frac{p (1 - kr^2/R_0^2)}{a^2} = \frac{\dot{a}}{a} \frac{p}{c^2},$$

with the 22 and 33 components giving identical contributions, so that $\Gamma_{ij}^0 T^{ij} = 3(\dot{a}/a)(p/c^2)$. Combining these results reproduces the continuity equation (2.39).

A.2 Fluid Equations in Comoving Coordinates

Here we derive the comoving forms of the fluid equations quoted in Section 2.5.1, Eqs. 2.97, 2.98 and 2.99. For a function $f = f(t, \vec{r})$, the chain rule with $\vec{\chi} = \vec{r}/a$ gives

$$\frac{\partial f}{\partial r_i} = \sum_j \frac{\partial \chi_j}{\partial r_i} \frac{\partial f}{\partial \chi_j} = \sum_j \frac{1}{a} \delta_{ij} \frac{\partial f}{\partial \chi_j} = \frac{1}{a} \frac{\partial f}{\partial \chi_i},$$

which is the spatial rule (Eq. 2.95), $\vec{\nabla}_{\vec{r}} = a^{-1} \vec{\nabla}$. The time derivative requires more care, since differentiating at fixed $\vec{r}$ is not the same as at fixed $\vec{\chi}$. Writing $f(t, \vec{r}) = f(t, a(t)\vec{\chi})$ and applying the chain rule at fixed $\vec{r}$ [2, 102],

$$\left(\frac{\partial f}{\partial t} \right)_{\vec{r}} = \left(\frac{\partial f}{\partial t} \right)_{\vec{\chi}} + \sum_i \frac{\partial f}{\partial \chi_i} \left(\frac{\partial \chi_i}{\partial t} \right)_{\vec{r}}.$$

Differentiating $\vec{\chi} = \vec{r}/a$ at fixed $\vec{r}$,

$$\left(\frac{\partial \vec{\chi}}{\partial t} \right)_{\vec{r}} = \vec{r} \frac{d}{dt} \left(\frac{1}{a} \right) = -\frac{\dot{a}}{a^2} \vec{r} = -\frac{\dot{a}}{a} \vec{\chi} = -H \vec{\chi},$$

and using $\sum_i \chi_i \partial / \partial \chi_i = \vec{\chi} \cdot \vec{\nabla}$ gives the time-derivative rule (Eq. 2.96),

$$\left(\frac{\partial f}{\partial t} \right)_{\vec{r}} = \left(\frac{\partial f}{\partial t} \right)_{\vec{\chi}} - H \vec{\chi} \cdot \vec{\nabla} f.$$

Applying the velocity decomposition (Eq. 2.67), the spatial rule (Eq. 2.95), and the time-derivative rule (Eq. 2.96) to the continuity equation in Eq. 2.92, the flux divergence splits into a Hubble-flow and a peculiar part. Using the relation between comoving and physical coordinates and $\vec{\nabla} \cdot \vec{\chi} = 3$,

$$\vec{\nabla}_{\vec{r}} \cdot (\rho \vec{v}) = H \vec{\nabla} \cdot (\rho \vec{\chi}) + \frac{1}{a} \vec{\nabla} \cdot (\rho \vec{v}_{\text{pec}}),$$

where $H \vec{\nabla} \cdot (\rho \vec{\chi}) = H \vec{\chi} \cdot \vec{\nabla} \rho + 3H\rho$, and the time derivative gives the contribution $(\partial_t \rho)_{\vec{r}} = \partial_t \rho - H \vec{\chi} \cdot \vec{\nabla} \rho$ (with ∂_t now at fixed $\vec{\chi}$). The two terms $\pm H \vec{\chi} \cdot \vec{\nabla} \rho$ cancel, leaving the comoving continuity equation presented in Eq. 2.97.

For the physical Euler equation (Eq. 2.93), we transform each term directly. The right-hand side follows from the spatial rule in Eq. 2.95,

$$-\frac{1}{\rho} \vec{\nabla}_{\vec{r}} p - \vec{\nabla}_{\vec{r}} \Phi = -\frac{1}{a} \frac{\vec{\nabla} p}{\rho} - \frac{1}{a} \vec{\nabla} \Phi.$$

The time derivative transforms as

$$\left(\frac{\partial \vec{v}}{\partial t} \right)_{\vec{r}} = \frac{\partial \vec{v}}{\partial t} - H \vec{\chi} \cdot \vec{\nabla} \vec{v},$$

while the term $(\vec{v} \cdot \vec{\nabla}_{\vec{r}}) \vec{v}$, rewritten using the spatial rule and the velocity decomposition

from Eq. 2.67, becomes

$$(\vec{v} \cdot \vec{\nabla}_{\vec{r}}) \vec{v} = \frac{1}{a} (\vec{v} \cdot \vec{\nabla}) \vec{v} = \frac{\dot{a}}{a} (\vec{\chi} \cdot \vec{\nabla}) \vec{v} + \frac{1}{a} (\vec{v}_{\text{pec}} \cdot \vec{\nabla}) \vec{v} = H \vec{\chi} \cdot \vec{\nabla} \vec{v} + \frac{\vec{v}_{\text{pec}}}{a} \cdot \vec{\nabla} \vec{v}.$$

Adding all contributions, the terms $\pm H \vec{\chi} \cdot \vec{\nabla} \vec{v}$ cancel, giving the comoving Euler equation (Eq. 2.98).

Finally, the Poisson equation follows immediately by applying $\nabla_{\vec{r}}^2 = a^{-2} \nabla^2$ to Eq. 2.94,

$$\frac{1}{a^2} \nabla^2 \Phi = 4\pi G \rho \quad \Rightarrow \quad \nabla^2 \Phi = 4\pi G a^2 \rho,$$

which was presented in Eq. 2.99.

Appendix B

Robustness Tests

This appendix presents a series of robustness tests performed to assess the stability of the BAO measurement under variations of the analysis choices. The tests explore the dependence of the recovered acoustic-scale parameter on the multipole binning scheme, fitting range, broadband parametrization, template construction, covariance calibration procedure, and mock catalog generation. In each case, the analysis was repeated while modifying a single component of the pipeline and keeping all other ingredients fixed. These tests complement the results presented in Chapter 6.

B.1 Binning Dependence

To assess the impact of the bandpower width on the BAO measurement, the analysis was repeated for a range of bin sizes between $\Delta\ell = 4$ and $\Delta\ell = 15$. For each choice of $\Delta\ell$, the angular power spectrum and the corresponding mock realizations were rebinned consistently, and the acoustic scale was determined from the profile likelihood analysis described in Section 5.5. The covariance matrix was estimated directly from the rebinned mock catalogs and iteratively recalibrated following the procedure of Section 5.4.1. For computational efficiency, this test employed a subset of 200 mock realizations rather than the full ensemble of 1000 mocks used in the fiducial analysis. The recovered values of α are shown in Figure B.1.

B.2 Fitting Range Dependence

To assess the dependence of the recovered acoustic scale on the adopted fitting interval, the lower and upper multipole limits were varied independently. For the $\ell_{\min}$ test, the minimum multipole was changed while keeping $\ell_{\max} = 200$ fixed. Conversely, for the $\ell_{\max}$ test, the maximum multipole was varied while keeping $\ell_{\min} = 10$ fixed. In each case, the corresponding matrix of the converged fiducial covariance matrix was extracted, the

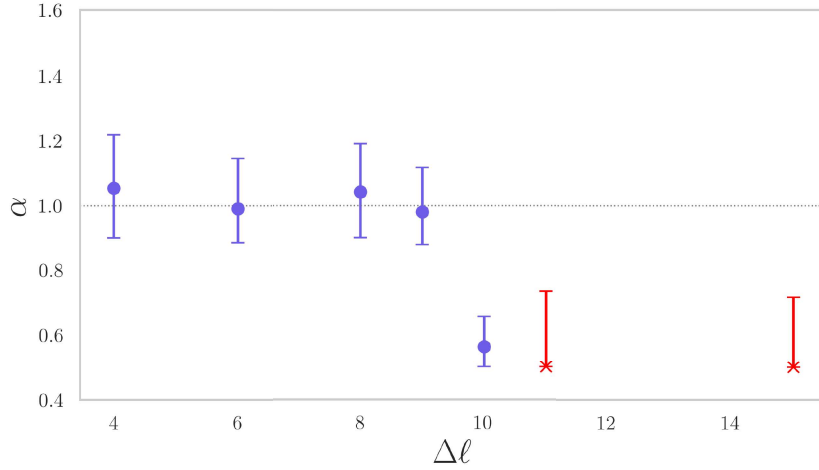


Figure B.1: For each binning configuration, the angular power spectrum was re-estimated and fitted using the same procedure as in the fiducial analysis. The dotted line indicates the expected value $\alpha = 1$. Red crosses denote configurations for which the profile-likelihood minimization did not converge reliably.

Hartlap correction was recomputed using the number of bandpowers included in the fit, and the profile-likelihood analysis was repeated. The resulting values of α are shown in Figure B.2.

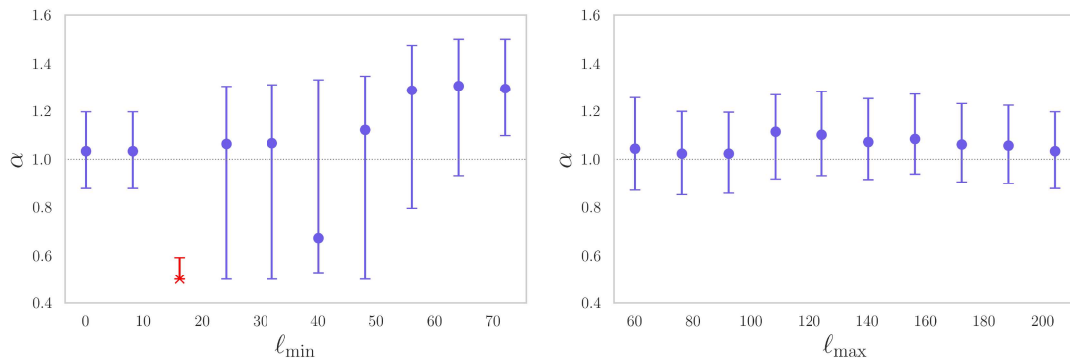


Figure B.2: Recovered values of the shift parameter α obtained when varying the minimum (left) and maximum (right) multipoles included in the fit. For each configuration, the profile-likelihood analysis was repeated using the corresponding converged covariance matrix. The dotted line indicates the expected value $\alpha = 1$. The red marker indicates the fiducial fitting range adopted in the main analysis. The red star denotes the non-convergent fit obtained for that choice of fitting range.

B.3 Template Construction Dependence

To evaluate the sensitivity of the BAO measurement to the theoretical template construction, the analysis was repeated using four template configurations. Two choices were considered for the input matter power spectrum: the linear spectrum and damped expression from Eq. 5.10. Each case was projected into angular space using both the Limber

approximation and the full spherical Bessel integration. The resulting constraints on the BAO shift parameter are shown in Figure B.3.

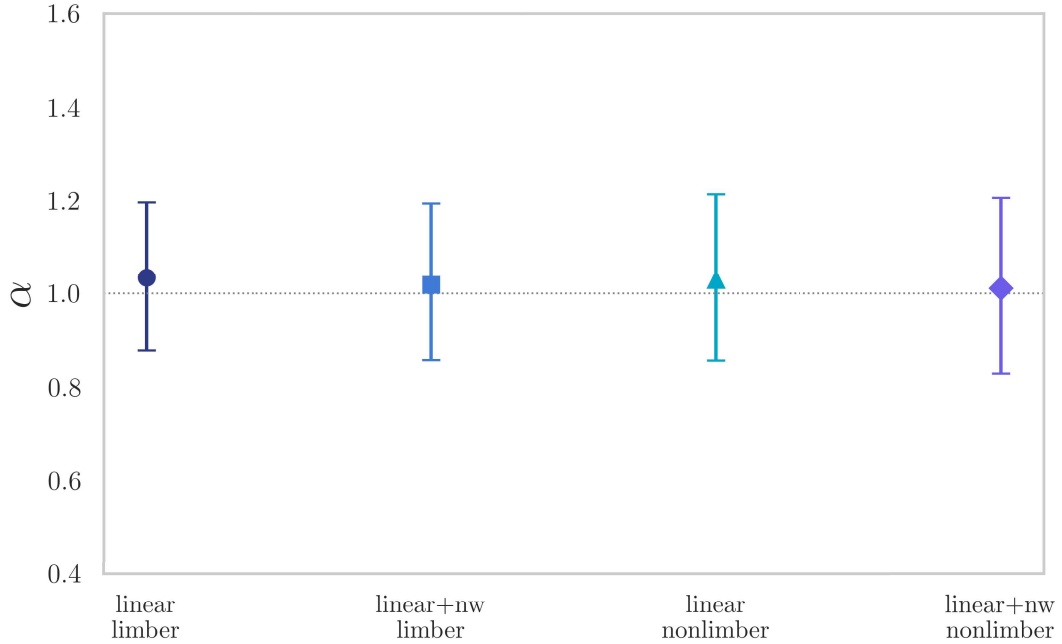


Figure B.3: Recovered values of the BAO shift parameter α obtained using different combinations of projection method template power spectrum. The labels indicate whether the angular power spectrum template was computed using the Limber approximation or the full spherical Bessel integration (non-Limber), and whether the input power spectrum was purely linear or included BAO damping. Error bars correspond to the 68% confidence intervals from the profile-likelihood analysis. The horizontal dotted line marks the expected value $\alpha = 1$. All fits were performed using the same fitting range, broadband parametrization, and iteratively calibrated covariance matrix adopted in the fiducial analysis.

B.4 Broadband Parametrization Dependence

To assess the dependence of the BAO measurement on the broadband parametrization, the profile-likelihood analysis was repeated using four alternative forms for the smooth component of the model,

$$B(\ell) = B_0^2 + \frac{a}{\ell}, \quad (\text{B.1})$$

$$B(\ell) = B_0^2, \quad (\text{B.2})$$

$$B(\ell) = B_0^2 + a\ell, \quad (\text{B.3})$$

$$B(\ell) = \left(B_0 + \frac{a}{\ell}\right)^2. \quad (\text{B.4})$$

For each parametrization, the complete fitting procedure was repeated using the same fitting range, BAO template, and iteratively calibrated covariance matrix adopted in the fiducial analysis. The recovered values of the BAO shift parameter α are shown in Figure B.4.

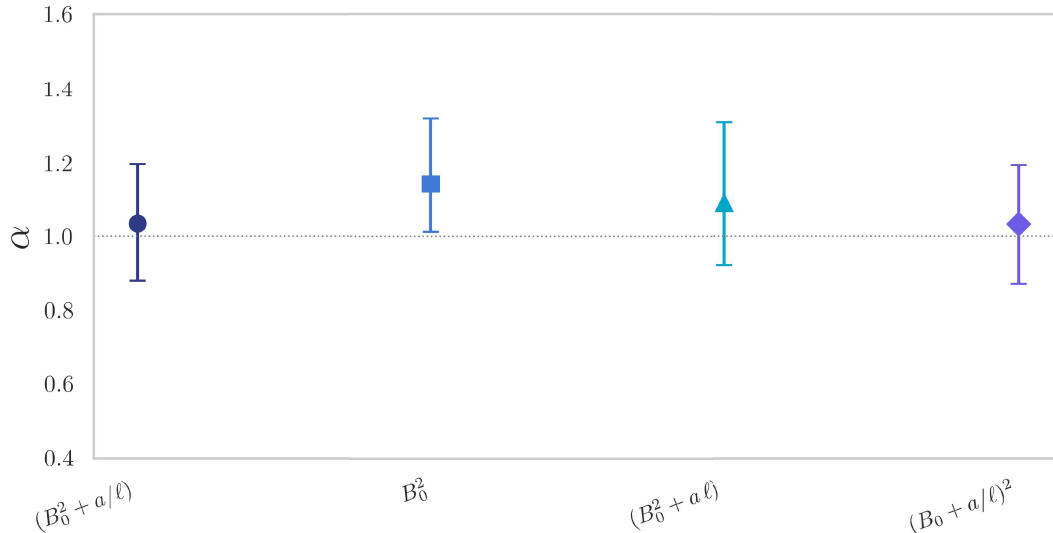


Figure B.4: Recovered values of the BAO shift parameter α for different broadband parametrizations. Error bars correspond to the 68% confidence intervals obtained from the profile-likelihood analysis. The horizontal dotted line marks the expected value $\alpha = 1$. All fits were performed using the same fitting range, BAO template, and iteratively calibrated covariance matrix, varying only the functional form adopted for the broadband component.

B.5 Covariance Calibration Convergence

The covariance matrix was iteratively recalibrated following the procedure described in Chapter 5. At each iteration, the covariance was rescaled according to the best-fit model obtained in the previous step and the profile-likelihood analysis was repeated. Figure B.5 and Table B.1 summarize the evolution of the recovered BAO parameters. Convergence is achieved after the first recalibration, with negligible changes between the second and third iterations.

Table B.1: Results obtained at each step of the iterative covariance calibration procedure.

Iteration	α	σ_α	$\Delta\chi^2$	Significance
0	1.036	0.087	4.34	2.08σ
1	1.036	0.159	1.39	1.18σ
2	1.034	0.159	1.38	1.17σ

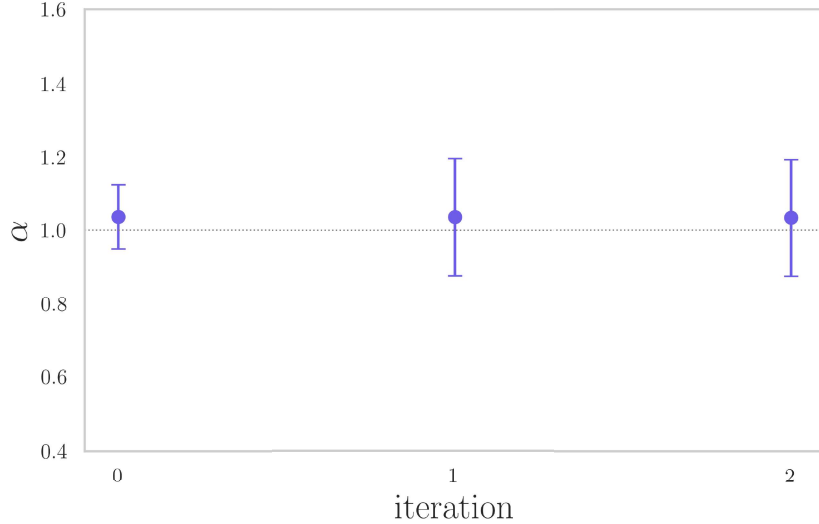


Figure B.5: Recovered values of the BAO shift parameter α as a function of the covariance calibration iteration. At each step, the covariance matrix is rescaled according to the best-fit model obtained in the previous iteration and the profile-likelihood analysis is repeated. Error bars correspond to the 68% confidence intervals. The horizontal dotted line indicates $\alpha = 1$.

B.6 Linear and Non-linear Mock Catalog Validation

To assess the sensitivity of the covariance estimation to the assumed clustering model, the complete analysis was repeated using two independent mock ensembles generated with GLASS. In the first case, the mocks were constructed from the linear matter power spectrum, while in the second the input spectrum included non-linear corrections. For both ensembles, the covariance matrix was iteratively recalibrated following the procedure adopted in the fiducial analysis. The resulting BAO constraints are summarized in Table B.2.

Table B.2: Comparison of the recovered BAO parameters using covariance matrices estimated from mock catalogs generated with either a linear or a non-linear matter power spectrum.

Mock type	α_{profile}	α_{MCMC}	$\Delta\chi^2$	Significance
Linear	$1.049^{+0.161}_{-0.180}$	$0.980^{+0.197}_{-0.226}$	1.22	1.10σ
Non-linear	$1.034^{+0.162}_{-0.155}$	$0.974^{+0.194}_{-0.248}$	1.38	1.17σ

The recovered values of α , as well as the BAO significance, remain statistically consistent between the two mock constructions, indicating that the measurement is not strongly sensitive to the choice of linear or non-linear input power spectrum used in the covariance estimation.

B.7 Distribution of Recovered BAO Shift Parameter in Mock Catalog

To validate the BAO fitting pipeline and assess potential systematic biases introduced by the use of linear perturbation theory templates, we apply the analysis described in Section 5.5 to two ensembles of 1 000 mock angular power spectra each. The first ensemble is generated from a linear matter power spectrum, and the second from a non-linear power spectrum, with all other survey properties held fixed (redshift range $z \in [0.15, 0.25]$, sky mask, and mean galaxy surface density). In both cases the fitting procedure is identical to that applied to the data: a linear theory template is used throughout.

For each mock realization i , the angular power spectrum $C_\ell^{(i)}$ is fit using the 4-parameter model from Eq. 5.9. The linear nuisance parameters (B_0^2 , a , A_0) are marginalized analytically at each value of α via a generalized least-squares profile likelihood, yielding the profiled $\chi^2(\alpha)$ for each mock. The covariance matrix used in all fits is the analytical Gaussian expression given by Eq. 5.6. This covariance is constructed once from the ensemble mean and reused for all 1000 fits within each case. The best-fit shift parameter for each mock is taken as the minimizer of $\chi^2(\alpha)$ on a uniform grid of 400 points in $\alpha \in [0.5, 1.5]$.

Mocks that do not yield a well-constrained measurement are excluded from the distributions shown below. Specifically, we discard realizations in which α rails against the boundary of the grid, where the χ^2 profile does not cross $\Delta\chi^2 = 1$ on both sides of the minimum, or in which $\alpha < 0.65$. This last threshold corresponds to a shift in the acoustic scale exceeding 35% from the fiducial value, well outside any cosmologically motivated range. Such realizations are noise-dominated and do not yield a meaningful BAO measurement. The fraction of retained mocks is $N_{\text{lin}} = 436$ for the linear case and $N_{\text{nonlin}} = 346$ for the non-linear case.

The resulting distributions of $\hat{\alpha}$ are shown in Figure B.6. Both ensembles are broadly centered on $\alpha = 1$, confirming that the linear template does not introduce a significant systematic bias even when applied to non-linear mocks. The dispersion of both distributions, $\sigma_\alpha^{\text{lin}} = 0.14$ and $\sigma_\alpha^{\text{nonlin}} = 0.15$, reflects the statistical uncertainty expected for a single realization at $z \in [0.15, 0.25]$ with the survey geometry of the blue galaxy sample. The large scatter and the fraction of unconstrained mocks are a consequence of the limited survey volume at this redshift, which reduces the effective BAO signal-to-noise per realization.

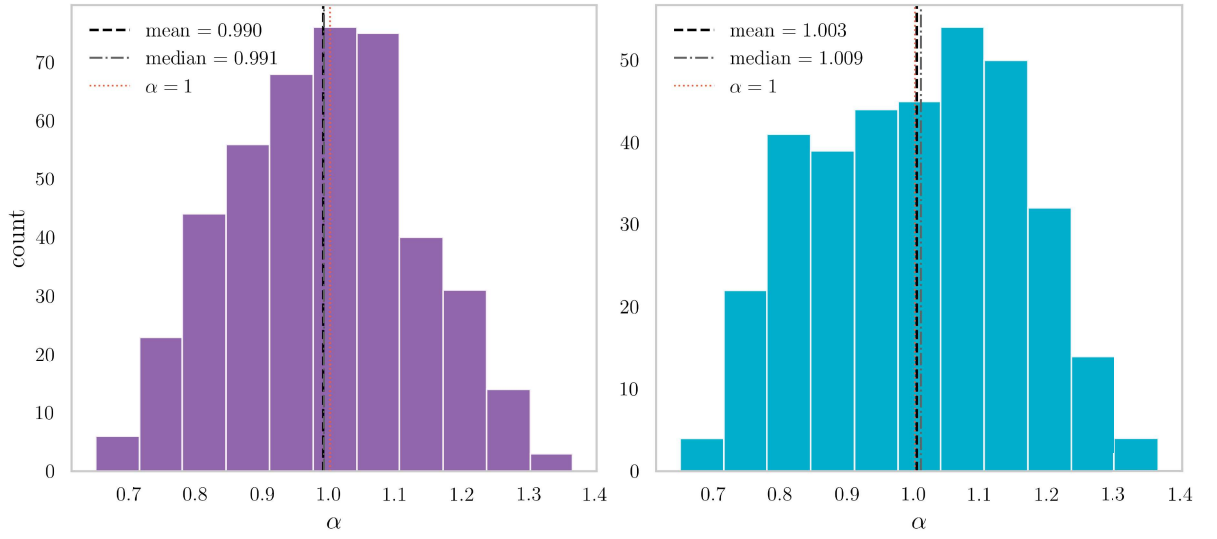


Figure B.6: Distribution of $\hat{\alpha}$ over mocks generated from a linear (left, $N = 436$) and non-linear (right, $N = 346$) matter power spectrum, fitted with a linear template. Only realizations with a well-constrained profile likelihood and $\alpha \geq 0.65$ are shown. Both distributions are consistent with $\alpha = 1$, with mean values $\langle \hat{\alpha} \rangle = 0.990$ and 1.003 respectively, confirming an unbiased recovery of the acoustic scale.